How does a Dipole universe will look like?

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What is Dipole universe?

- Dipole universe is the simplest **homogeneous anisotropic** model after FLRW, and it allows to coherent flows to exist.
- The metric is :

$$ds^2 = -dt^2 + X^2(t)dx^2 + e^{-2A_0x}Y^2(t)(dy^2 + dz^2) \tag{1}$$

- The perfect fluids can moves along $x-$axis:

$$T^\nu_\mu = (\rho + p)u^\mu u_\nu + \delta^\mu_\nu p; \quad u^\mu = (\cosh(\beta(t)), \sinh(\beta(t))/X(t), 0, 0) \tag{2}$$

Cosmological Model

Every Cosmological model can be specified with these:

- Underling (smoothed) geometry.
- Matter content and interactions.
- Relation between matter and geometry (Gravity).

Such models will helps us to derive observable quantities and predict their values.

Λ-CDM model

In this sense, the Standard model of cosmology specified by:

- Underling (smoothed) geometry $\rightarrow$ flat FLRW
- Matter content and interactions $\rightarrow$ Cold Dark Matter + Λ+SM of Particles
- Relation between matter and geometry (Gravity) $\rightarrow$ General relativity (Minimally coupled)

If everything is fine, then Λ-CDM can solve all the ”puzzles”.

Λ-CDM challenges

Maybe everything is NOT fine!

- H_0 tension
- S_8 tension
- Missing baryon problem
- CDM discrepancies
  - Core-cusp problem
  - Missing satellite problem
  - ...
- ...

Possibilities

Solution→ Modify the cosmological model:

1. Matter content and interactions → other physics for CDM and Dark Energy
2. Relation between matter and geometry (Gravity) → modified gravity
3. Underling (smoothed) geometry → non-FLRW metric

Most solutions is about 1 and 2, but why?

Why FLRW?

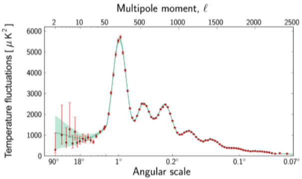
- FLRW agrees with Copernican principle.
- Greatly simplifies Einsteins equations
- Early universe (inflation) can isotropize the geometry (Cosmic no-hair theorem)

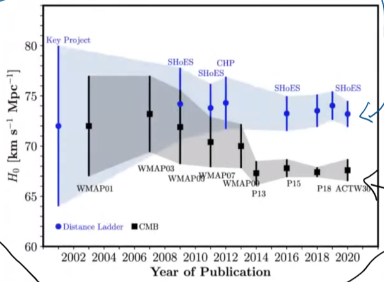
- However it is hard to directly observe the FLRW; calculating an observable always involves the whole cosmological model not just the geometry.
- In the case of a cosmological model, geometry is more like a core assumption, than something could be tested.

If current puzzles persists in every models which is built upon FLRW assumption, it could signal a crisis and cause a paradigm shift, which in this case is going beyond FLRW.

A persistent puzzle: Hubble Tension

$$d_L(z) = \frac{z}{H_0}\left(1 + \frac{(1-q_0)}{2} z\right)$$





Maybe it's time to check the FLRW itself!

Is the observable Universe consistent with the cosmological principle?

Of course after subtracting CMB dipole, our universe shows a remarkable isotropy, But

- Great structures (Giant Arc, Quasar groups, great wall,..)
- CMB anomalies
- Bulk flow and dark flow
- Radio galaxy and QSO dipole
- H_0 and q_0 dipole
- Dipole in the distribution of galaxy spin directions

Data is not clear but it seems there is a Dipole Mystery

P. K. Aluri, P. Cea, P. Chingangbam, M. C. Chu, R. G. Clowes, D. Hutsemékers, J. P. Kochappan, A. M. Lopez, L. Liu and N. C. M. Martens, *et al.* Class. Quant. Grav. **40** (2023) no.9, 094001 doi:10.1088/1361-6382/acbefc [arXiv:2207.05765 [astro-ph.CO]].

