NEUTRINO OSCILLATION AND THE ERA OF PRECISION DATA``

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HIDDeν

Hunting Invisibles: Dark sectors, Dark matter and Neutrinos

Kinetic term

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi \qquad \bar{\psi} = \psi^\dagger \gamma^0$$

Peskin's notation

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma^\mu = (1_{2\times 2}, \sigma^i) \quad \bar{\sigma}^\mu = (1_{2\times 2}, -\sigma^i)$$

Chirality

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1_{2\times 2} & 0 \\ 0 & 1_{2\times 2} \end{pmatrix}$$

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad \psi_L = \frac{1-\gamma_5}{2}\psi \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

Kinetic term $\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi$ $\qquad \psi_L^\dagger \vdots \psi_L \quad \psi_R^\dagger \vdots \psi_R$

Gauge interaction

$$A_\mu \bar{\psi}(\gamma^\mu + b\gamma^\mu\gamma^5)\psi \qquad \psi_L^\dagger \vdots \psi_L \quad \psi_R^\dagger \vdots \psi_R$$

Non-chiral interaction : Electrodynamics; QCD $\quad b = 0$

$$A_\mu(\bar{\psi}_L\gamma^\mu\psi_L + \bar{\psi}_R\gamma^\mu\psi_R)$$

Chiral interaction: Electroweak

$$b \neq 0 \quad A_\mu\left((1-b)\bar{\psi}_L\gamma^\mu\psi_L + (1+b)\bar{\psi}_R\gamma^\mu\psi_R\right)$$

Conclusion

- With only left-handed fermions we can write both the kinetic terms and gauge interaction terms.

- With only right-handed fermions we can write both the kinetic terms and gauge interaction terms.

DIRAC MASS TERM

$$m\bar{\psi}\psi = m(\psi_L^\dagger\ \psi_R^\dagger)\begin{pmatrix} 0_{2\times 2} & 1_{2\times 2} \\ 1_{2\times 2} & 0_{2\times 2} \end{pmatrix}\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = m(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L)$$

We need both left-handed and right-handed fermions to write a Dirac mass Term.

Chirality flipping

Any other sort of mass term

- Dirac mass term

$$\bar{\psi}\psi = \psi^\dagger \gamma^0 \psi$$

- Invariance under Lorentz transformation

$$\cancel{\psi^\dagger \psi}$$

- Any other form of mass term?

Any other sort of mass term

- Dirac mass term

$$\bar{\psi}\psi = \psi^\dagger \gamma^0 \psi$$

- Invariance under Lorentz transformation

$$\cancel{\psi^\dagger \psi}$$

- Any other form of mass term?

Majorana mass term

Any other sort of mass term

- Dirac mass term

$$\bar{\psi}\psi = \psi^\dagger\gamma^0\psi$$

- Invariance under Lorentz transformation

$$\cancel{\psi^\dagger\psi}$$

- Any other form of mass term?

Majorana mass term

To be discussed later

Solution of Dirac equation

$$(i\partial_\mu \gamma^\mu - m)\psi = 0$$

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \sum_s (a_p^s u^s(p) e^{-ip\cdot x} + b_p^{s\dagger} v^s(p) e^{ip\cdot x})$$

Spinor $$u^s = \begin{pmatrix} \sqrt{p\cdot\sigma}\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\xi^s \end{pmatrix}$$ $\sigma_3\xi(\uparrow) = \xi(\uparrow) \quad \sigma_3\xi(\downarrow) = -\xi(\downarrow)$

Ultra-relativistic limit

$$p^\mu = (p, 0, 0, p)$$

$$\sqrt{p\cdot\sigma} = \sqrt{2p}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \sqrt{p\cdot\bar{\sigma}} = \sqrt{2p}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$u^s = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^s \\ \sqrt{p\cdot\bar{\sigma}}\,\xi^s \end{pmatrix}$$

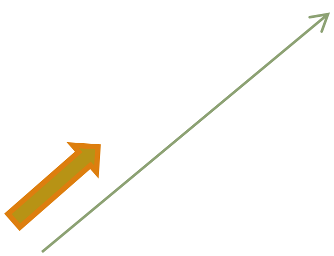
$$\psi(\uparrow) \propto \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \psi(\downarrow) \propto \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Right-handed	Left-handed
$(\vec{S}\cdot\hat{p} = +\frac{1}{2})$	$(\vec{S}\cdot\hat{p} = -\frac{1}{2})$

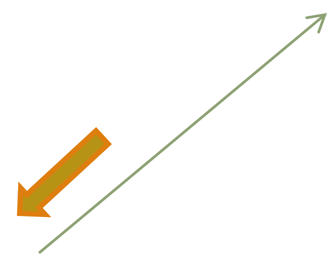
Helicity versus chirality

Helicity= $\vec{S} \cdot \hat{p}$

Ultra-relativistic limit: helicity $\Leftrightarrow$ chirality



Right-handed



Left-handed

Anti-fermion

$$v^s = \begin{pmatrix} \sqrt{p\cdot\sigma}\,\xi^s \\ -\sqrt{p\cdot\bar{\sigma}}\,\xi^s \end{pmatrix}$$

ANTI FERMION $\quad \sigma_3\xi(\uparrow) = -\xi(\uparrow) \qquad \sigma_3\xi(\downarrow) = \xi(\downarrow)$

Notice reversed sign!

Reminder of fermion: $\quad \sigma_3\xi(\uparrow) = \xi(\uparrow) \qquad \sigma_3\xi(\downarrow) = -\xi(\downarrow)$

Anti-fermion

$$v^s = \begin{pmatrix} \sqrt{p \cdot \sigma}\xi^s \\ -\sqrt{p \cdot \bar{\sigma}}\xi^s \end{pmatrix}$$

$$\sigma_3 \xi(\uparrow) = -\xi(\uparrow) \qquad \sigma_3 \xi(\downarrow) = \xi(\downarrow)$$

