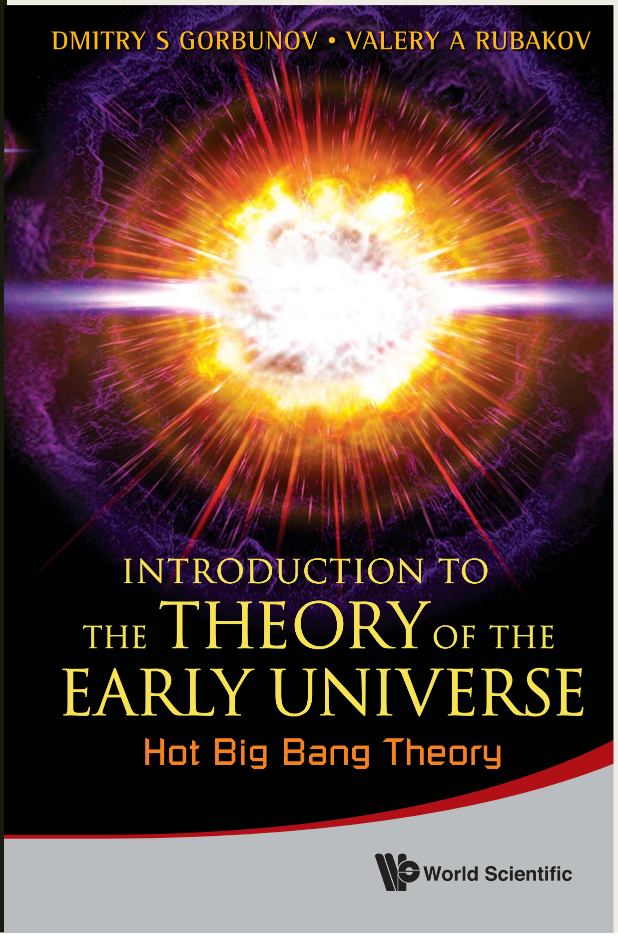
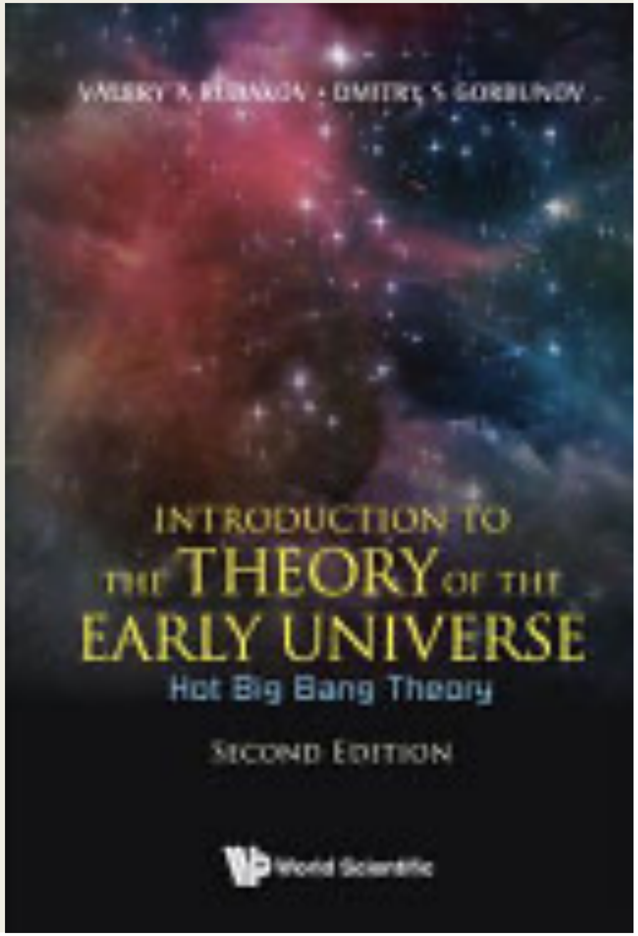
DARK MATTER

Introduction

Yasaman Farzan
Physics school, IPM

Standard cosmology in a nutshell





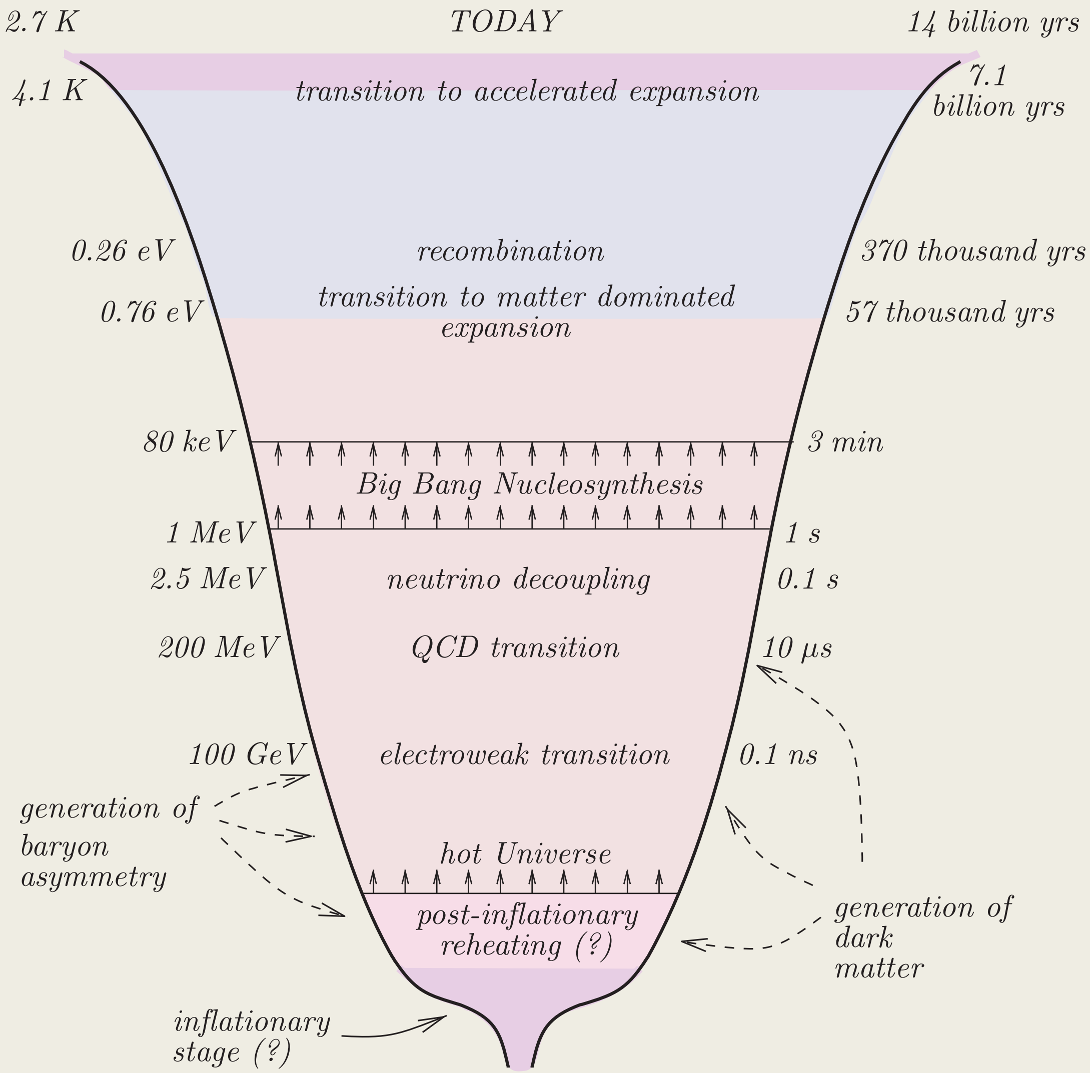


Fig. 1.9 Stages of the evolution of the Universe.

Friedmann–Lemaître–Robertson–Walker Metric

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j.$$

$$H(t) = \frac{\dot{a}(t)}{a(t)}.$$

$$z(t) = \frac{a_0}{a(t)} - 1$$

$$z = H_0 r, \quad z \ll 1.$$

Massless particle:

$$\mathbf{p}(t) = \frac{\mathbf{k}}{a(t)}, \quad \omega(t) = \frac{k}{a(t)}.$$

$$\mathbf{p}(t) = \frac{\mathbf{k}}{a(t)}, \quad \omega(t) = \frac{k}{a(t)}.$$

The number of particles in an element of comoving phase space

$$f(k) d^3\mathbf{x} d^3\mathbf{k} = \text{const.}$$

$$\boxed{d^3\mathbf{x} d^3\mathbf{k} = d^3(a\mathbf{x}) d^3\left(\frac{\mathbf{k}}{a}\right) = d^3\mathbf{X} d^3\mathbf{p}.}$$

Friedmann Equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}.$$

Ricci tensor,

$$R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\nu\lambda} + \Gamma^\lambda_{\mu\nu}\Gamma^\sigma_{\lambda\sigma} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\lambda\nu}$$

homogeneous and isotropic Universe,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{\varkappa}{a^2}.$$