So we wish to depart from FLRW to next most simple metric which allows for a Dipole

Bianchi Classification

Bianchi Classification of Lie Algerba in 3D:

I: $[\mathbf{e}_i, \mathbf{e}_j] = 0$	II: $[\mathbf{e}_2, \mathbf{e}_3] = \mathbf{e}_1$	III: $[\mathbf{e}_1, \mathbf{e}_2] = e_1$
IV: $[\mathbf{e}_1, \mathbf{e}_3] = \mathbf{e}_1$ $[\mathbf{e}_2, \mathbf{e}_3] = \mathbf{e}_1 + \mathbf{e}_2$	V: $[\mathbf{e}_1, \mathbf{e}_3] = \mathbf{e}_1$ $[\mathbf{e}_2, \mathbf{e}_3] = \mathbf{e}_2$	VI: $[\mathbf{e}_1, \mathbf{e}_3] = \mathbf{e}_1$ $[\mathbf{e}_2, \mathbf{e}_3] = \alpha\mathbf{e}_1$ $-1 \leq \alpha < 1$
VII: $[\mathbf{e}_1, \mathbf{e}_3] = \beta\mathbf{e}_1 - \mathbf{e}_2$ $[\mathbf{e}_2, \mathbf{e}_3] = \mathbf{e}_1 + \beta\mathbf{e}_2$	VIII: $[\mathbf{e}_1, \mathbf{e}_2] = \mathbf{e}_1$ $[\mathbf{e}_2, \mathbf{e}_3] = \mathbf{e}_3$ $[\mathbf{e}_1, \mathbf{e}_3] = 2\mathbf{e}_1$	IX: $SO(3)$

For any orthogonal tetrad base there is an associated Lie algebra:

$$[\mathbf{e}_a, \mathbf{e}_b] = \gamma^c_{ab}(t)\mathbf{e}_c$$

More Symmetry

Some Bianchi universes have more symmetric subspace:

Table 18.2 The Bianchi models permitting higher symmetry subcases. The parameter c is zero if and only if the preferred spatial vector is hypersurface-orthogonal.

	Isotropic Bianchi models	
FLRW $k = +1$:	Bianchi IX [two commuting groups]	
FLRW $k = 0$:	Bianchi I, Bianchi VII_0	
FLRW $k = -1$:	Bianchi V, Bianchi VII_h	
	LRS Bianchi models	
Orthogonal	$c = 0$	$c \neq 0$
Taub-NUT 1	[KS $K = 1$: no subgroup]	Bianchi IX
Taub-NUT 3	Bianchi I, VII_0 [KS $K = 0$]	Bianchi II
Taub-NUT 2	Bianchi III [KS $K = -1$] (Farnsworth)	Bianchi VII, III
Tilted		
	Bianchi V, VII_h (Collins–Ellis)	

Figure: Ellis, G., Maartens, R., & MacCallum, M. (2012). Relativistic Cosmology. Cambridge: Cambridge University Press. doi:10.1017/CBO9781139014403

Metric

- Assume following metric of the Dipole universe

$$ds^2 = -dt^2 + X^2(t)dx^2 + e^{-2A_0 x} Y^2(t)(dy^2 + dz^2) \tag{3}$$

- Symmetries: 3 translation + rotation around x axis, Killing vectors:

$$\xi_1 = \mathbf{e}_y \tag{4}$$

$$\xi_2 = \mathbf{e}_z \tag{5}$$

$$\xi_3 = \frac{1}{A_0}\mathbf{e}_x + (y\mathbf{e}_y + z\mathbf{e}_z) \tag{6}$$

$$\xi_4 = y\mathbf{e}_z - z\mathbf{e}_y \tag{7}$$

$$[\xi_1, \xi_3] = \xi_1; \quad [\xi_2, \xi_3] = \xi_2 \implies \mathrm{BianchiV} \tag{8}$$