Anti-particle of fermion with positive helicity

$\Updownarrow$

Anti-fermion with negative helicity

Anti-particle of fermion with negative helicity

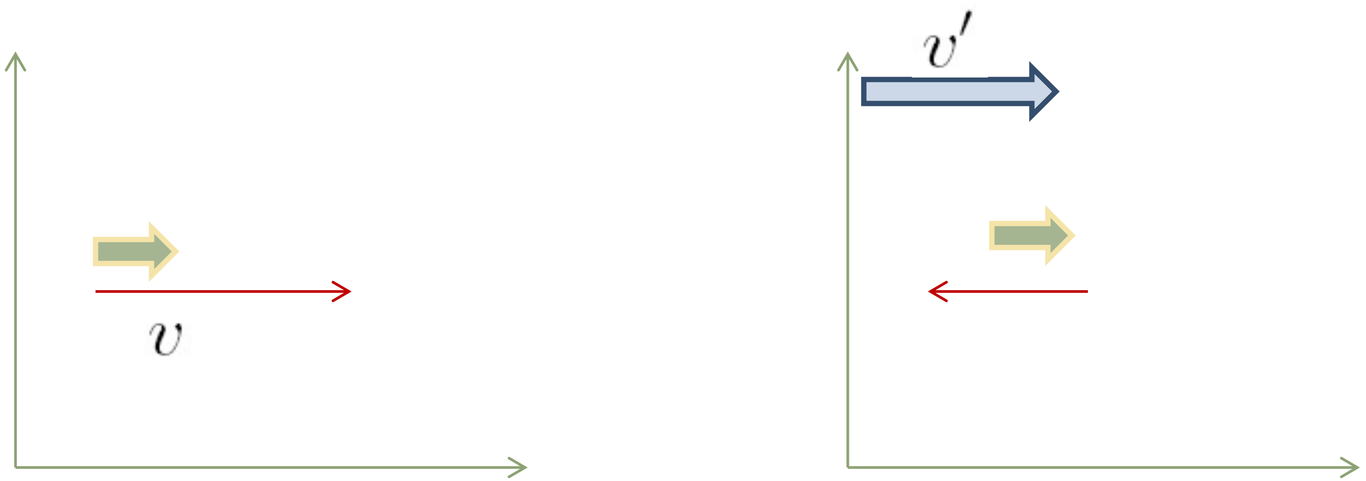
$\Updownarrow$

Anti-fermion with positive helicity

Reminder

- Helicity=spin in the direction of momentum
- For massless particles, helicity is invariant.
- But, for massive particles, is not

$$v' > v$$



Observation

- In the weak interactions,
- Neutrinos have negative helicity
- Anti-neutrinos have positive helicity

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^*$$

$$^{152}Sm^* \to {}^{152}Sm + \gamma$$

M. Goldhaber et al., Phys. Rev. 109 (1957) 1015

Angular momentum conservation

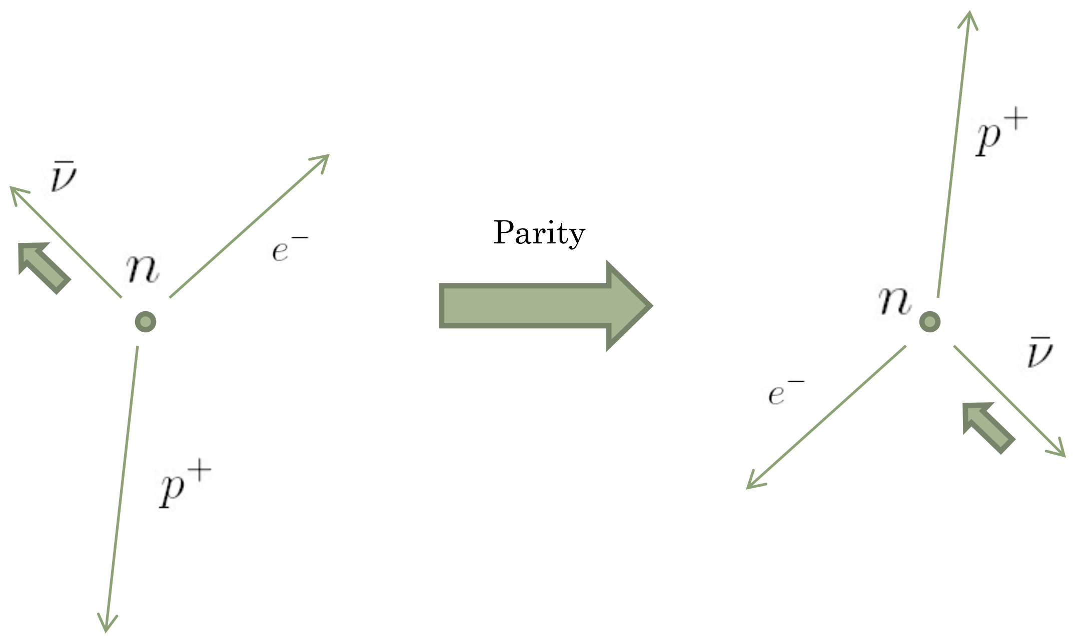
Four-fermion effective fermi interaction

$$H_W = \frac{G_\beta}{\sqrt{2}}(\bar{p}\gamma_\mu(1 - g_A\gamma_5)n)(\bar{e}\gamma_\mu\underbrace{(1 - \gamma_5)\nu}_{\text{Left-handed}})$$