In the spatially flat model,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho.$$

$$f(\mathbf{p},t) = f_i\left(\frac{a(t)}{a_i}\mathbf{p}\right).$$

$$f_i(\mathbf{p}) = f_{\mathrm{Pl}}\left(\frac{|\mathbf{p}|}{T_i}\right) = \frac{1}{(2\pi)^3}\frac{1}{e^{|\mathbf{p}|/T_i}-1}.$$

$$\boxed{T_{\mathit{eff}}(t) = \frac{a_i}{a(t)}T_i.}$$

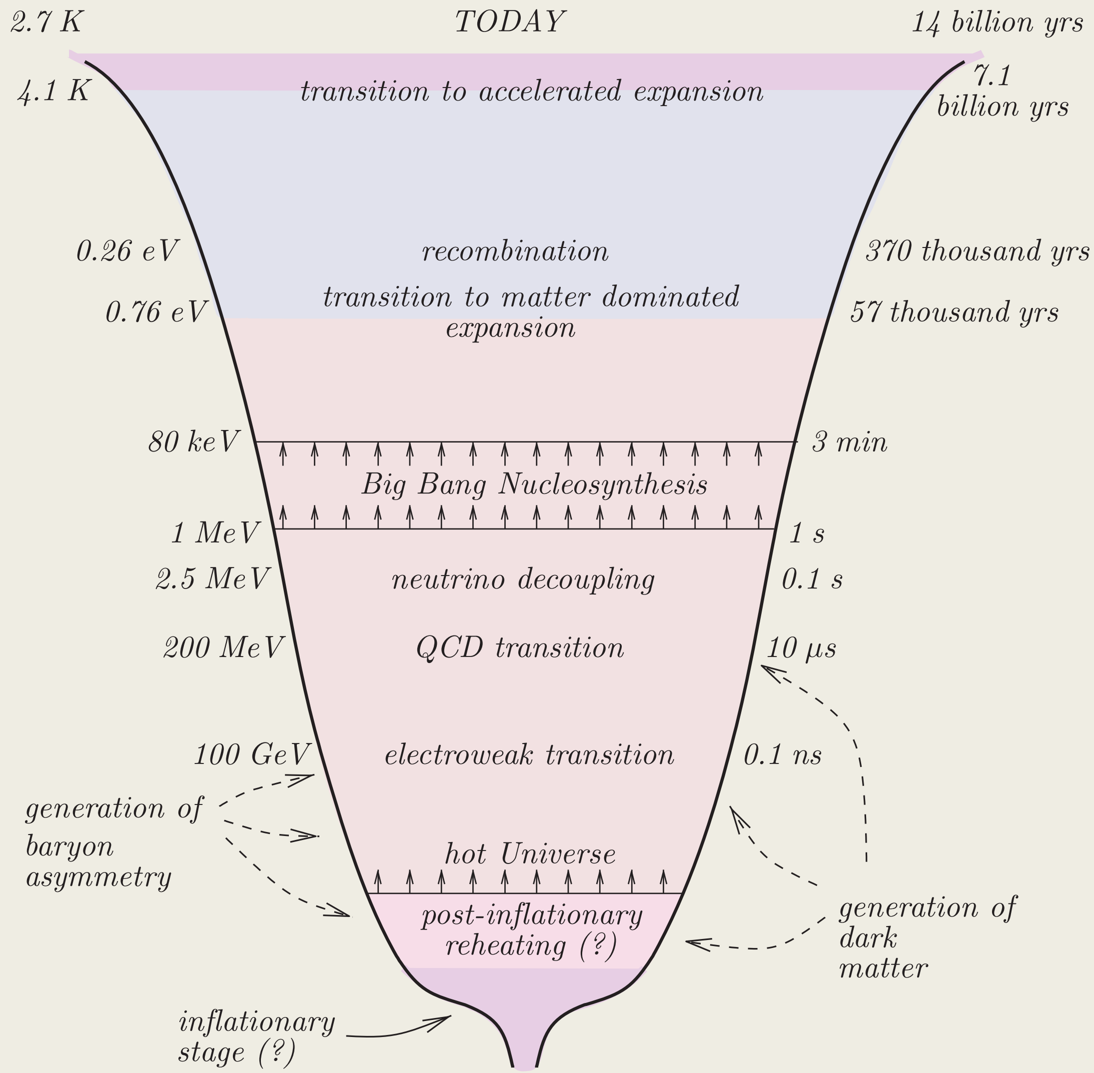


Fig. 1.9 Stages of the evolution of the Universe.

Non-relativistic matter ("dust")

$$p = 0. \qquad \rho = \frac{\text{const}}{a^3}.$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\text{const}}{a^3} \qquad a(t) = \text{const} \cdot (t - t_s)^{2/3},$$

$$H(t) = \frac{\dot{a}}{a}(t) = \frac{2}{3t}.$$

Relativistic matter ("radiation")

$$p = \frac{1}{3}\rho.$$

$$\rho = \frac{\text{const}}{a^4}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\text{const}}{a^4}$$

$$H \equiv \frac{\dot{a}}{a} = \frac{1}{2t},$$

$$\rho = \frac{\pi^2}{30} g_* T^4,$$

$$\boxed{g_* = \sum_b g_b + \frac{7}{8}\sum_f g_f}$$

$$H = \frac{T^2}{M_{Pl}^*},$$

$$M_{Pl}^* = \sqrt{\frac{90}{8\pi^3 g_*}} M_{Pl} = \frac{1}{1.66\sqrt{g_*}} M_{Pl}$$

Vacuum

$$T_{\mu\nu} = \rho_{vac}\eta_{\mu\nu}.$$

$$p = -\rho_{vac}.$$

General barotropic equation of state $\boldsymbol{p = w\rho}$

$$p = w\rho,$$

ΛCDM: Cosmological Model with Dark Matter and Dark Energy

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G(\rho_M + \rho_{rad} + \rho_\Lambda + \rho_{curv}),$$

$$\frac{8\pi}{3} G\rho_{curv} = -\frac{\varkappa}{a^2}$$

$$\rho_c \equiv \frac{3}{8\pi G} H_0^2.$$

$$\Omega_M = \frac{\rho_{M,0}}{\rho_c}, \quad \Omega_{rad} = \frac{\rho_{rad,0}}{\rho_c}, \quad \Omega_\Lambda = \frac{\rho_{\Lambda,0}}{\rho_c}, \quad \Omega_{curv} = \frac{\rho_{curv,0}}{\rho_c}.$$

$$\sum_i \Omega_i \equiv \Omega_M + \Omega_{rad} + \Omega_\Lambda + \Omega_{curv} = 1.$$

$$H_0 = h \cdot 100 \, \frac{\mathrm{km}}{\mathrm{s} \cdot \mathrm{Mpc}},$$

$$h = 0.705 \pm 0.013.$$

CMB

$$\rho_{\gamma,0} = 2\frac{\pi^2}{30}T_0^4, \qquad T_0 \;=\; 2.726\,\mathrm{K}.$$

$$\rho_{\gamma,0} = 2.6 \cdot 10^{-10}\,\frac{\mathrm{GeV}}{\mathrm{cm}^3},$$

$$\Omega_\gamma = 2.5 \cdot 10^{-5} h^{-2} = 5.0 \cdot 10^{-5}, \quad h = 0.705.$$

Dominant contributions

$$\Omega_M \approx 0.27, \quad \Omega_\Lambda \approx 0.73$$

$$\Omega_M = \Omega_B + \Omega_{DM},$$

$$\Omega_B = 0.046$$

$$\Omega_{DM} = 0.23.$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_M \left(\frac{a_0}{a}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{curv} \left(\frac{a_0}{a}\right)^2\right]$$

Transition from Deceleration to Acceleration

$$\dot{a}^2 = \frac{8\pi}{3} G\rho_c \left(\frac{\Omega_M a_0^3}{a} + \Omega_\Lambda a^2 \right).$$

$$\ddot{a} = a\frac{4\pi}{3} G\rho_c \left(2\Omega_\Lambda - \Omega_M \left(\frac{a_0}{a} \right)^3 \right).$$

$$\left(\frac{a_0}{a_{ac}} \right)^3 = \frac{2\Omega_\Lambda}{\Omega_M},$$

$$z_{ac} = \left(\frac{2\Omega_\Lambda}{\Omega_M} \right)^{1/3} - 1.$$

$$\Omega_M = 0.27,\ \Omega_\Lambda = 0.73,$$

$$z_{ac} \approx 0.76.$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho_c\left[\Omega_M\left(\frac{a_0}{a}\right)^3 + \Omega_{rad}\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{curv}\left(\frac{a_0}{a}\right)^2\right]$$

Transition from Radiation Domination to Matter Domination

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma, \qquad T_\gamma \equiv T. \qquad \rho_\nu = 3 \cdot 2 \cdot \frac{7}{8}\frac{\pi^2}{30}T_\nu^4,$$

$$\rho_{rad} = \rho_\gamma + \rho_\nu = \left[2 + \frac{21}{4}\left(\frac{4}{11}\right)^{4/3}\right]\frac{\pi^2}{30}T^4,$$

$$\rho_M = \left(\frac{a_0}{a}\right)^3 \Omega_M \rho_c.$$

$$1 + z_{eq} = 3.2 \cdot 10^3. \qquad T_{eq} = 0.76 \text{ eV}$$

Thermodynamics in Expanding Universe

$$A_1 + A_2 + \cdots + A_n \leftrightarrow B_1 + B_2 + \cdots + B_{n'},$$

$$\mu_{A_1} + \mu_{A_2} + \cdots + \mu_{A_n} = \mu_{B_1} + \mu_{B_2} + \cdots + \mu_{B_{n'}}.$$

$$f(\mathbf{p}) = \frac{1}{(2\pi)^3} \frac{1}{e^{(E(\mathbf{p})-\mu)/T} \mp 1}.$$

$$\mu_\gamma = 0.$$

$$e^+ + e^- \leftrightarrow 2\gamma.$$

$$\mu_{e^-} + \mu_{e^+} = 0.$$

$$\mu_i = 0.$$

Expression (5.10) gives then the Stefan–Boltzmann law,

$$\rho_i = \frac{g_i}{2\pi^2} \int \frac{E^3}{e^{E/T} \mp 1} dE = \begin{cases} g_i \dfrac{\pi^2}{30} T^4 & - \text{Bose} \\ \dfrac{7}{8} g_i \dfrac{\pi^2}{30} T^4 & - \text{Fermi} \end{cases}$$

$$\rho = g_* \frac{\pi^2}{30} T^4,$$

$$g_* = \sum_{\substack{\text{bosons} \\ \text{with } m \ll T}} g_i + \frac{7}{8} \sum_{\substack{\text{fermions} \\ \text{with } m \ll T}} g_i$$

$$\Omega_B = \rho_B/\rho_c = m_B n_B/\rho_c,$$

$$\Omega_B h^2 \simeq 0.023, \quad \Omega_B \simeq 0.046,$$

$$n_\gamma = 2\frac{\zeta(3)}{\pi^2}T_0^3 = 411\,\mathrm{cm}^{-3}.$$

Relic Neutrinos

$$\sigma_\nu \sim G_F^2 E^2, \qquad G_F = 1.17 \cdot 10^{-5}\,\mathrm{GeV}^{-2}.$$

$$\tau_\nu = \frac{1}{\langle \sigma_\nu n v \rangle}, \qquad \tau_\nu \sim \frac{1}{G_F^2 T^5}. \qquad H^{-1} = \frac{M_{Pl}^*}{T^2}.$$