More about metric

In $X = Y = a$ (no shear) it is FLRW universe with $k = -A_0^2$. Take this metric

$$ds^2 = -dt^2 + a^2(t)(dx^2 + e^{-2A_0 x}(dr^2 + r^2 d\phi^2)) \tag{9}$$

and use this coordinate transformation:

$$e^{-A_0 x} = \cosh(A_0 R) + \cos\theta \sinh(A_0 R) \tag{10}$$

$$-A_0 r = \frac{1}{\sin(\theta)}\left(\frac{\tanh(A_0 R) + \cos\theta}{1 + \cos\theta \tanh(A_0 R)} - \cos(\theta)\right) \tag{11}$$

then the metric will take this form:

$$ds^2 = -dt^2 + a(t)^2(dR^2 + \frac{1}{A_0^2}\sinh^2(A_0 R)(d\theta^2 + \sin^2\theta d\phi^2)) \tag{12}$$

Dynamical equations

Here are the dynamical equations of this universe:

$$\begin{aligned}
\frac{\ddot{X}}{X} + 2\frac{\dot{X}\dot{Y}}{XY} - 2\frac{A_0^2}{X^2} &= \frac{1}{2}(\rho - p) + (\rho + p)\sinh^2\beta + \Lambda && (13) \\
\frac{\ddot{Y}}{Y} + \frac{\dot{Y}^2}{Y^2} + \frac{\dot{X}\dot{Y}}{XY} - 2\frac{A_0^2}{X^2} &= \frac{1}{2}(\rho - p) && (14) \\
\frac{2A_0}{X}\left(\frac{\dot{X}}{X} - \frac{\dot{Y}}{Y}\right) &= (\rho + p)\sinh\beta\cosh\beta && (15) \\
2\frac{\dot{X}\dot{Y}}{XY} + \frac{\dot{Y}^2}{Y^2} - \frac{3A_0^2}{X^2} &= \rho + (\rho + p)\sinh^2\beta + \Lambda && (16)
\end{aligned}$$

$$T_\mu^\nu = (\rho + p)u^\mu u_\nu + \delta_\nu^\mu p; \quad u^\mu = (\cosh(\beta(t)), \sinh(\beta(t))/X(t), 0, 0) \qquad (17)$$

Dynamical equations

By change of the variables$X(t) = a(t)e^{-2b(t)}; \quad Y(t) = a(t)e^{b(t)}$ we will have:

$$\begin{aligned}
\dot{H} + 3H^2 - \tfrac{2A_0^2}{a^2}e^{-4b} &= \frac{1}{2}(\rho - p) + \frac{1}{3}(\rho + p)\sinh^2\beta + \Lambda && (18)\\
\dot{\sigma} + 3H\sigma &= (\rho + p)\sinh^2\beta && (19)\\
H^2 - \tfrac{1}{9}\sigma^2 - \tfrac{A_0^2}{a^2}e^{-4b} &= \frac{1}{3}(\rho + (\rho + p)\sinh^2\beta) + \frac{\Lambda}{3} && (20)\\
\tfrac{2A_0}{a}e^{-2b}\sigma &= (\rho + p)\sinh\beta\cosh\beta && (21)
\end{aligned}$$

$$\sigma(t) = 3\dot{b}, \quad H(t) = \frac{\dot{a}}{a} \qquad (22)$$

Dynamic of fluids

Here are the $\nabla^\mu T_{\mu\nu} = 0$ equations:

$$\dot\rho + 3H(\rho + p) = -(\rho + p)\tanh\beta(\dot\beta - \frac{2A_0}{ae^{2b}}) \quad (23)$$

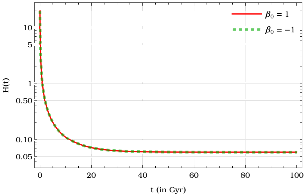
$$\dot p + H(\rho + p) = -(\rho + p)(\frac{2}{3}\sigma + \dot\beta\coth\beta) \quad (24)$$

assuming $p = w\rho$ will result in:

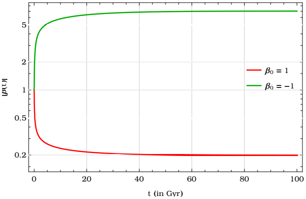
$$\Big(\coth\beta - w\tanh\beta\Big)\dot\beta = (3w-1)H - \frac{2}{3}\sigma - \frac{2A_0 w}{X}\tanh\beta \quad (25)$$

sign of the LHS is the same with $\frac{d}{dt}\beta^2$ so sign of the RHS can determine whether $|\beta|$ tends to grow or not.

β can grow even for $w = 1/3$



(a)



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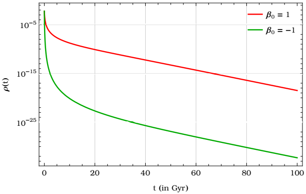

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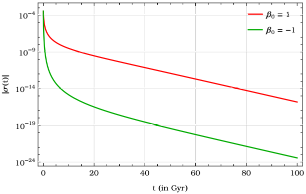
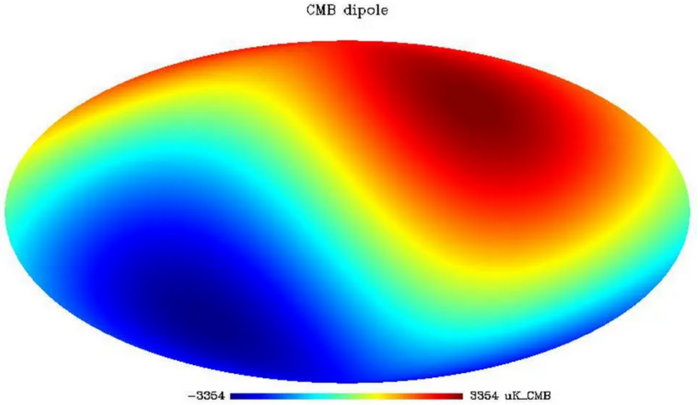


(d)

Observational Consequences

We are interested in how this universe will looks like



(Credit: J. Delabrouille et al., A&A, 2013)

Source and Observer

Relative motion between source and observer could result in dipole and multipoles in sky, this could be obtain in 2 way:

1. Both source and observer are moving with u^μ:

$$u^\mu = (\cosh\beta(t), \frac{\sinh\beta(t)}{X(t)}, 0, 0) \tag{26}$$

   They will have different u^μ if u^μ changes with time.

2. Observer moves with different speed than sources:

$$u_o^\mu = (\cosh\beta_o(t), \frac{\sinh\beta_o(t)}{X(t)}, 0, 0) \tag{27}$$

$$u_s^\mu = (\cosh\beta_s(t), \frac{\sinh\beta_s(t)}{X(t)}, 0, 0) \tag{28}$$

Definition of Redshift

If k^μ is the wavenumber vector of light ray, then $u^\mu k_\mu$ is the frequency measured by an observer moving with u^μ so:

$$1 + z = \frac{(u^\mu k_\mu)_s}{(u^\nu k_\nu)_o} \tag{29}$$

Evolution of the electromagnetic field $F_{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$ in free space follows from:

$$\nabla_\nu \nabla^\nu A_\mu + R_{\mu\nu} A^\nu = 0; \quad \nabla_\mu A^\mu = 0 \tag{30}$$

If we assume $A_\mu = g(\psi)\alpha_\mu$, which $g' \nabla\psi\alpha \gg g\nabla\alpha$ then:

$$k_\mu := \nabla_\mu \psi \implies k^\mu k_\mu = k^\mu \nabla_\mu \psi = 0 \& 2k^\nu \nabla_\nu \alpha_\mu = -\alpha_\mu \nabla^\nu k_\nu \tag{31}$$

$$\alpha_\mu k^\mu = 0; \quad k^\mu = c\frac{dx^\mu}{d\lambda} \quad \lambda \text{ is affine parameter of null geodesic} \tag{32}$$