$$E \ll m_W \sim 80\ \mathrm{GeV}$$

Parity violation

$$\vec{x} \to -\vec{x} \quad \vec{p} \to -\vec{p} \quad J = S + L \to J$$



Parity violation

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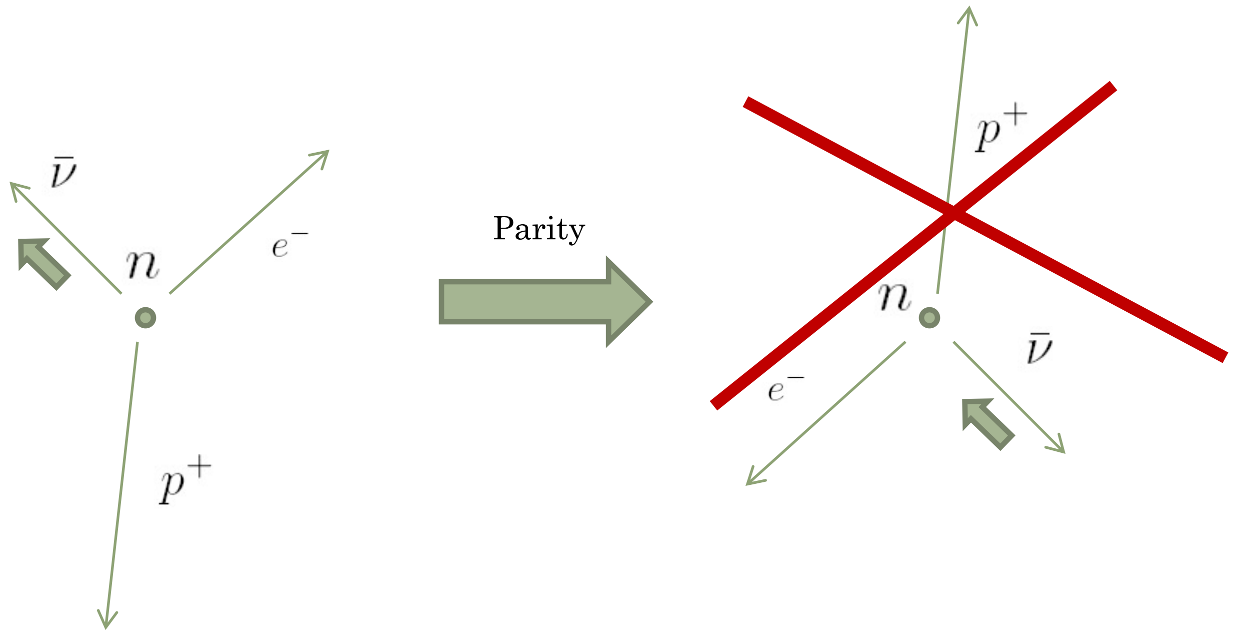
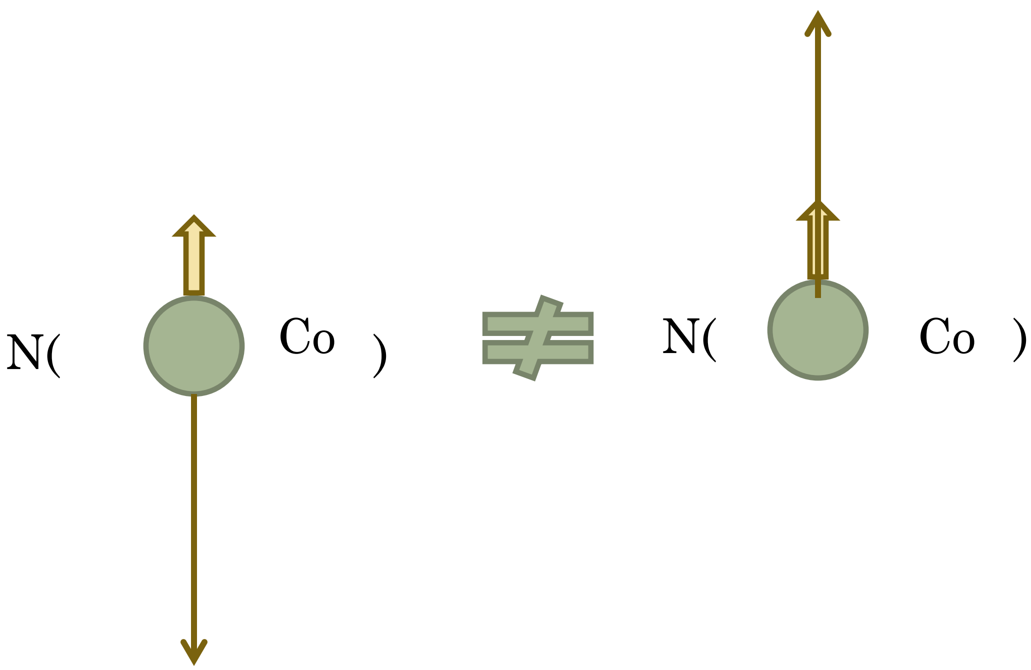


Parity is violated

DISCOVERY OF PARITY VIOLATION

$$\mathrm{Co} \longrightarrow \mathrm{Ni} + e^{-} + \bar{\nu}_e$$



Wu

This discovery resulted in her colleagues Tsung-Dao Lee and Chen-Ning Yang winning the 1957 Nobel Prize in Physics, while Wu herself was awarded the inaugural Wolf Prize in Physics in 1978.

- Effective four-Fermi interaction

$$\Downarrow$$

$$SU(2) \times U(1)$$

- Effective four-Fermi interaction

$$SU(2) \times U(1)$$

Only left-handed neutrinos
And right-handed antineutrinos

Lagrangian of the leptons

EM:

$$-eA_\mu \sum_\alpha (\bar{\ell}_{\alpha L}\gamma^\mu \ell_{\alpha L} + \bar{\ell}_{\alpha R}\gamma^\mu \ell_{\alpha R})$$

NC:

$$\frac{eZ_\mu}{\sin\theta_w \cos\theta_w}\left[\sum_\alpha \left(\frac{\bar{\nu}_{\alpha L}\gamma^\mu \nu_{\alpha L}}{2} + \frac{\bar{\ell}_{\alpha L}(2\sin^2\theta_w - 1)\gamma^\mu \ell_{\alpha L}}{2} + \sin^2\theta_w \bar{\ell}_{\alpha R}\gamma^\mu \ell_{\alpha R}\right)\right]$$

CC:

$$\frac{e}{\sqrt{2}\sin\theta_w \cos\theta_w}\left[\sum_\alpha (\bar{\nu}_{\alpha L}\gamma^\mu \ell_{\alpha L} W_\mu^+ + \bar{\ell}_{\alpha L}\gamma^\mu \nu_{\alpha L} W_\mu^-)\right]$$

Mass:

$$m_{\alpha\beta}\bar{\ell}_{\alpha R}\ell_{\beta L} + \text{H.c.}$$

In the general basis

In SM:

$$\cancel{\bar{\nu}_R \nu_L}$$

$$m_\nu = 0$$

Change of basis

$$\ell_{\alpha L} \to \sum_\beta U_{\alpha\beta} \ell_{\beta L} \qquad \ell_{\alpha R} \to \sum_\beta V_{\alpha\beta} \ell_{\beta R}$$

$$\nu_{\alpha L} \to \sum_\beta U_{\alpha\beta} \nu_{\beta L}$$

- Gauge interactions are invariant but

$$m_\ell \to V^T m_\ell U$$

$$m_e \bar{e} e + m_\mu \bar{\mu} \mu + m_\tau \bar{\tau} \tau$$

- In this basis,

$$\boxed{\bar{\nu}_{eL}} \gamma^\nu e_L + \boxed{\bar{\nu}_{\mu L}} \gamma^\nu \mu_L + \boxed{\bar{\nu}_{\tau L}} \gamma^\nu \tau_L$$