$$\tau_\nu(T) \sim H^{-1}(T).$$

$$T_{\nu,f} \sim \left(\frac{1}{G_F^2 M_{Pl}^*} \right)^{1/3} \sim 2 - 3\,\mathrm{MeV}.$$

$$e^- e^+ \to \gamma\gamma$$

entropy conservation of the electron-photon

$$g_*(T)a^3T^3 = \text{const},$$

$$g_*(T_{\nu,f}) = 2 + \frac{7}{8}(2+2) = \frac{11}{2},$$

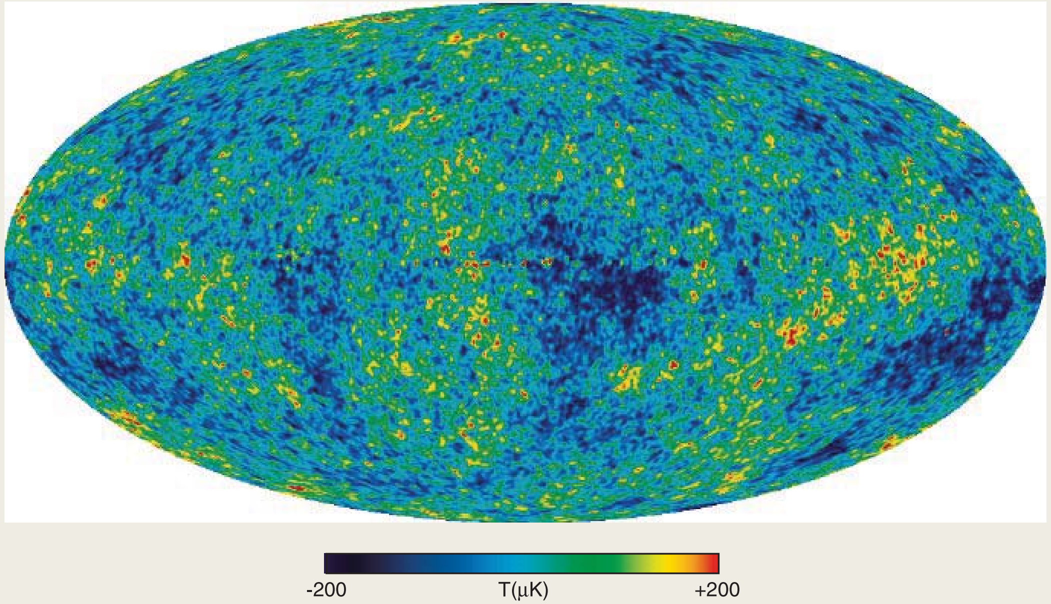
$$\Rightarrow$$

$$g_* = 2,$$

$$\frac{T_{\gamma,0}}{T_{\nu,0}} = \left(\frac{g_*(T_{\nu,f})}{g_*(T_0)}\right)^{1/3} = \left(\frac{11}{4}\right)^{1/3} \simeq 1.4. \quad n_{\nu,0} = \frac{3}{4}\cdot 2\cdot \frac{\zeta(3)}{\pi^2}T_\nu^3(t_0) \simeq 112\ \text{cm}^{-3}.$$

CMB and Neutrino distribution

$$\delta T \sim 100\ \mu\mathrm{K},\ \text{i.e.},\ \delta T/T_0 \sim 10^{-4} - 10^{-5}.$$

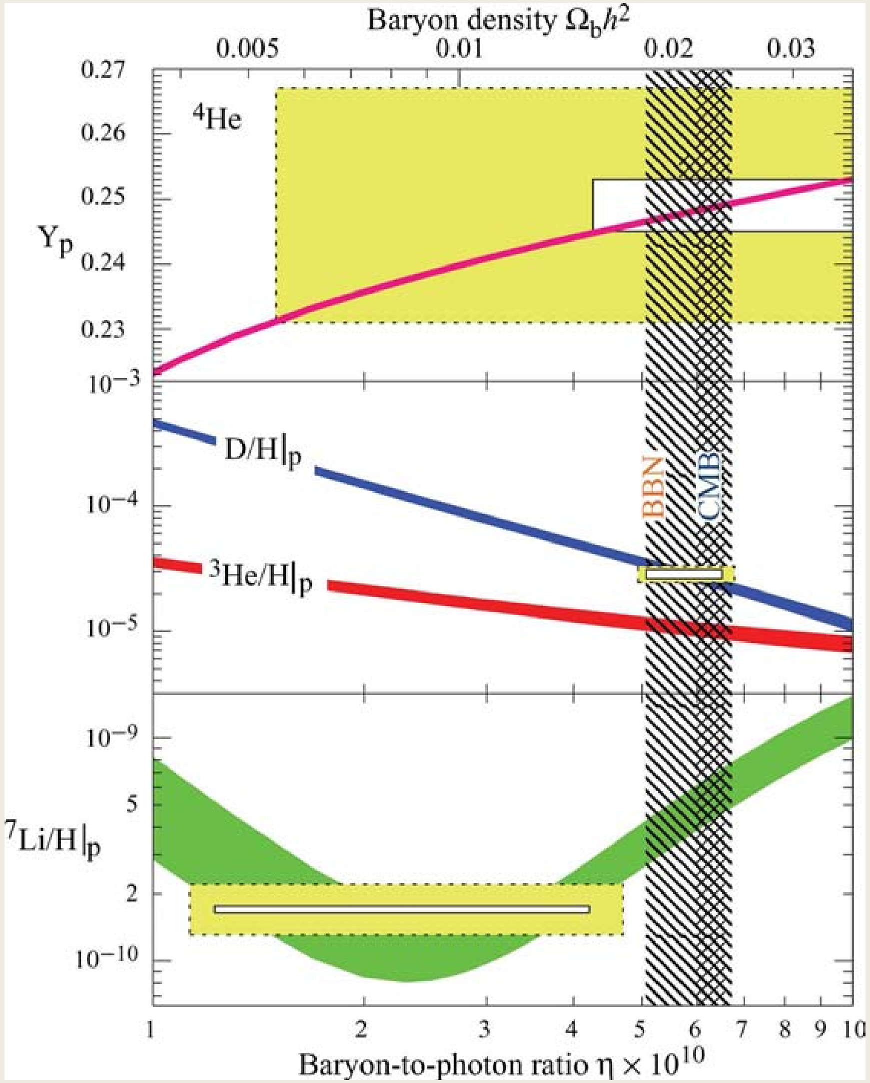


Neutrinos?? Structure formation. Matter –radiation equality. Neutrino velocity

Nucleosynthesis

$$p + n \longrightarrow \mathrm{D} + \gamma. \qquad p(n,\gamma)\mathrm{D}.$$

$$\mathrm{D} + \mathrm{D} \to {}^3\mathrm{He} + n \quad \text{and} \quad \mathrm{D} + \mathrm{D} \to \mathrm{T} + p$$



$$Y_p = \frac{n_{^4\mathrm{He}} \cdot m_{^4\mathrm{He}}}{n_H \cdot m_H + n_{^4\mathrm{He}} \cdot m_{^4\mathrm{He}}},$$

Neutron decoupling

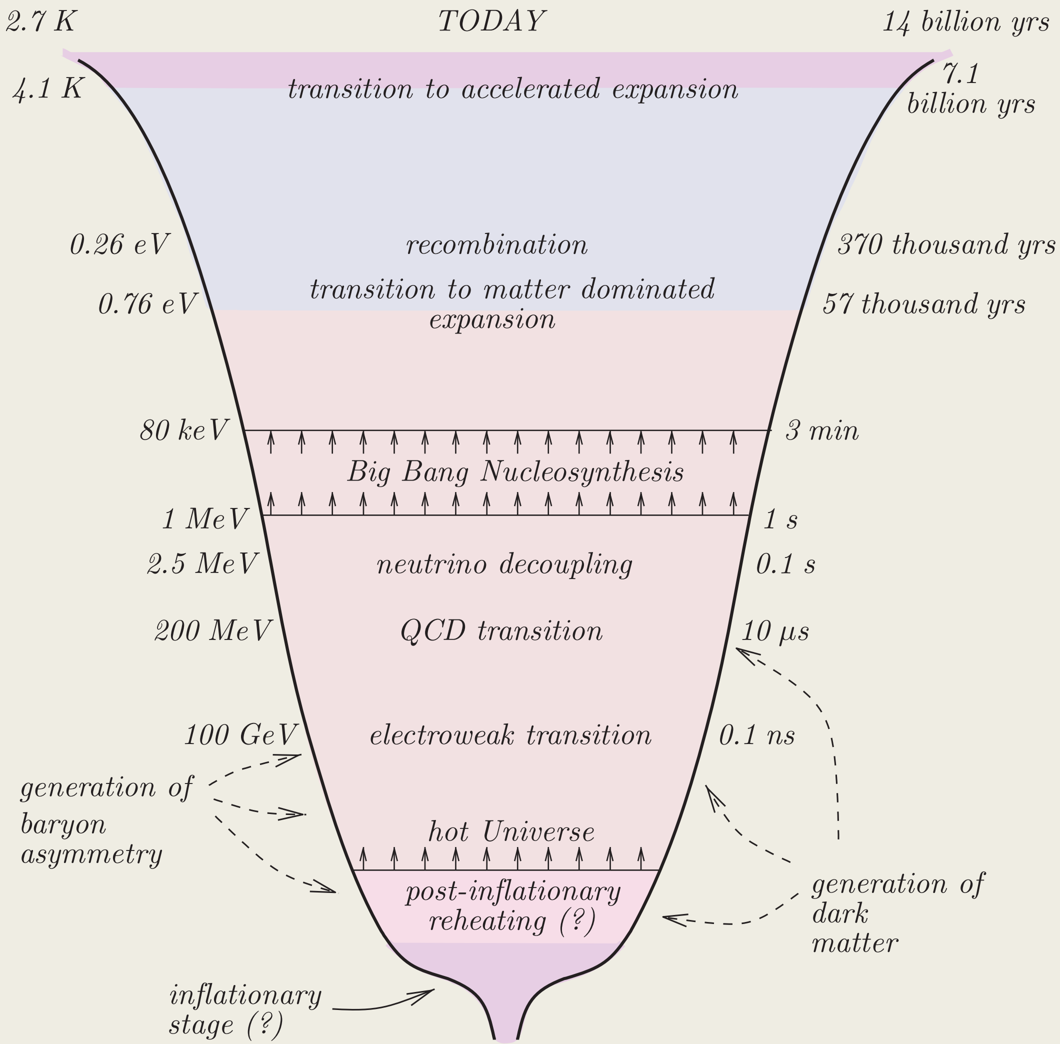
$$p + e \leftrightarrow n + \nu_e\,, \qquad \Delta m \equiv m_n - m_p = 1.3\,\mathrm{MeV}$$