Geodesic-equation

- Define γ like:

$$\dot{x} = -\frac{\tanh\gamma}{X}, \quad \dot{r} = -\frac{e^{A_0 x}}{Y\cosh\gamma} \tag{33}$$

Putting this into geodesic equations implies:

$$\implies \dot{\gamma} - \frac{A_0}{X} + \sigma\tanh\gamma = 0; \quad \sigma = \frac{\dot{X}}{X} - \frac{\dot{Y}}{Y} \tag{34}$$

- θ is the local physical angle between $x-$axis and the light ray:

$$\tanh\gamma = \cos\theta \tag{35}$$

Taking the aberration to account:

$$\tan\theta_{LoS} = \frac{1}{\sinh(\gamma_o + \beta o)} \quad , \cos\theta_{LoS} = \tanh(\gamma_o + \beta o) \quad , \sin\theta_{LoS} = \frac{1}{\cosh(\gamma_o + \beta o)} \tag{36}$$

Finding redshifts

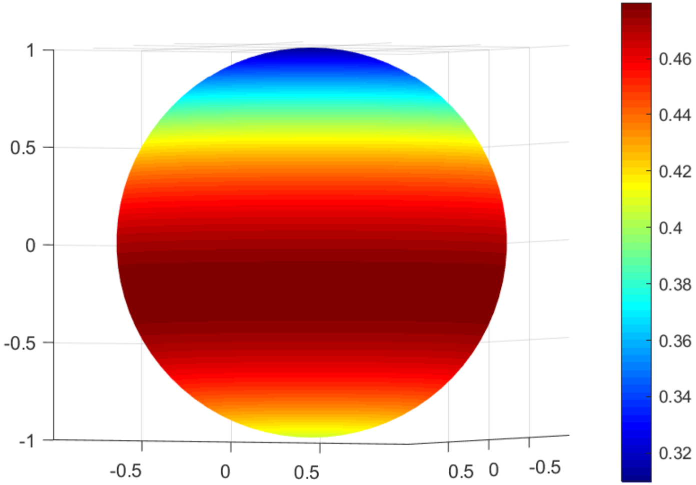
- Wavenumber four-vector can be found from geodesic equations:

$$k^\mu \propto \frac{dx^\mu}{d\lambda} \implies k^\mu = \frac{C_0 e^{A_0 x}}{Y}(-\cosh\gamma, \frac{\sinh\gamma}{X}, \frac{e^{A_0 x}}{Y}, 0) \tag{37}$$

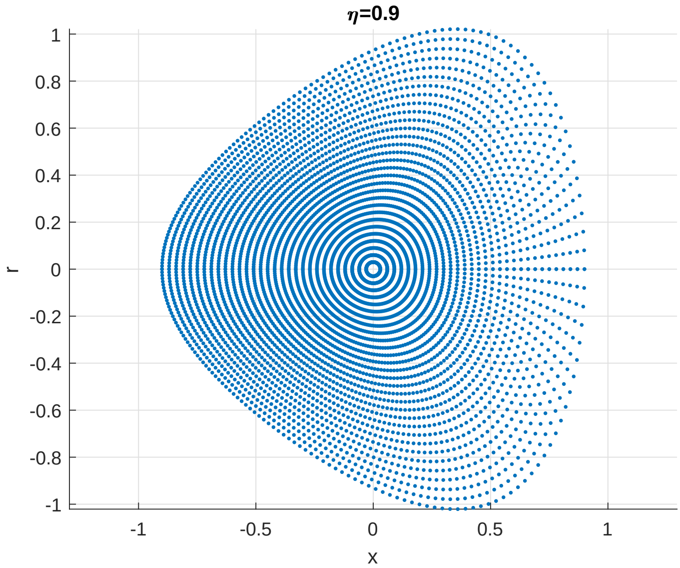
- So we have:

$$\boxed{1 + z = \frac{\cosh(\gamma_s + \beta_s)}{\cosh(\gamma_o + \beta_o)} \frac{Y_o}{Y_s} e^{A_0(x_s - x_o)}} \tag{38}$$