Lepton Flavor

$$L_e : \begin{cases} e^-_R,\ e^-_L\ \nu_{eL} \Longrightarrow L_e = 1 \\ e^+_R\ , e^+_L\ , \bar{\nu}_{eL} \Longrightarrow L_e = -1 \end{cases}$$

$$L_\mu : \begin{cases} \mu^-_R,\ \mu^-_L\ , \nu_{\mu L} \Longrightarrow L_\mu = 1 \\ \mu^+_R\ , \mu^+_L\ , \bar{\nu}_{\mu L} \Longrightarrow L_\mu = -1 \end{cases}$$

$$L_\tau : \begin{cases} \tau^-_R,\ \tau^-_L,\ \nu_{\tau L} \Longrightarrow L_\tau = 1 \\ \tau^+_R,\ \tau^+_L,\ \bar{\nu}_{\tau L} \Longrightarrow L_\tau = -1 \end{cases}$$

In the SM, L_e , L_μ and L_τ are conserved.
Decay modes conserving lepton flavor:

$$\mu^- \to \bar{\nu}_e \nu_\mu e^- \qquad \tau^- \to \nu_\tau \bar{\nu}_\mu \mu^- \qquad \tau^- \to \pi^- \nu_\tau$$

Decay modes violating lepton flavor:

Lepton flavor conservation

$$\cancel{\mu^- \to e^- \gamma} \qquad \cancel{\mu^- \to e^- e^+ e^-}$$

$$\cancel{\mu^- N \to e^- N} \qquad \cancel{\tau^- \to e^- \gamma}$$

Propagation

- Within Old SM with $m_\nu = 0$ the flavor of neutrino do not change in propagation.

ν_e remains ν_e

ν_μ remains ν_μ

ν_τ remains ν_τ

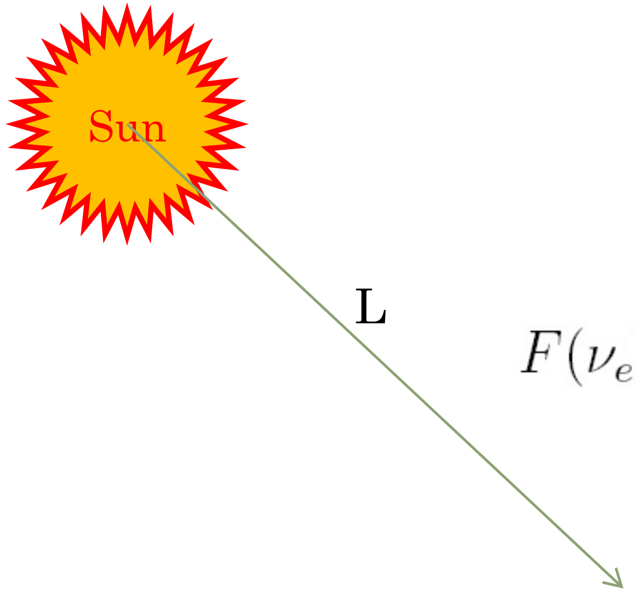
Propagation in OLD SM

- Within Old SM with $m_\nu = 0$ the flavor of neutrino do not change in propagation.

ν_e remains ν_e

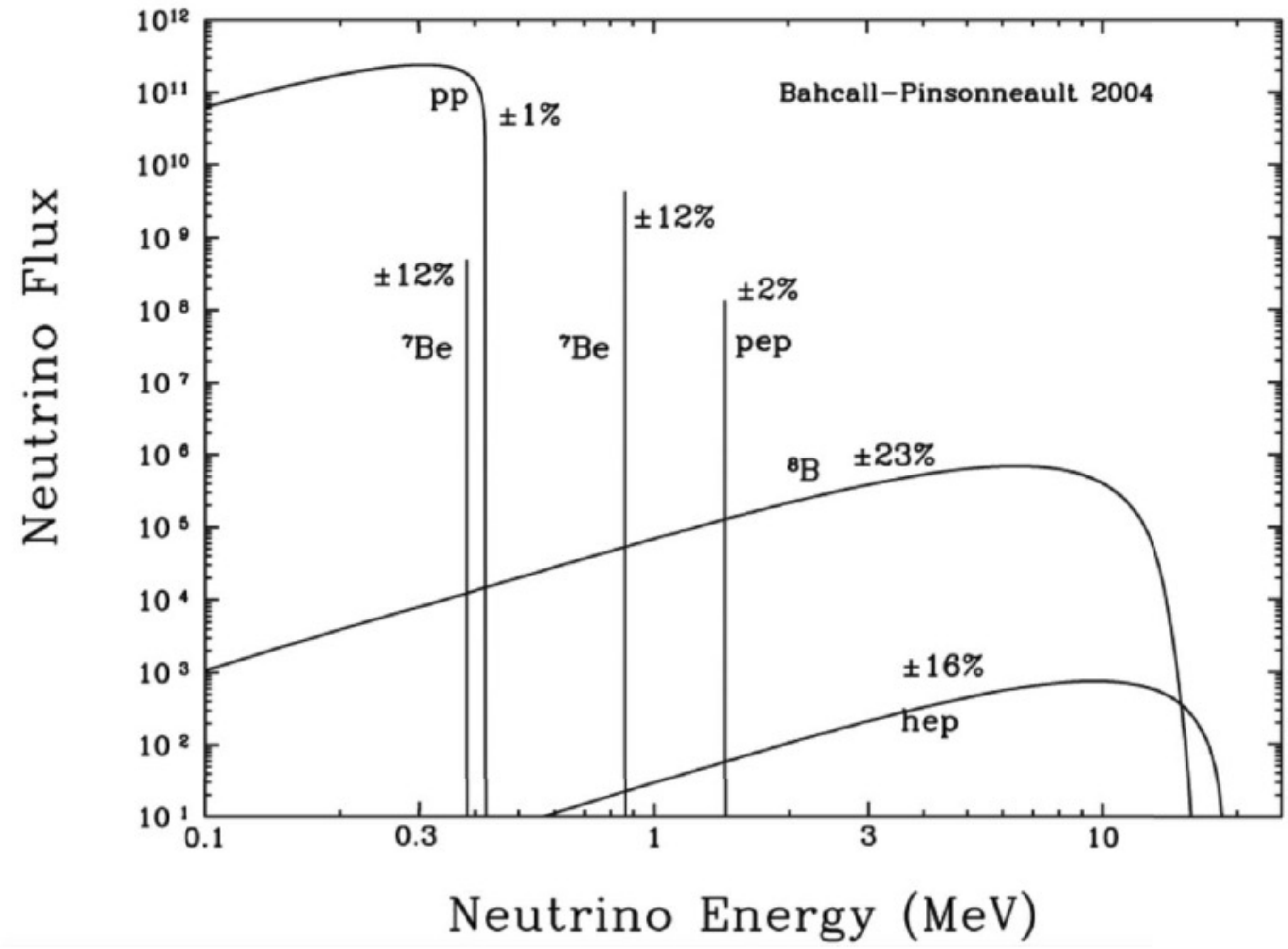
ν_μ remains ν_μ

ν_τ remains ν_τ



$$F(\nu_e) \propto \frac{1}{r^2}$$

Solar model



Bahcall–Pinsonneault 2004
pp
±1%
±12%
^{7}Be
±12%
^{7}Be
±2%
pep
^{8}B
±23%
±16%
hep
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
0.1
0.3
1
3
10
Neutrino Flux
Neutrino Energy (MeV)