$$\tau_n = \Gamma_n^{-1}, \quad \Gamma_n = C_n G_F^2 T^5,$$

$$\Gamma_n(T) \sim H(T) = \frac{T^2}{M_{Pl}^*}. \qquad M_{Pl}^* = \frac{M_{Pl}}{1.66\sqrt{g_*}},$$

$$g_* = 2 + \frac{7}{8}\cdot 4 + \frac{7}{8}\cdot 2\cdot N_\nu.$$

Dark ages and cosmic dawn



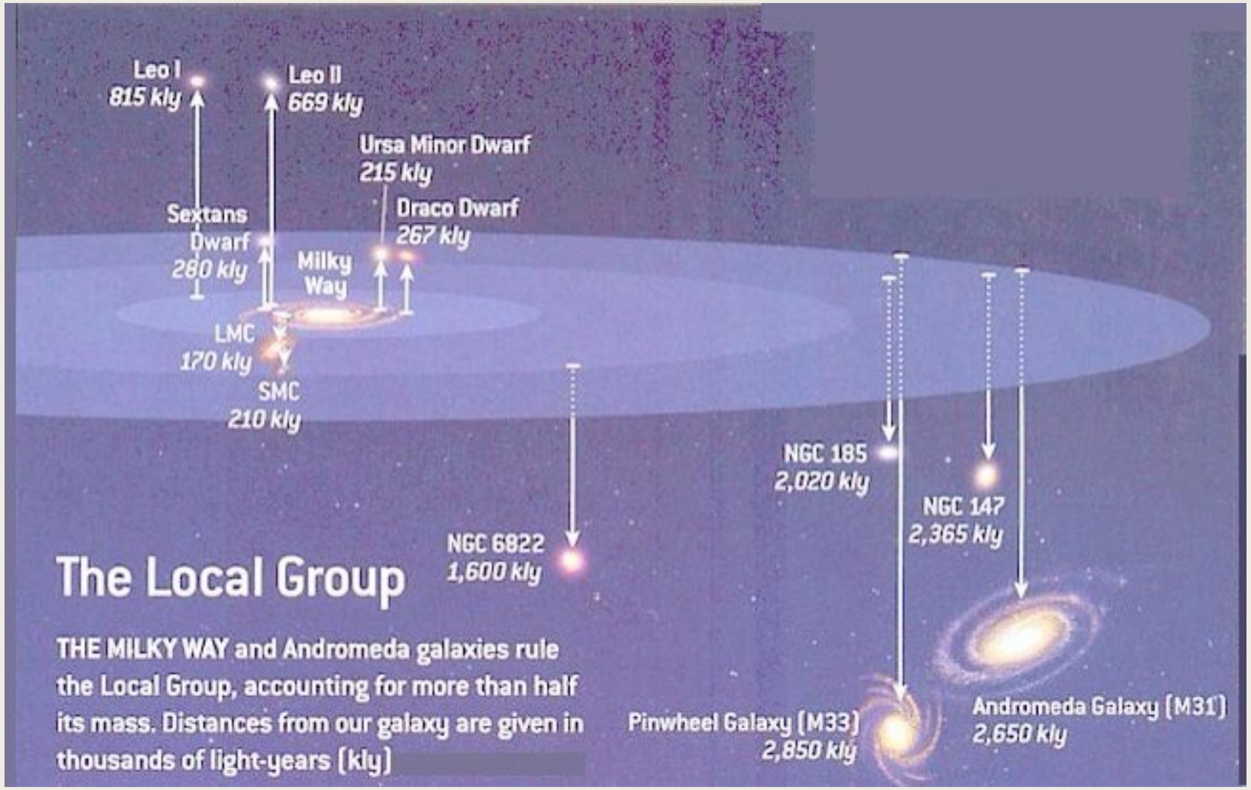
astronomical unit	au	$149\,597\,870\,700$ m
parsec (1 au/1 arcsec)	pc	$3.085\,677\,581\,49\ldots \times 10^{16}$ m $= 3.261\,56\ldots$ ly
light year (deprecated unit)	ly	$0.306\,601\ldots$ pc $= 0.946\,073\ldots \times 10^{16}$ m

H_0	$100\ h\ \mathrm{km\,s^{-1}\,Mpc^{-1}} = h \times (9.777\,752\ \mathrm{Gyr})^{-1}$
h	0.674(5) from CMB anisotropies (*Planck*) *or* 0.730(10) from the distance ladder (SH0ES)

$$z = H_0 r, \quad z \ll 1,$$

4 Gpc. ⟺ Z=1

Some scales


Leo I
815 kly
Leo II
669 kly
Ursa Minor Dwarf
215 kly
Draco Dwarf
267 kly
Sextans
Dwarf
280 kly
Milky
Way
LMC
170 kly
SMC
210 kly
NGC 185
2,020 kly
NGC 147
2,365 kly
NGC 6822
1,600 kly
The Local Group
THE MILKY WAY and Andromeda galaxies rule the Local Group, accounting for more than half its mass. Distances from our galaxy are given in thousands of light-years (kly)
Pinwheel Galaxy (M33)
2,850 kly
Andromeda Galaxy (M31)
2,650 kly

Dark matter

Dark Matter

Marco Cirelli

Laboratoire de Physique Théorique et Hautes Énergies (LPTHE), CNRS & Sorbonne Université, 4 Place Jussieu, Paris, France

Alessandro Strumia

Dipartimento di Fisica dell'Università di Pisa, Italia

Jure Zupan

Department of Physics, University of Cincinnati, Cincinnati, Ohio 45221,USA

Abstract

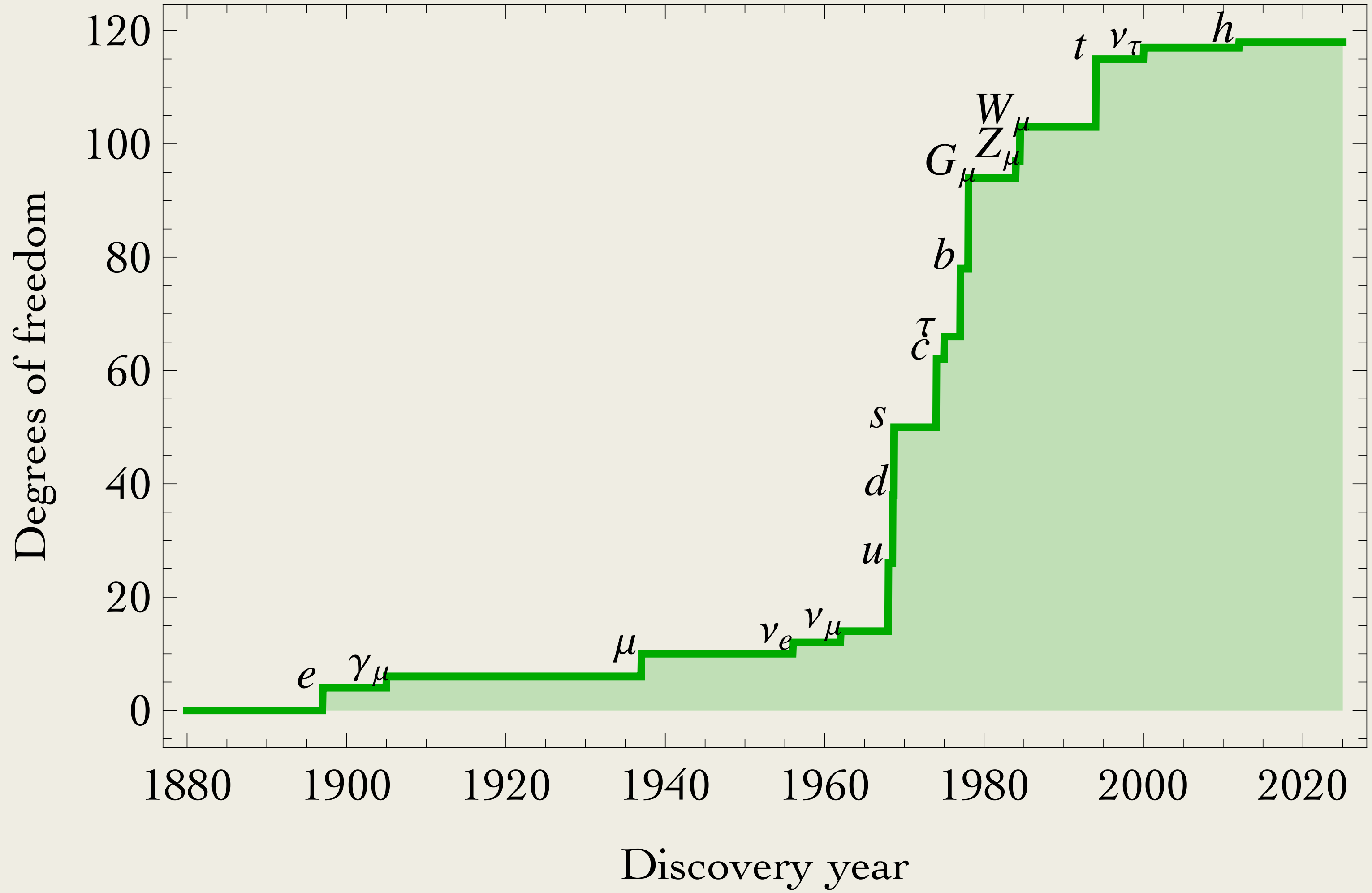
We review observational, experimental and theoretical results related to Dark Matter.