Redshift Form (Not Observable)



Wavefront



Expansion and acceleration parameters

One can define the expansion and acceleration along the line of sight:

$$\mathbb{H} := -\frac{1}{E}\frac{d\ln E}{d\lambda} = -\frac{E'}{E^2}, \qquad \mathbb{Q} = -1 + \frac{(\ln \mathbb{H})'}{(\ln E)'} \tag{39}$$

At the Observer's position:

$$\mathbb{H}_0 := -\frac{E'_o}{E_o^2}, \qquad \mathbb{Q}_0 = -1 + \frac{(\ln \mathbb{H})'_o}{(\ln E)'_o} \tag{40}$$

Since $E = k.u$ we have:

$$\mathbb{H}_0 := \frac{k^\mu k^\nu \nabla_\mu u_\nu|_o}{(k \cdot u)_o^2} \tag{41a}$$

$$\mathbb{Q}_0 = -3 + \frac{k^\mu k^\nu k^\rho \nabla_\mu \nabla_\nu u_\rho\Big|_o}{(k \cdot u)^3 \mathbb{H}_0^2} \tag{41b}$$

Distance vs Redshift

- Angular diameter distance along a geodesic path d_A:

$$\frac{\mathrm{d}}{\mathrm{d}\lambda} d_A = \frac{1}{2} \nabla^{\mu} k_{\mu} d_A \tag{42}$$

and redshift evolution:

$$\frac{\mathrm{d}z}{\mathrm{d}\lambda} = \frac{1}{(k^{\mu} u_{\mu})_o} k^{\mu} k^{\nu} \nabla_{\mu} u_{\mu} \tag{43}$$

- Together will result in:

$$\frac{\mathrm{d}d_A}{\mathrm{d}z} = \frac{(k^{\mu} u_{\mu})_o}{k^{\mu} k^{\nu} \nabla_{\mu} u_{\nu}} \times \frac{1}{2} \nabla^{\mu} k_{\mu} d_A \tag{44}$$

- It helps us to calculate the Taylor series $d_A(z)$

Taylor series of $d_L(z)$

one can define $\mathbb{H}_0, \mathbb{Q}_0$ as:

$$d_L(z) = \frac{z}{\mathbb{H}_0}\Big(1 + \frac{1}{2}(1 - \mathbb{Q}_0)z\Big) + O(z^3) \tag{45}$$

Which:

$$\mathbb{H}_0 := \frac{k^\mu k^\nu \nabla_\mu u_\nu|_o}{(k_\mu u^\mu)_o^2} \tag{46}$$

$$\mathbb{Q}_0 = -3 + \frac{k^\mu k^\nu k^\rho \nabla_\mu \nabla_\nu u_\rho\Big|_o}{(k^\rho u_\rho)^3 \mathbb{H}_0^2} \tag{47}$$

Note that:

$$\begin{aligned}\mathbb{H}_0 =& (H_0 - \dot{\beta}_0 \cos\theta)(\cosh\beta_0 - \cos\theta \sinh\beta_0) \\ &+ \frac{\sigma_0}{3}\cosh\beta_0 (3\cos^2\theta - 1 - 2\cos\theta\tanh\beta_0) \\ &- \frac{A}{X_o}\sinh\beta_0 \sin^2\theta\end{aligned} \tag{48}$$

Linear order

to simplify, assume $A_0 = \sigma_0 = \dot{\sigma}_0 = 0$:

$$\mathbb{Q}_0 \mathbb{H}_0^2 = q(t_0) H_0 + \dot{\beta_0}^2 (1 - 3\cos^2\theta) + (3H_0\dot{\beta}_0 + \ddot{\beta}_0)\cos\theta \tag{49}$$

$$\mathbb{H}_0 = (H_0 - \dot{\beta}_0 \cos\theta)(\cosh\beta_0 - \cos\theta \sinh\beta_0) \tag{50}$$