	BP00	BP04	BSB05(GS98)	BSB05(AGS05)
$\Phi_{pp} / 10^{10}$	$5.95\,(1 \pm 0.01)$	$5.94\,(1 \pm 0.01)$	$5.99\,(1 \pm 0.009)$	$6.06\,(1 \pm 0.007)$
$\Phi_{pep} / 10^{8}$	$1.40\,(1 \pm 0.015)$	$1.40\,(1 \pm 0.02)$	$1.42\,(1 \pm 0.015)$	$1.45\,(1 \pm 0.011)$
$\Phi_{hep} / 10^{3}$	9.3	$7.88\,(1 \pm 0.16)$	$7.93\,(1 \pm 0.155)$	$8.25\,(1 \pm 0.155)$
$\Phi_{^7\mathrm{Be}} / 10^{9}$	$4.77\,(1 \pm 0.10)$	$4.86\,(1 \pm 0.12)$	$4.84\,(1 \pm 0.105)$	$4.34\,(1 \pm 0.093)$
$\Phi_{^8\mathrm{B}} / 10^{6}$	$5.05\,(1\,^{+0.20}_{-0.16})$	$5.79\,(1 \pm 0.23)$	$5.69\,(1\,^{+0.173}_{-0.147})$	$4.51\,(1\,^{+0.127}_{-0.113})$
$\Phi_{^{13}\mathrm{N}} / 10^{8}$	$5.48\,(1\,^{+0.21}_{-0.17})$	$5.71\,(1\,^{+0.37}_{-0.35})$	$3.05\,(1\,^{+0.366}_{-0.268})$	$2.00\,(1\,^{+0.145}_{-0.127})$
$\Phi_{^{15}\mathrm{O}} / 10^{8}$	$4.80\,(1\,^{+0.25}_{-0.19})$	$5.03\,(1\,^{+0.43}_{-0.39})$	$2.31\,(1\,^{+0.374}_{-0.272})$	$1.44\,(1\,^{+0.165}_{-0.142})$
$\Phi_{^{17}\mathrm{F}} / 10^{6}$	$5.63\,(1 \pm 0.25)$	$5.91\,(1\,^{+0.44}_{-0.44})$	$5.83\,(1\,^{+0.724}_{-0.420})$	$3.25\,(1\,^{+0.166}_{-0.142})$

C. Giunti and C. W. Kim
Fundamentals of neutrino physics and astrophysics

observation= (1/3) of expected

Davis, Harmer and Hoffman, PRL (1968)

Solution of the puzzle

- Neutrino oscillation

- Pontecorvo proposed in 1957 in analogy of

$$K^0 \leftrightarrows \bar{K}^0$$

Even before solar neutrinos were discovered!!!

Neutrino mixing

$$\boxed{\bar{\nu}_{eL}}\gamma^{\nu} e_L + \boxed{\bar{\nu}_{\mu L}}\gamma^{\nu} \mu_L + \boxed{\bar{\nu}_{\tau L}}\gamma^{\nu} \tau_L$$

Charged leptons have definite mass

NEUTRINO MIXING

$$\boxed{\bar{\nu}_{eL}}\gamma^{\nu} e_L + \boxed{\bar{\nu}_{\mu L}}\gamma^{\nu} \mu_L + \boxed{\bar{\nu}_{\tau L}}\gamma^{\nu} \tau_L$$

Flavor neutrinos do not have definite mass

Neutrino mixing

$$\boxed{\bar{\nu}_{eL}}\gamma^{\nu}e_L + \boxed{\bar{\nu}_{\mu L}}\gamma^{\nu}\mu_L + \boxed{\bar{\nu}_{\tau L}}\gamma^{\nu}\tau_L$$

Flavor neutrino do not have definite mass

They are admixtures of neutrinos with definite mass.

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i$$

Flavor

Mass

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i$$

$U_{\alpha i}$ is a unitary matrix.

$$\sum_i U_{\alpha i} U^*_{\beta i} = \delta_{\alpha\beta} \qquad \sum_\alpha U_{\alpha i} U^*_{\alpha j} = \delta_{ij}$$

From fields to Hilbert space states

$$\psi = ...a + ...b^\dagger$$

$$\psi|0\rangle \propto |\bar{\psi}\rangle \qquad \bar{\psi}|0\rangle \propto |\psi\rangle$$

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i$$

From fields to Hilbert space states

$$\psi = ...a + ...b^\dagger$$

$$\psi|0\rangle \propto |\bar{\psi}\rangle \qquad \bar{\psi}|0\rangle \propto |\psi\rangle$$

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i$$

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \qquad |\bar{\nu}_\alpha\rangle = \sum_i U_{\alpha i}|\bar{\nu}_i\rangle$$

Unitarity

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \qquad |\bar{\nu}_\alpha\rangle = \sum_i U_{\alpha i}|\bar{\nu}_i\rangle$$

$$\sum_i U_{\alpha i}U^*_{\beta i} = \delta_{\alpha\beta} \qquad \sum_\alpha U_{\alpha i}U^*_{\alpha j} = \delta_{ij}$$

$$\langle\nu_i|\nu_j\rangle = \delta_{ij} \quad \langle\bar{\nu}_i|\bar{\nu}_j\rangle = \delta_{ij} \quad \langle\nu_\alpha|\nu_\beta\rangle = \delta_{\alpha\beta} \quad \langle\bar{\nu}_\alpha|\bar{\nu}_\beta\rangle = \delta_{\alpha\beta}$$