DARK MATTER

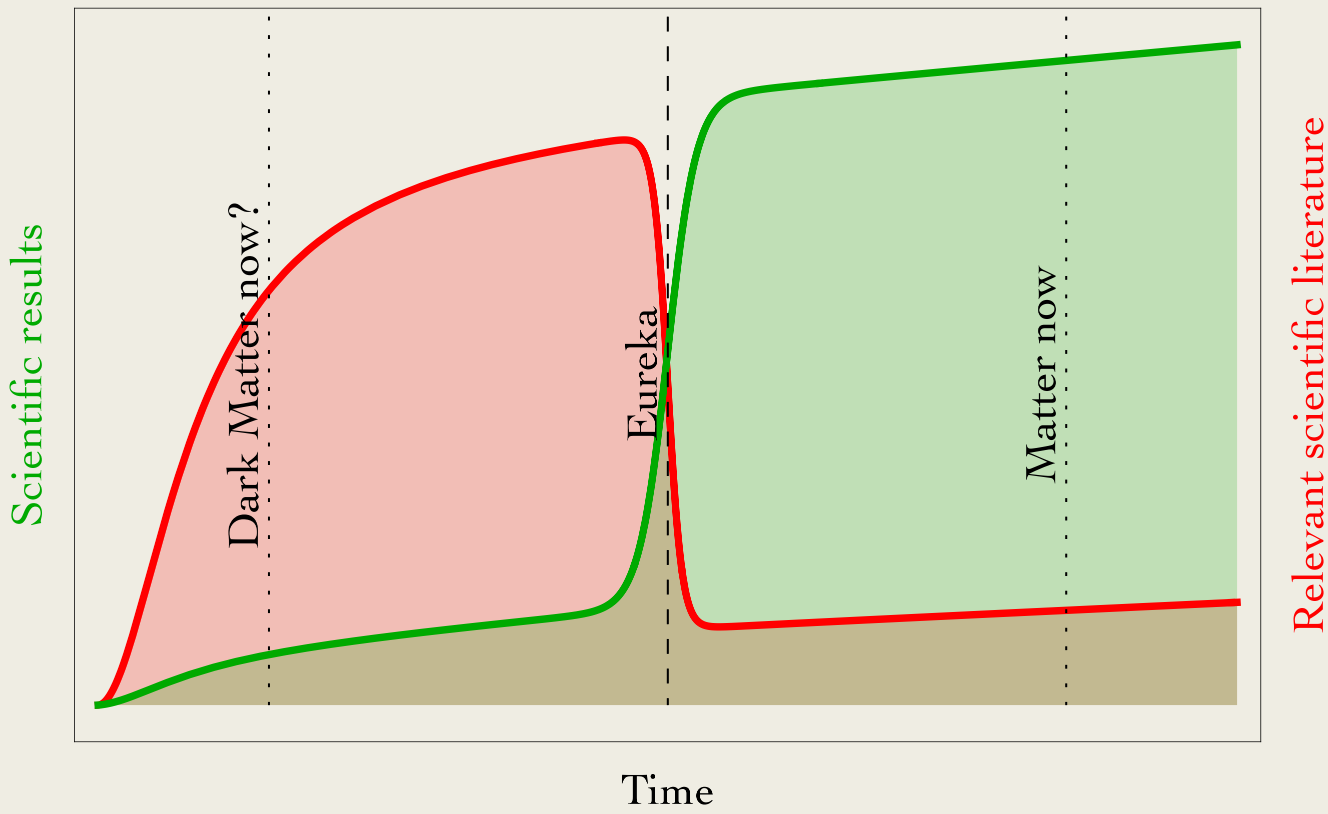
• • • Needs confirmation • • •

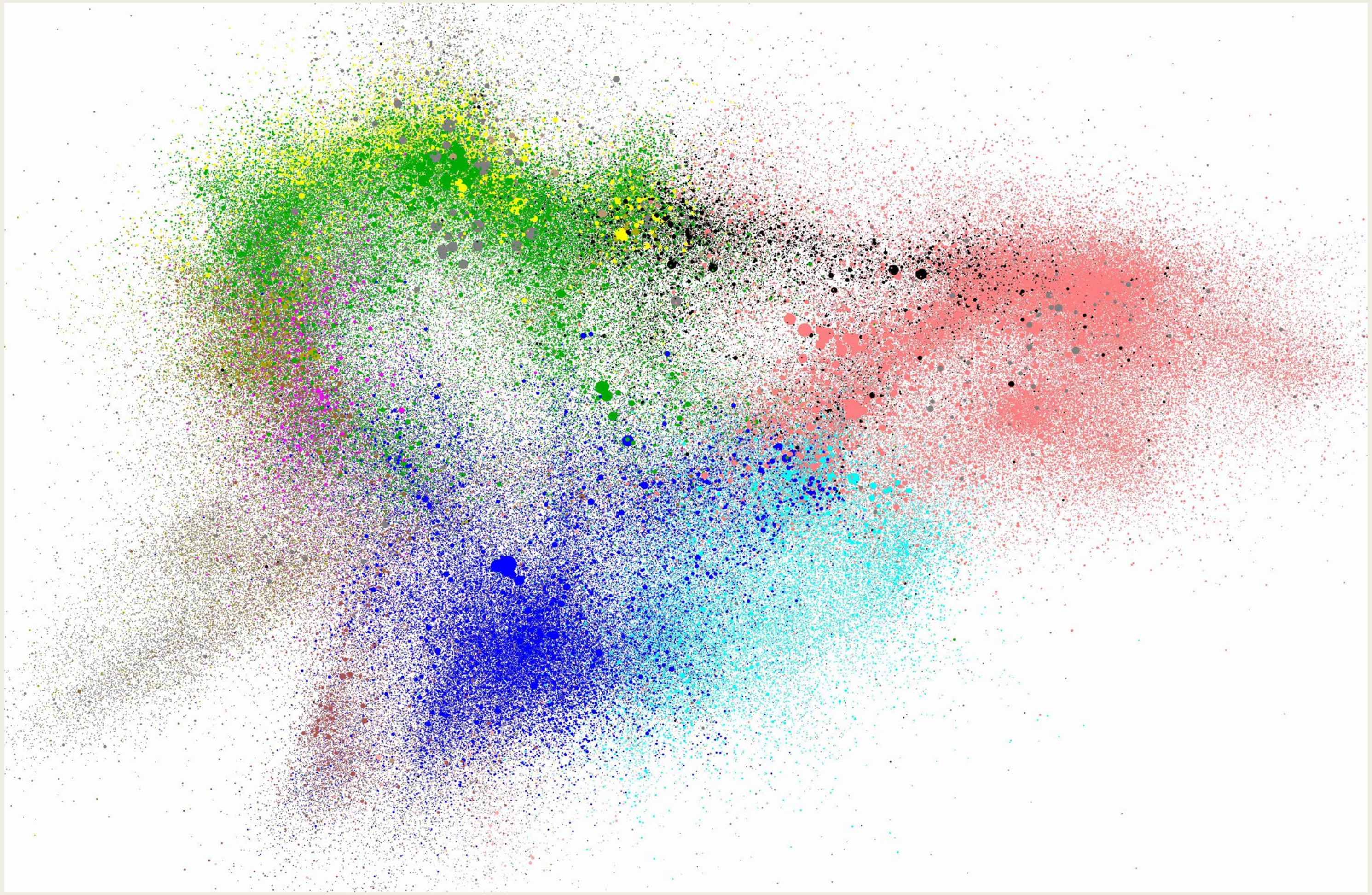
PROPERTIES

$I(J^{PC})$	MASS	WIDTH	DECAY MODES	PRODUCTION
$?(?^{??})$	$? \pm ?$	$? \pm ?$	STABLE ?	$\sigma(?? \to ??) = ?$

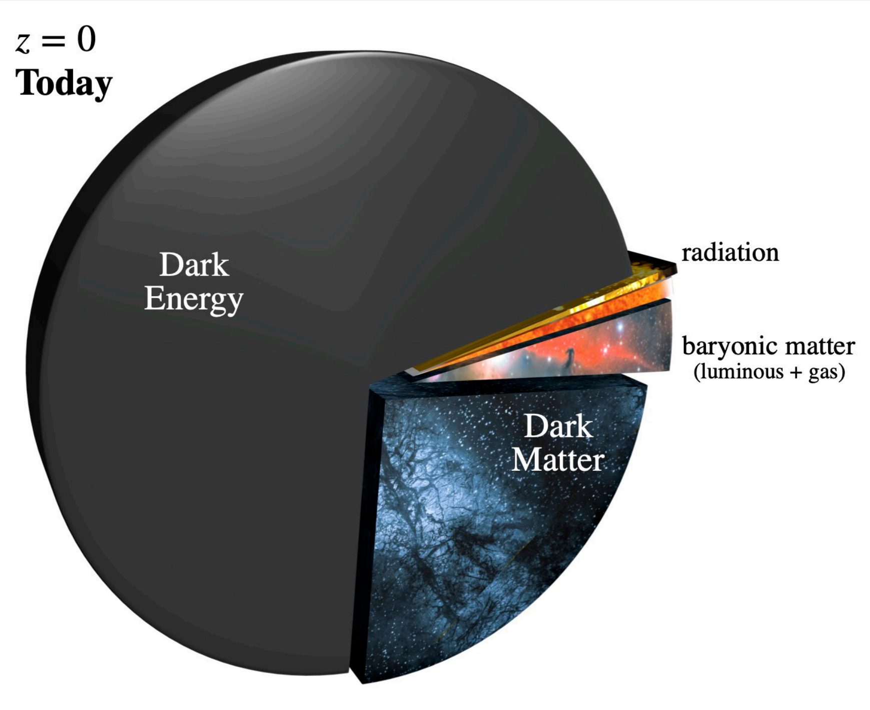


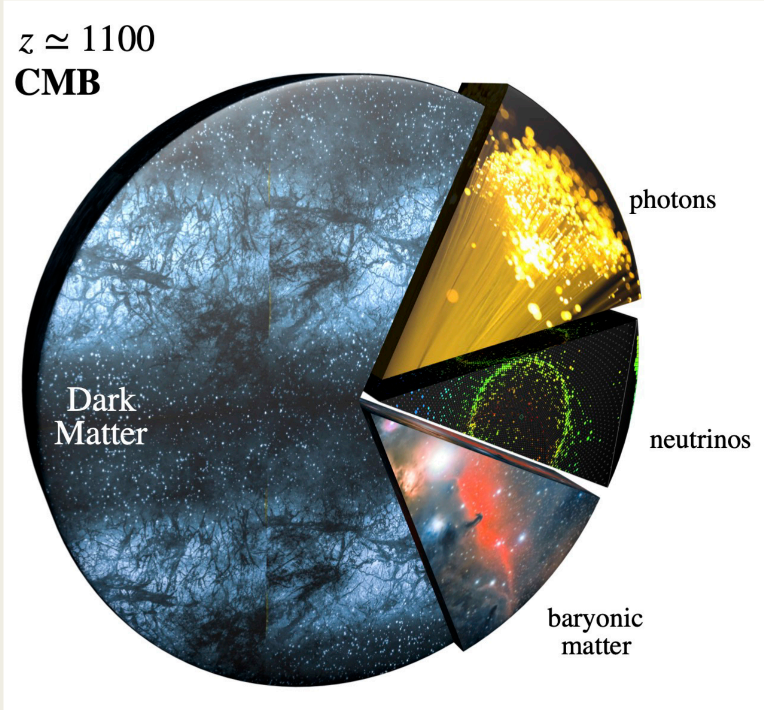
Degrees of freedom
Discovery year
0
20
40
60
80
100
120
1880
1900
1920
1940
1960
1980
2000
2020
e
γ
μ
μ
ν_e
ν_μ
u
d
s
c
τ
b
G_μ
Z_μ
W_μ
t
ν_τ
h