This clearly shows the posiblity of getting $\mathbb{Q}_0$ in some line of sights

Number Counts

Define $\mathcal{N}(z, \theta_0)$ for a desired light source as follow:

$$\mathcal{N}(z, \theta_o) = \frac{dN(z, \theta_0)}{d\Omega dz} \tag{51}$$

$dN(z, \theta_o)$ is the number of that source at the observation angle θ_o between z and $z + dz$ and inside the solid angle $d\Omega$.
We can assume a coherent source particle flow with u_p^μ four-velocity and rest number density, n :

$$N^\mu = n u_p^\mu; \quad n = -u_p^\mu N_\mu \tag{52}$$

and in case of the number conservation, we will have: $\nabla_\mu N^\mu = 0$ Thus we will have:

$$\mathcal{N}(z, \theta_o) = n(t(z, \theta_o)) D_A^2 k.u_p \frac{d\lambda}{dz} = n(t(z, \theta_o))(1 + z) D_A^2 \frac{(k.u)_o^2}{k^\mu k_\nu \nabla_\mu u_p^\nu} \tag{53}$$

An example

Let's assume a universe with: $A_0 = 1, \rho_m(a = 1) = 0.1, \Lambda = 0, \beta_m(1) = -0.1$ then there is some plots:

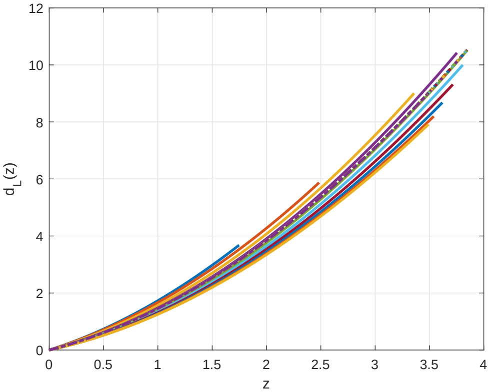


Figure: $d_L(z)$ for various line of sights

An example

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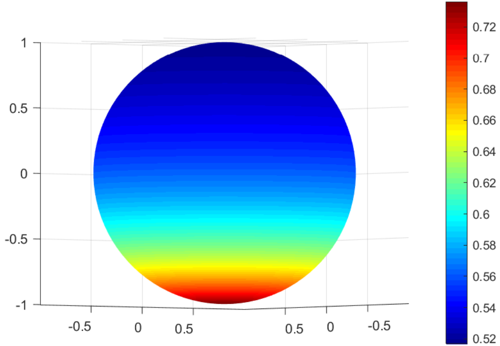

Figure: $d_L(z = 0.5)$

An example

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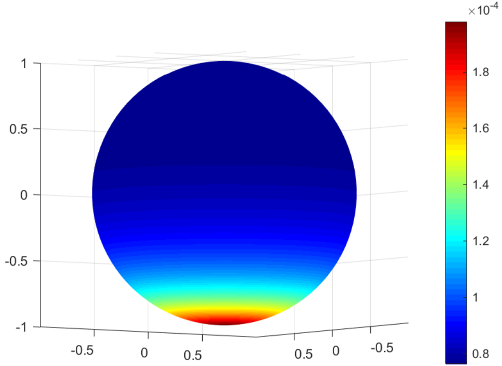

Figure: $\mathcal{N}(z = 0.5)$

An example

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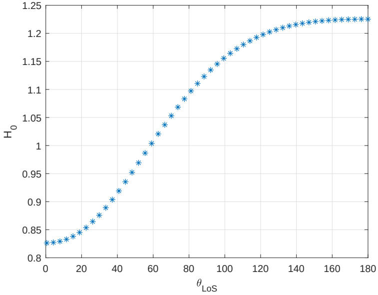
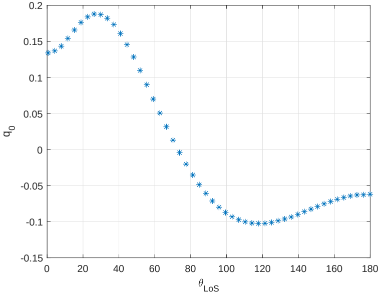


(a)



(b)

Conclusion and Outlook

- It seems that the Dipole Cosmology "can" provide a solution to H_0 tension, and even to the cosmological constant problem.
- All data analysis must be revisited in this setup
- We need more numerical analysis regarding Dipole cosmology.
- Confront this model with the data.