Evolution in time

$$|\psi\rangle \to e^{-iHt}|\psi\rangle$$

$$|\nu_i\rangle \to e^{-iE_it}|\nu_i\rangle \qquad E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}$$

$$|\nu_\alpha; t\rangle = \sum_i U^*_{\alpha i} e^{-iE_it}|\nu_i\rangle = e^{-ipt}\sum_i U^*_{\alpha i} e^{-im_i^2 t/(2p)}|\nu_i\rangle$$

$$|\bar{\nu}_\alpha; t\rangle = \sum_i U_{\alpha i} e^{-iE_it}|\bar{\nu}_i\rangle = e^{-ipt}\sum_i U_{\alpha i} e^{-im_i^2 t/(2p)}|\bar{\nu}_i\rangle$$

Oscillation probability

$$|\nu_\alpha;t\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t}|\nu_i\rangle = e^{-ipt}\sum_i U^*_{\alpha i} e^{-im_i^2 t/(2p)}|\nu_i\rangle$$

$$|\bar{\nu}_\alpha;t\rangle = \sum_i U_{\alpha i} e^{-iE_i t}|\bar{\nu}_i\rangle = e^{-ipt}\sum_i U_{\alpha i} e^{-im_i^2 t/(2p)}|\bar{\nu}_i\rangle$$

$$P(\nu_\alpha \to \nu_\beta) = |\ \langle \nu_\beta|\nu_\alpha;t\rangle\ |^2 = \sum_{i,j} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} e^{i\frac{\Delta m^2_{ji}}{2p}t}$$

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = |\ \langle \bar{\nu}_\beta|\bar{\nu}_\alpha;t\rangle\ |^2 = \sum_{i,j} U^*_{\beta i} U_{\alpha i} U_{\beta j} U^*_{\alpha j} e^{i\frac{\Delta m^2_{ji}}{2p}t}$$

- We have taken plane waves.
- We have taken the same momentum for all mass eigenstates.

$$|\nu_\alpha;t\rangle = \sum_i U^*_{\alpha i} e^{-iE_i t}|\nu_i\rangle = e^{-ipt}\sum_i U^*_{\alpha i} e^{-im_i^2 t/(2p)}|\nu_i\rangle$$

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Why?

Controversy

- Some groups have questioned the standard treatments.

Controversy

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Lots of papers in recent (!!) years

Controversy

- Some groups have questioned the standard treatments.

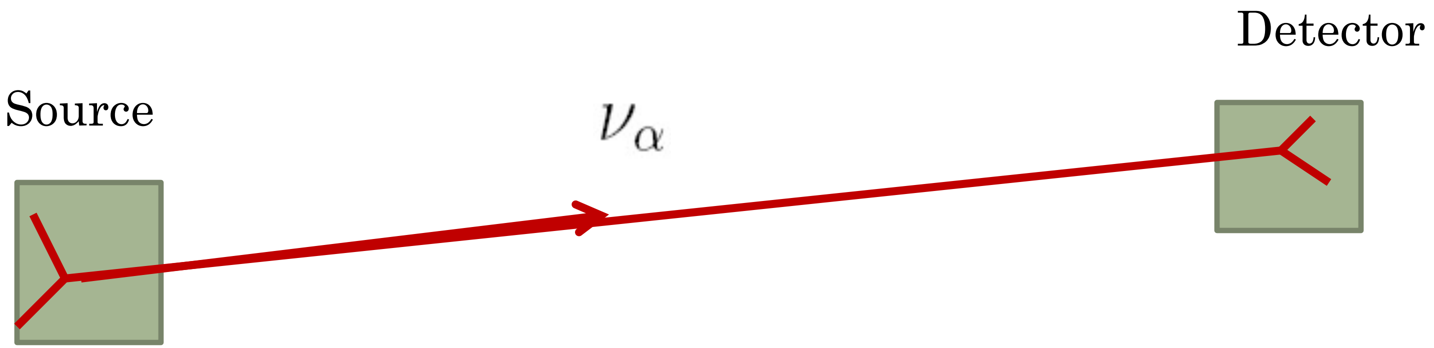
Lots of papers in recent years

Some terribly wrong!

Reliable and clarifying sources

- Akhmedov
- Giunti
- Smirnov

WHAT HAPPENS IN PRACTICE

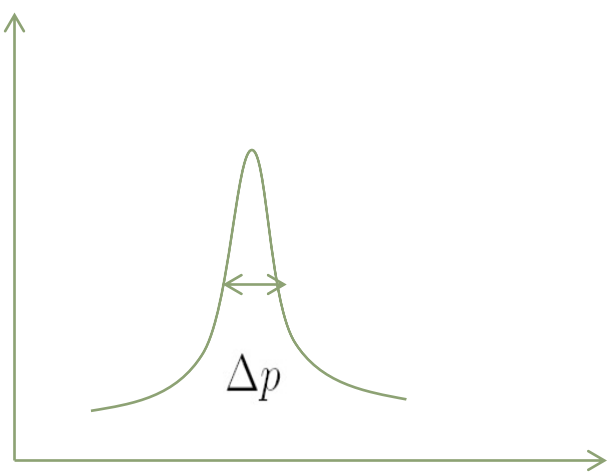


Plane waves → Wave-packets

Wave packets

$$|\nu_\alpha; \vec{x}, t\rangle_S = \int \frac{d^3p}{(2\pi)^3} g_S(p) e^{-i[E_i(t-t_S) - \vec{p}\cdot(\vec{x}-\vec{x}_S)]} U^*_{\alpha i} |\nu_i; p\rangle$$

$g_S(p)$



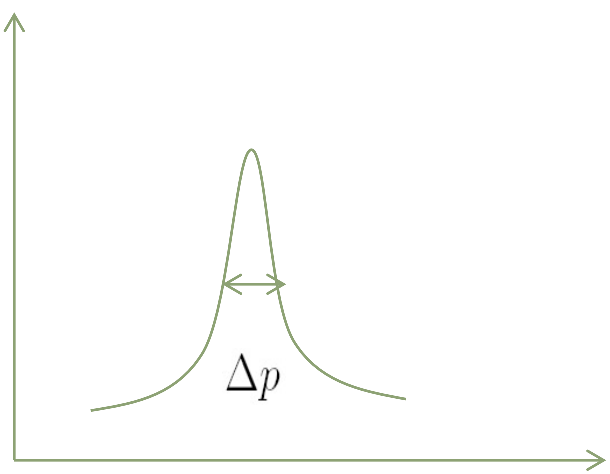
$$\int \frac{d^3p}{(2\pi)^3} |g_S(p)|^2 = 1$$

p

Wave packets

$$|\nu_\beta; \vec{x}, t\rangle_D = \int \frac{d^3p}{(2\pi)^3} g_D(p) e^{-i[E_i(t-t_D) - \vec{p}\cdot(\vec{x}-\vec{x}_D)]} U^*_{\beta i} |\nu_i; p\rangle$$