Scientific results
Relevant scientific literature
Dark Matter now?
Eureka
Matter now
Time



hep-ph, *astro-ph*, *hep-th*, *gr-qc*, *hep-ex*, *nucl-th*, *hep-lat*,





$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho_c \left[\Omega_M \left(\frac{a_0}{a}\right)^3 + \Omega_{rad} \left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda + \Omega_{curv} \left(\frac{a_0}{a}\right)^2\right]$$

$$\Omega_{\mathrm{DM}} h^2 = 0.1200 \pm 0.0012.$$

$$\Omega_{\mathrm{DM}} = 0.264 \pm 0.003,$$

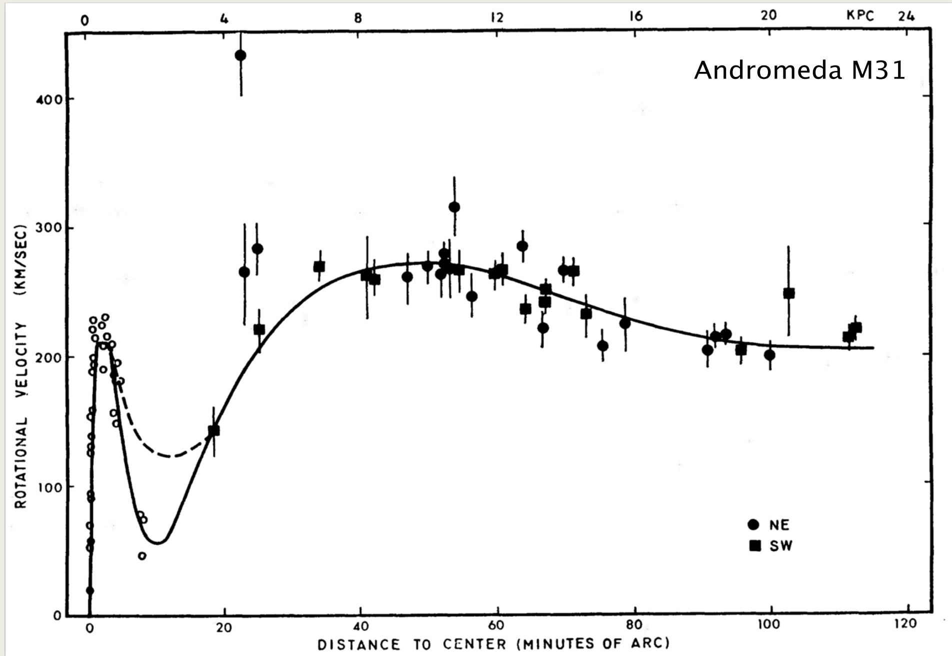
Characteristics

- Non-relativistic
- Cold
- Non-interacting and dissipationless
- Stable

Evidence for dark matter

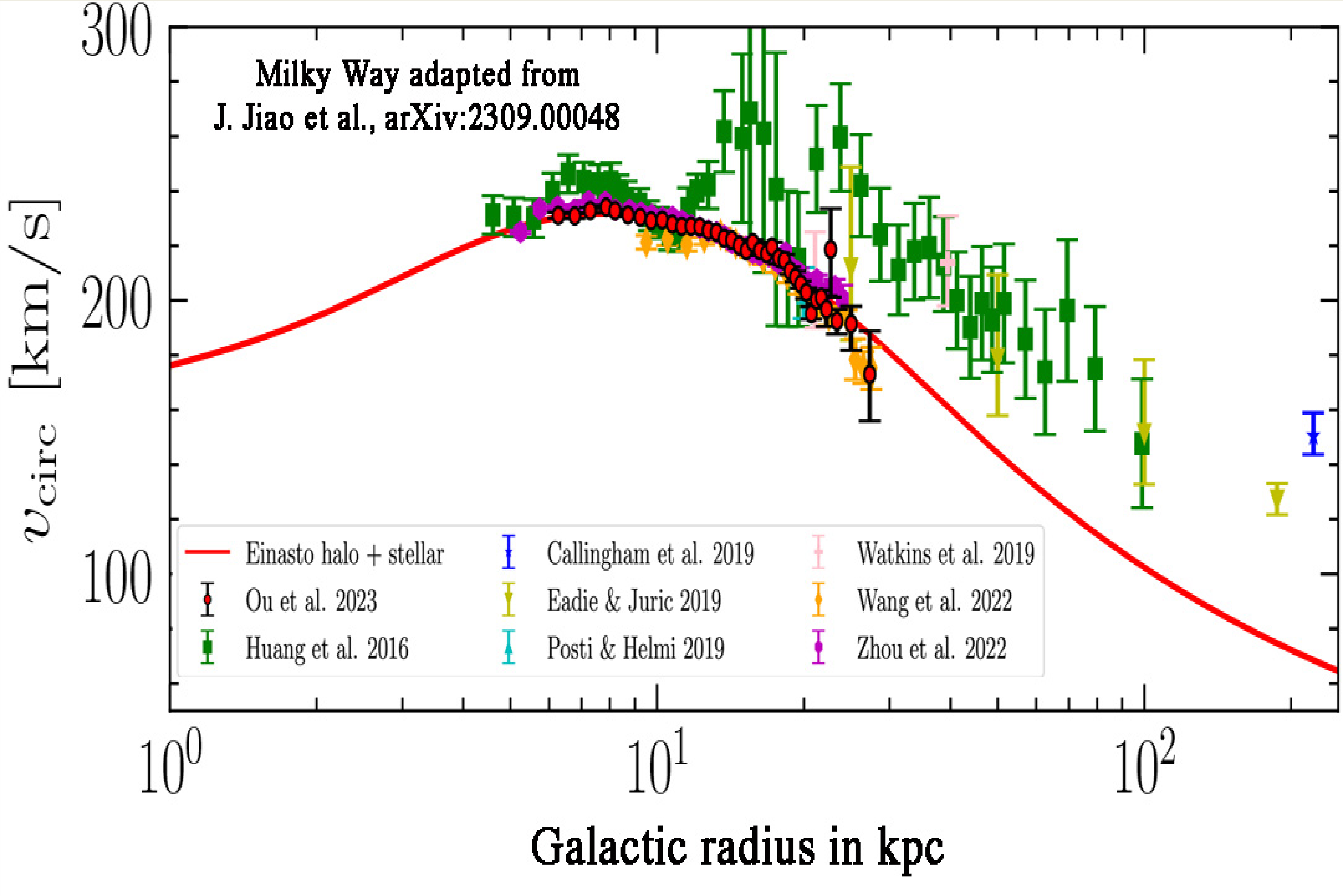
- Galaxy scale
- Galaxy cluster scale
- Cosmological scale