Thank You!

$$
\begin{aligned}
\mathfrak{H}_{\mathfrak{o}}{}^2\mathfrak{Q}_0 = \quad & q(t)H(t)^2 && \Big(\cos^2\theta\sinh^2\beta - 2\sinh\beta\cosh\beta\cos\theta + \cosh^2\beta\Big) && (54)\\
& +\tfrac{1}{3}\dot{\sigma} && \Big(\cosh^2\beta + \cos^2\theta(2 - 5\cosh^2\beta) + \sinh\beta\cosh\beta\cos\theta(1 + 3\cos^2\theta)\Big) && (55)\\
& +\ddot{\beta}\cos\theta && \Big(\cosh^2\beta + \cos^2\theta\sinh^2\beta - 2\sinh\beta\cosh\beta\cos\theta\Big) && (56)\\
& +\tfrac{1}{3}\sigma(t)H(t) && \Big(5\cosh^2\beta - 3 + 7\cos^2\theta - 13\cosh^2\beta\cos^2\theta + \sinh\beta\cosh\beta\cos\theta(9\cos^2\theta - 1)\Big) && (57)\\
& +3\dot{\beta}H(t) && \Big(\sinh^2\beta\cos^2\theta + \cosh^2\beta - 2\sinh\beta\cosh\beta\cos\theta\Big)\cos\theta && (58)\\
& +\frac{2A}{a(t)}H(t) && \Big(\sinh\beta\cosh\beta - \cos\theta\sinh^2\beta\Big)\sin^2\theta && (59)\\
& +\dot{\beta}^2 && \Big(\cosh^2\beta - 3\cos^4\theta\sinh^2\beta - \cos^2\theta(1 + 2\cosh^2\beta) && (60)\\
& && +2\sinh\beta\cosh\beta\cos\theta(-1 + 3\cos^2\theta)\Big) && (61)\\
& +\sigma^2(t) && \Big(-\frac{2}{3} + \cosh^2\beta(\frac{14}{9}\cos^2\theta - 3\cos^4\theta + \frac{5}{9}) + \frac{10}{9}\cos^2\theta && (62)\\
& && +\sinh\beta\cosh\beta\cos\theta(4\cos^2\theta - \frac{28}{9})\Big) && (63)\\
& +\frac{A^2}{a(t)^2} && \Big(\sinh^2\beta(4\cos^2 - 3\cos^4\theta) - 2\sinh\beta\cosh\beta\cos\theta\sin^2\theta\Big) && (64)\\
& +\dot{\beta}\sigma(t) && \Big(\cos\theta(3 - 5\cos^2\theta)(1 - 2\cosh^2\beta) + 2\sinh\beta\cosh\beta(1 - 3\cos^4\theta)\Big) && (65)\\
& +\dot{\beta}\frac{2A}{a(t)} && \Big(-\sin^2\theta(3\cos^2\theta + 4\sinh\beta\cosh\beta\cos\theta) && (66)\\
& && +\cosh^2\beta(1 + 2\cos^2\theta - 3\cos^4\theta)\Big) && (67)\\
& +\frac{\sigma(t)}{3}\frac{2A}{a(t)} && \Big((5 - 8\cosh^2\beta)\cos\theta\sin^2\theta && (68)\\
& && +\sinh\beta\cosh\beta(9\cos^4\theta - 10\cos^2\theta - 1)\Big) && (69)
\end{aligned}
$$