$g_D(p)$



$$\int \frac{d^3p}{(2\pi)^3} |g_D(p)|^2 = 1$$

p

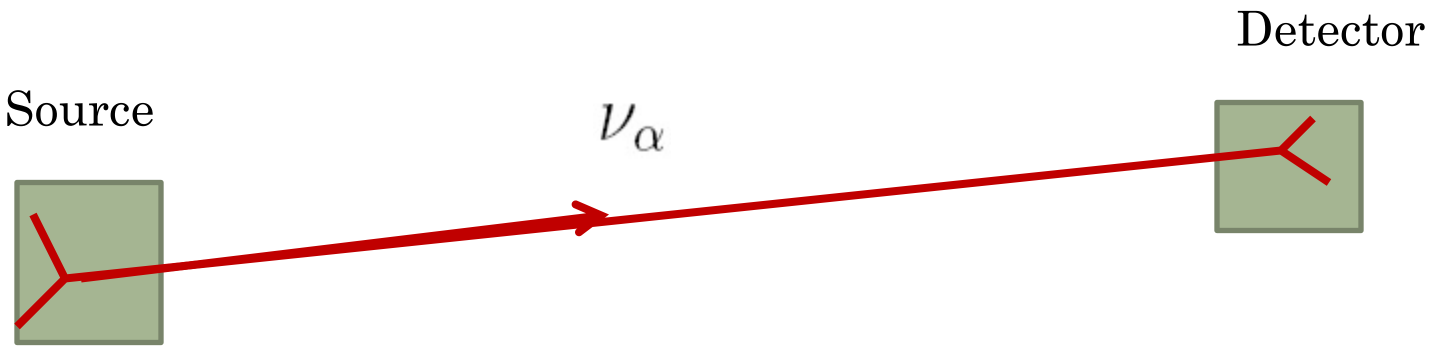
$$L = x_S - x_D \qquad T = t_D - t_S$$

$$P(\nu_\alpha \to \nu_\beta; L) = \int |_D\langle \nu_\beta | \nu_\alpha \rangle_S|^2 dT$$

After integration finite resolution
Of detector (summing over $g_D(p)$)

Standard formulas for wavepacket is reproduced.

Feynman diagram



$$\Delta p \gg \frac{1}{X_D - X_S} \Longrightarrow \nu_\alpha \text{ on-shell}$$

Quantum Field Theory $\Longrightarrow$ Quantum mechanics

Assumption

$$|\nu_\alpha;\vec{x},t\rangle_S = \int \frac{d^3p}{(2\pi)^3} g_S(p) e^{-i[E_i(t-t_S)-\vec{p}\cdot(\vec{x}-\vec{x}_S)]} U^*_{\alpha i}|\nu_i;p\rangle$$

$$|\nu_\beta;\vec{x},t\rangle_D = \int \frac{d^3p}{(2\pi)^3} g_D(p) e^{-i[E_i(t-t_D)-\vec{p}\cdot(\vec{x}-\vec{x}_D)]} U^*_{\beta i}|\nu_i;p\rangle$$

No dependence on ``i”

Approximation ⇨ assumption

At source:

$$a \to \nu_i + X \qquad g_S(p) \propto \mathcal{M}(a \to \nu_i X)$$

At detector:

$$b + \nu_i \to Y \qquad g_D(p) \propto \mathcal{M}(b + \nu_i \to Y)$$

$$\frac{d\mathcal{M}}{dm_i^2} \Delta m_{ij}^2 \ll \mathcal{M}$$

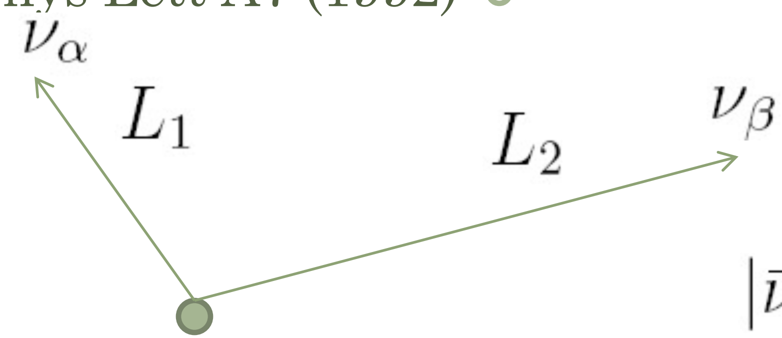
No dependence on ``i”

Coherent production

Can the neutrinos from Z0 decay oscillate?

Smirnov and Zatsepin

Mod Phys Lett A7 (1992)



$$\frac{|\bar{\nu}_1\nu_1\rangle + |\bar{\nu}_2\nu_2\rangle + |\bar{\nu}_3\nu_3\rangle}{\sqrt{3}}$$

$$P(\nu_\alpha \to \nu_\beta; L = L_1 + L_2)$$

$$\frac{\Delta m_{ij}^2 (L_1 + L_2)}{2E}$$

Do charged leptons oscillate?

- Electron, muon and the tau leptons are mass eigenstates.

$\Downarrow$

They do not oscillate!

Do charged leptons oscillate?

Electron, muon and the tau leptons are mass eigenstates.

They do not oscillate!

How about a coherent mixture of charged leptons?

Because of large mass difference producing such a state is Not practical.

Matter effects

$$P(\nu_\alpha \to \nu_\beta) = |\ \langle \nu_\beta | \nu_\alpha ; t \rangle\ |^2 = \sum_{i,j} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} e^{i \frac{\Delta m^2_{ji}}{2p} t}$$

Matter effect $$\mathcal{H}_{eff}(x) = \sum_{\alpha = e,\mu,\tau} V_\alpha \bar{\nu}_{\alpha L}(x) \gamma^0 \nu_{\alpha L}(x)$$

$$V_\alpha \equiv \begin{bmatrix} V_{NC} + V_{CC} & 0 & 0 \\ 0 & V_{NC} & 0 \\ 0 & 0 & V_{NC} \end{bmatrix}$$

$$V_{CC} = \sqrt{2} G_F N_e \quad V_{NC} = -\frac{\sqrt{2}}{2} G_F N_n$$

WHERE DOES THE MATTER EFFECT COME FROM?