Rotation curve

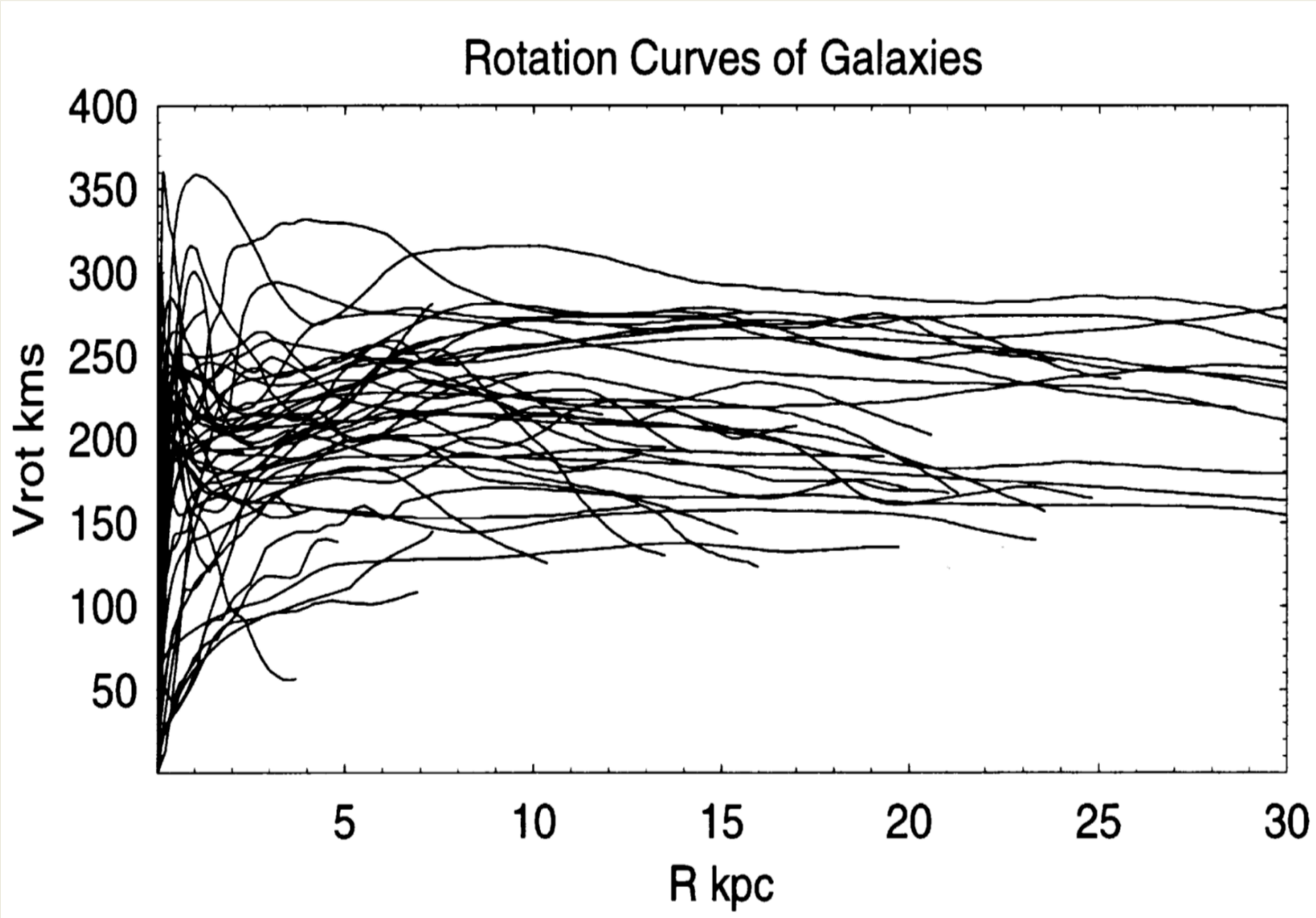


Rubin and Ford (1970).

Vera Rubin



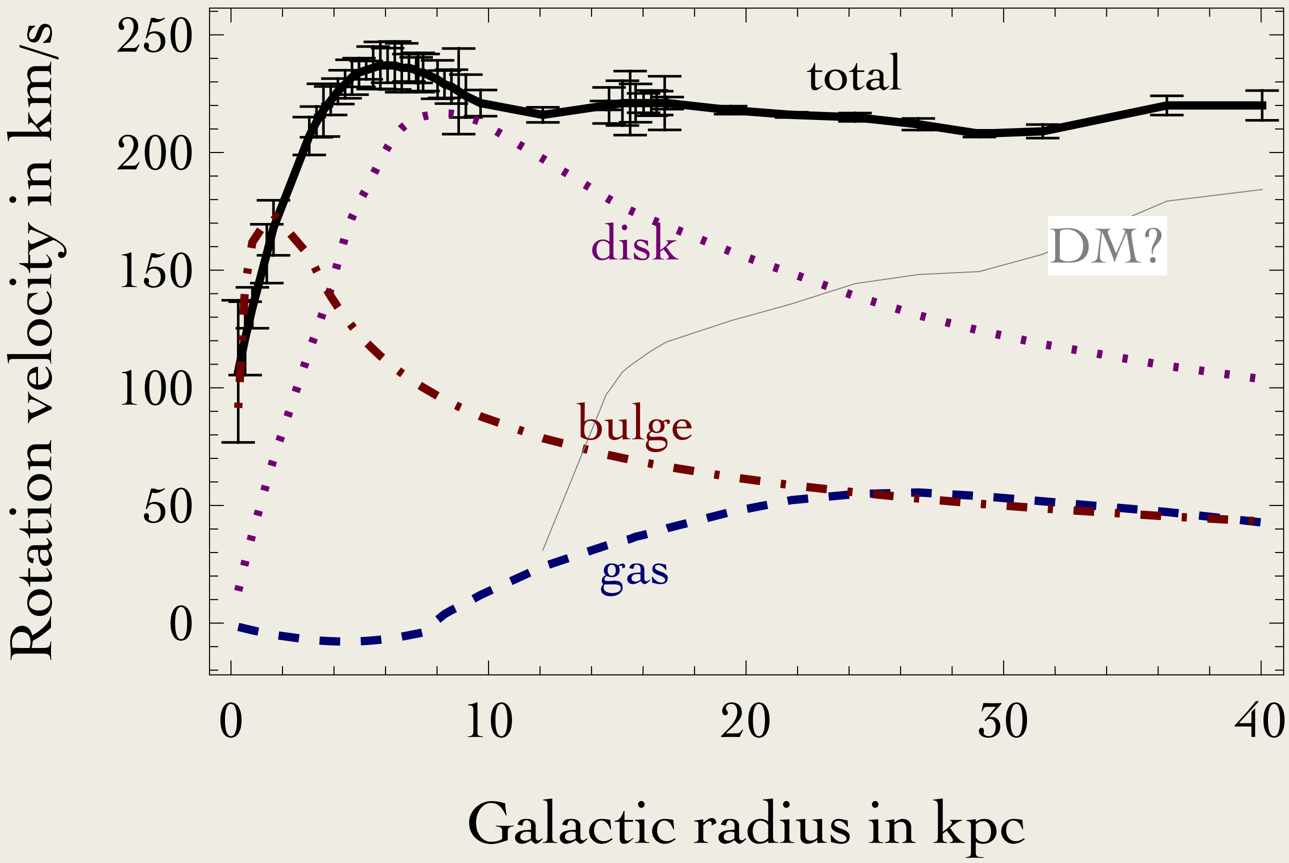
Milky Way adapted from
J. Jiao et al., arXiv:2309.00048
300
200
100
$v_{\rm circ}$ [km/s]
Einasto halo + stellar
Ou et al. 2023
Huang et al. 2016
Callingham et al. 2019
Eadie & Juric 2019
Posti & Helmi 2019
Watkins et al. 2019
Wang et al. 2022
Zhou et al. 2022
10^0
10^1
10^2
Galactic radius in kpc



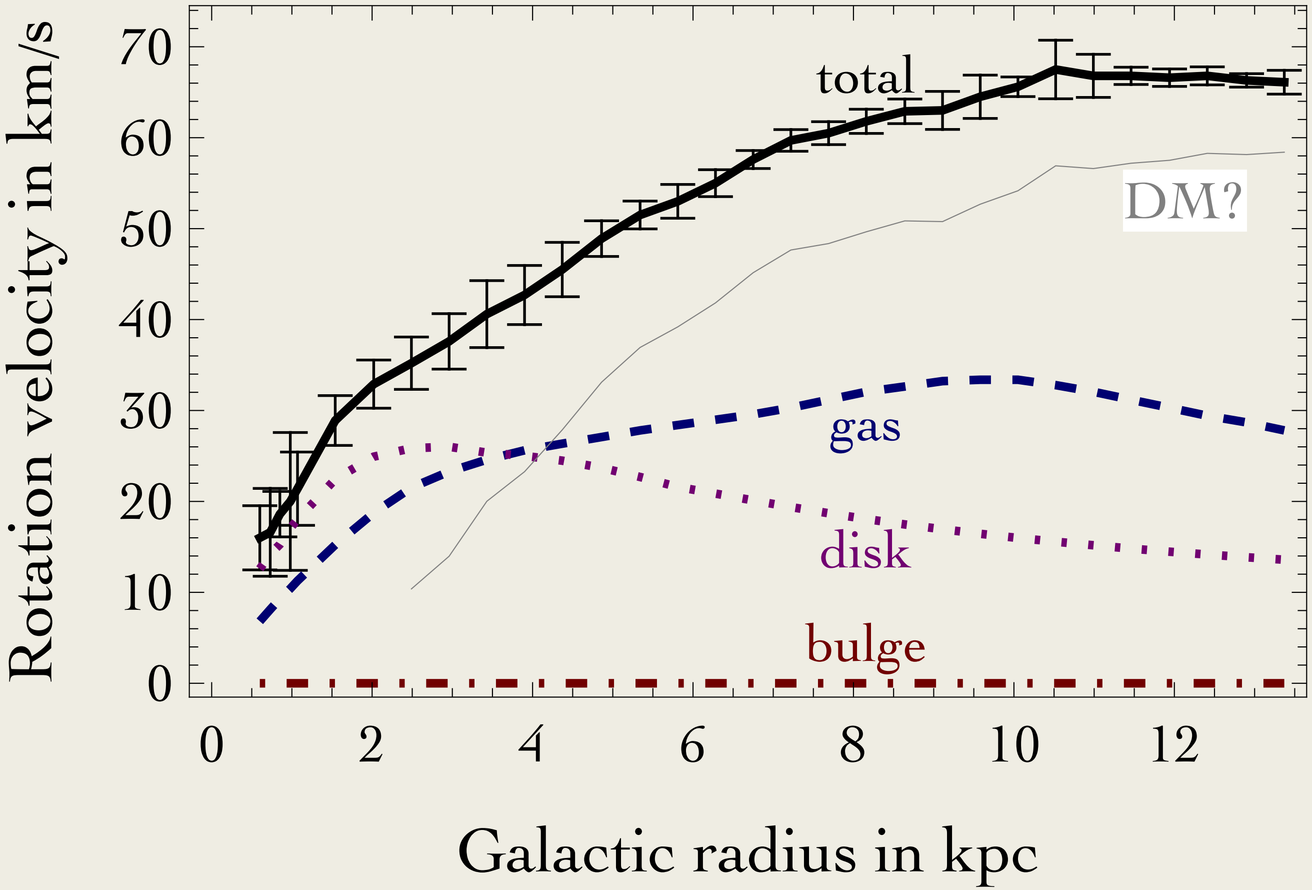
Rotation Curves of Galaxies
400
350
300
250
200
150
100
50
Vrot kms
5
10
15
20
25
30
R kpc

$$m\frac{v_{\rm circ}^2(r)}{r} = \frac{G\,m\mathcal{M}(r)}{r^2} \qquad \Rightarrow \qquad v_{\rm circ}(r) = \sqrt{\frac{G\mathcal{M}(r)}{r}}.$$

UGC03205 (denser, baryonic, more stars)



DDO161 (fainter, darker, more gas)