$$\mathcal{H}_{eff}(x) \equiv \mathcal{H}_{eff}^{NC}(x) + \mathcal{H}_{eff}^{CC}(x)$$

$$\mathcal{H}_{eff}^{CC}(x) = \frac{G_F}{\sqrt{2}}[\bar{\nu}_e(x)\gamma^\rho(1-\gamma^5)e(x)][\bar{e}(x)\gamma_\rho(1-\gamma^5)\nu_e(x)]$$

Fierz transformation

$$\mathcal{H}_{eff}^{CC}(x) = \frac{G_F}{\sqrt{2}}[\bar{\nu}_e(x)\gamma^\rho(1-\gamma^5)\nu_e(x)][\bar{e}(x)\gamma_\rho(1-\gamma^5)e(x)]$$

$$\mathcal{H}_{eff}^{NC}(x) = \sum_f \frac{G_F}{\sqrt{2}}[\bar{\nu}(x)\gamma^\rho(1-\gamma^5)\nu(x)][\bar{f}(x)\gamma_\rho(g_V - g_A\gamma^5)f(x)]$$

$$\langle \mathrm{matter} | \mathcal{H}_{eff}(x) | \mathrm{matter} \rangle$$

Un-polarized

$$\langle \vec{p} \rangle = 0$$

$$N_e = N_p$$

$$\mathcal{H}_{eff}(x) = \sum_{\alpha = e,\mu,\tau} V_\alpha \bar{\nu}_{\alpha L}(x) \gamma^0 \nu_{\alpha L}(x) \qquad V_\alpha \equiv \begin{bmatrix} V_{NC} + V_{CC} & 0 & 0 \\ 0 & V_{NC} & 0 \\ 0 & 0 & V_{NC} \end{bmatrix}$$

$$V_{CC} = \sqrt{2} G_F N_e \quad V_{NC} = -\frac{\sqrt{2}}{2} G_F N_n$$

Effective Lagrangian in a medium

$$\mathcal{L} = i\bar{\psi}\partial \cdot \gamma\psi - m\bar{\psi}\psi - \underbrace{V\bar{\psi}\gamma^0\psi}_{}$$

Is not Lorentz invariant

Written in rest frame of the medium

$$\boxed{\mathcal{L} = i\bar{\psi}\partial \cdot \gamma\psi - m\bar{\psi}\psi - V\bar{\psi}\gamma^0\psi}$$

$$(i\partial \cdot \gamma - m - V\gamma^0)\psi = 0$$

$$\boxed{\psi(x) = \int \frac{d^3p}{(2\pi)^3}\frac{1}{\sqrt{2E_p}}\sum_s [a_p^s u^s(p)e^{-ip\cdot x} + b_p^{s\dagger} v^s(p)e^{ip\cdot x}]}$$

Neutrino $$(+E\gamma^0 - \vec{p}\cdot\vec{\gamma} - m - V\gamma^0)u(p) = 0$$

Antineutrino $$(-E\gamma^0 + \vec{p}\cdot\vec{\gamma} - m - V\gamma^0)v(p) = 0$$

$$(\pm E\gamma^0 - V\gamma^0 - m)(u \text{ or } d) = \pm\vec{p}\cdot\vec{\gamma}(u \text{ or } d)$$

$$\boxed{(\pm E\gamma^0 - V\gamma^0 - m)(u \text{ or } d) = \pm\vec{p}\cdot\vec{\gamma}(u \text{ or } d)}$$

$$[(\pm E - V)\gamma^0 + m]\times \qquad \gamma^0\vec{\gamma} = -\vec{\gamma}\gamma^0$$

$$[(\pm E - V) + m][(\pm E - V) - m](u\ or\ d) = \pm(\vec{p}\cdot\vec{\gamma})(-(\pm E - V)\gamma^0 + m)(u\ or\ d) =$$

$$= -(\vec{p}\cdot\vec{\gamma})(\vec{p}\cdot\vec{\gamma})(u\ or\ d) = \pm(\vec{p}\cdot\vec{\gamma})(-(\pm E - V)\gamma^0 + m)(u\ or\ d) =$$

$$(\vec{p}\cdot\vec{p})(u\ or\ d) = p^2(u\ or\ d)$$

Neutrino

$$\boxed{E \simeq p + \frac{m^2}{2E} \pm V}$$

Antineutrino

J^f_μ f

$$\frac{G_F}{2} J^\mu_\nu J^f_\mu$$

$$J^\mu = \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5)\nu_\alpha \quad J^f_\mu = \bar{f}\gamma_\mu (g_V^{\alpha,f} + g_A^{\alpha,f}\gamma_5) f$$

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \mathcal{H}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$$\mathcal{H} \to \mathcal{H}' = \mathcal{H} + \Delta\mathcal{H} \cdot 1_{3\times 3}$$

$1_{3\times 3}$ $\mathcal{H}'$ $\mathcal{H}$

$(\nu_\alpha \ \nu_\beta)$

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = \mathcal{H}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\cdot\begin{pmatrix}0 & 0\\ 0 & \Delta_{21}\end{pmatrix}\cdot\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} + \begin{pmatrix}V & 0\\ 0 & 0\end{pmatrix}$$

$$\Delta_{21} = \frac{m_2^2 - m_1^2}{2E} \qquad \mathcal{H} \to \mathcal{H} + \Delta\mathcal{H} I_{2\times 2}$$

$$\mathrm{Diag}(m_1^2/2E, m_2^2/2E) \qquad \mathrm{Diag}(0, \Delta_{21})$$

$$\mathrm{Diag}(V_\alpha, V_\beta) \qquad \mathrm{Diag}(V = V_\alpha - V_\beta, 0)$$

$$\Delta_{21}^{eff} = \sqrt{(\Delta_{21}\cos 2\theta - V)^2 + (\Delta_{21}\sin 2\theta)^2}$$

$$\tan 2\theta_m = \frac{\Delta_{21}\sin 2\theta}{\Delta_{21}\cos 2\theta - V}$$

$$\mathcal{H} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 0 & \Delta_{21}^{eff} \end{pmatrix} \cdot \begin{pmatrix} \cos\theta_m & -\sin\theta_m \\ \sin\theta_m & \cos\theta_m \end{pmatrix}$$

$$P(\nu_\alpha \to \nu_\beta) = P(\nu_\beta \to \nu_\alpha) = 4\sin^2\theta_m \cos^2\theta_m \sin^2 \frac{\Delta_{21}^{eff} t}{2}$$

$$\Delta_{21}\cos\theta \to V$$

$$\Delta_{21}\cos 2\theta \to V$$