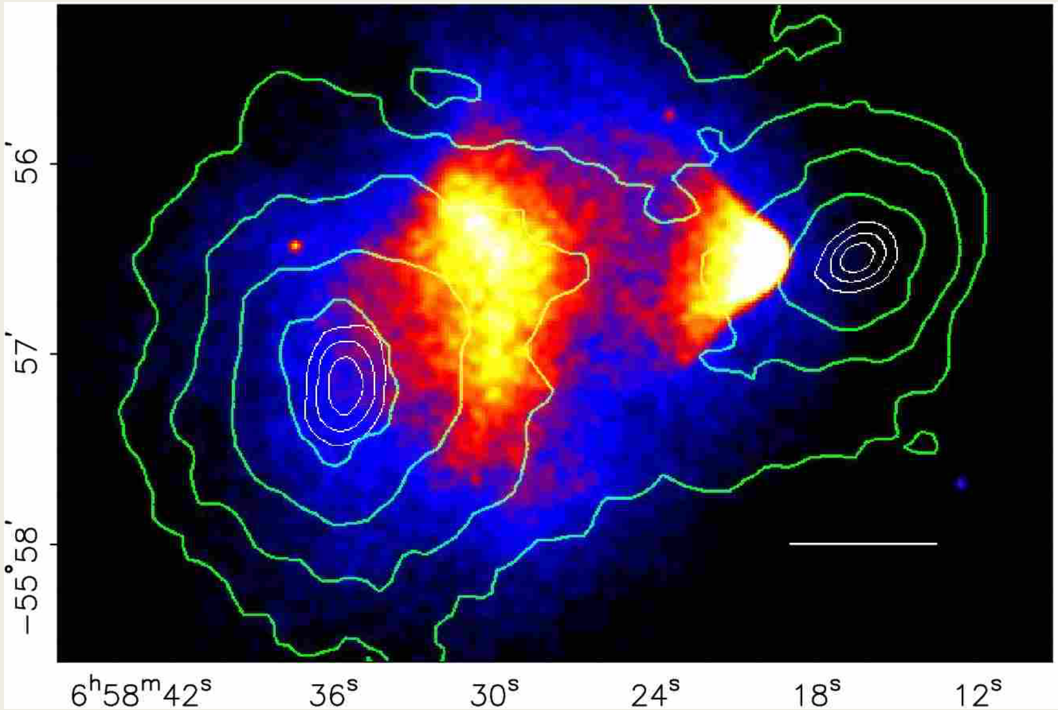


Midi: clusters of galaxies

velocity dispersion in the Coma cluster of galaxies,

in 1933.

Bullet cluster



bullet cluster located 3.7 Gyr

2006

The two objects collided 150 million years ago.

Harvey et al. (2015) [13] report the results on 72 of them and conclude that the existence of DM can be established with a significance of more than 7σ.

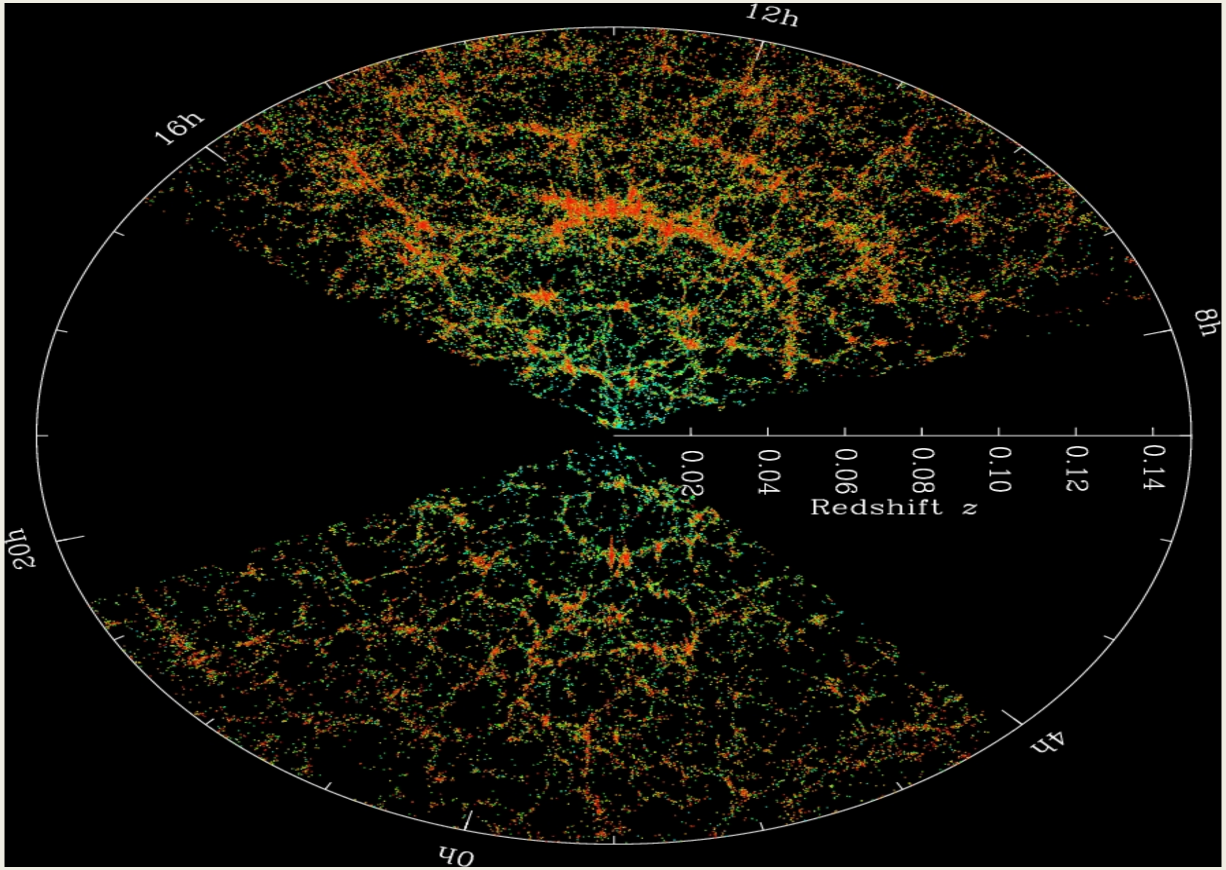
$$\frac{\sigma}{M} \lesssim 1\,\frac{\mathrm{cm}^2}{\mathrm{g}} = \frac{1.8\,\mathrm{mb}}{\mathrm{GeV}} = \frac{4580}{\mathrm{GeV}^3},$$

Cosmic Shear

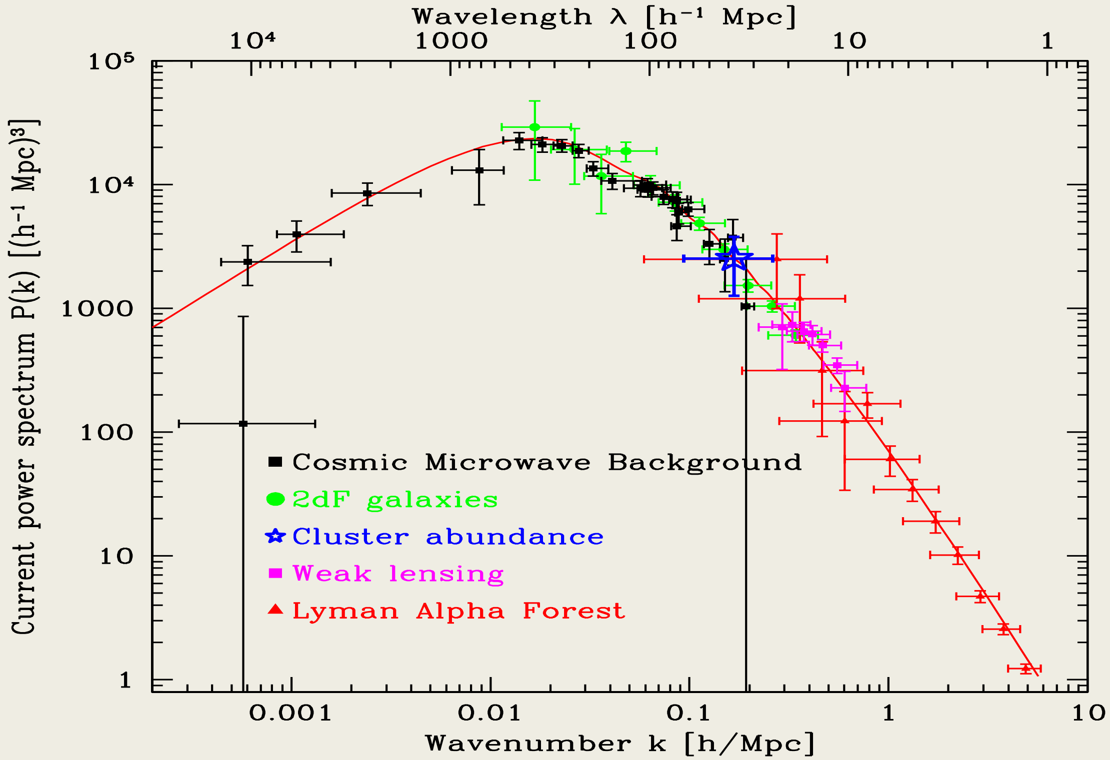
- Cosmic shear refers to the deflection of light from very distant galaxies by the gravitational attraction due to the foreground mass concentrations.

- vast filaments and loose clumps.

$$\Omega_{\mathrm{DM}} \approx 0.25.$$


12h
16h
8h
20h
4h
0h
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Redshift z

Matter power spectrum



$$\rho = \rho_0(t) + \rho_1(\boldsymbol{x}, t),$$

$$\delta(\boldsymbol{r}) \equiv \frac{\rho_1(\boldsymbol{r})}{\rho_0} = \int \frac{d^3k}{(2\pi)^3}\, \delta_k(t) \exp\left[-i\boldsymbol{k}\cdot\boldsymbol{x}\right], \qquad \text{where} \quad \boldsymbol{x} \equiv \frac{\boldsymbol{r}}{a(t)}.$$

$$\langle \delta_k \delta_{k'} \rangle = (2\pi)^3\, P(k)\, \delta^3(\boldsymbol{k} - \boldsymbol{k}'),$$

evolution ('Euler') equations

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0, & \text{continuity,} \\ \dfrac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = -\dfrac{\boldsymbol{\nabla} \wp}{\rho} - \boldsymbol{\nabla}\Phi, & \text{Newton law } \boldsymbol{a} = \boldsymbol{F}/m, \\ \nabla^2 \Phi = 4\pi G \rho, & \text{Poisson,} \end{cases}$$

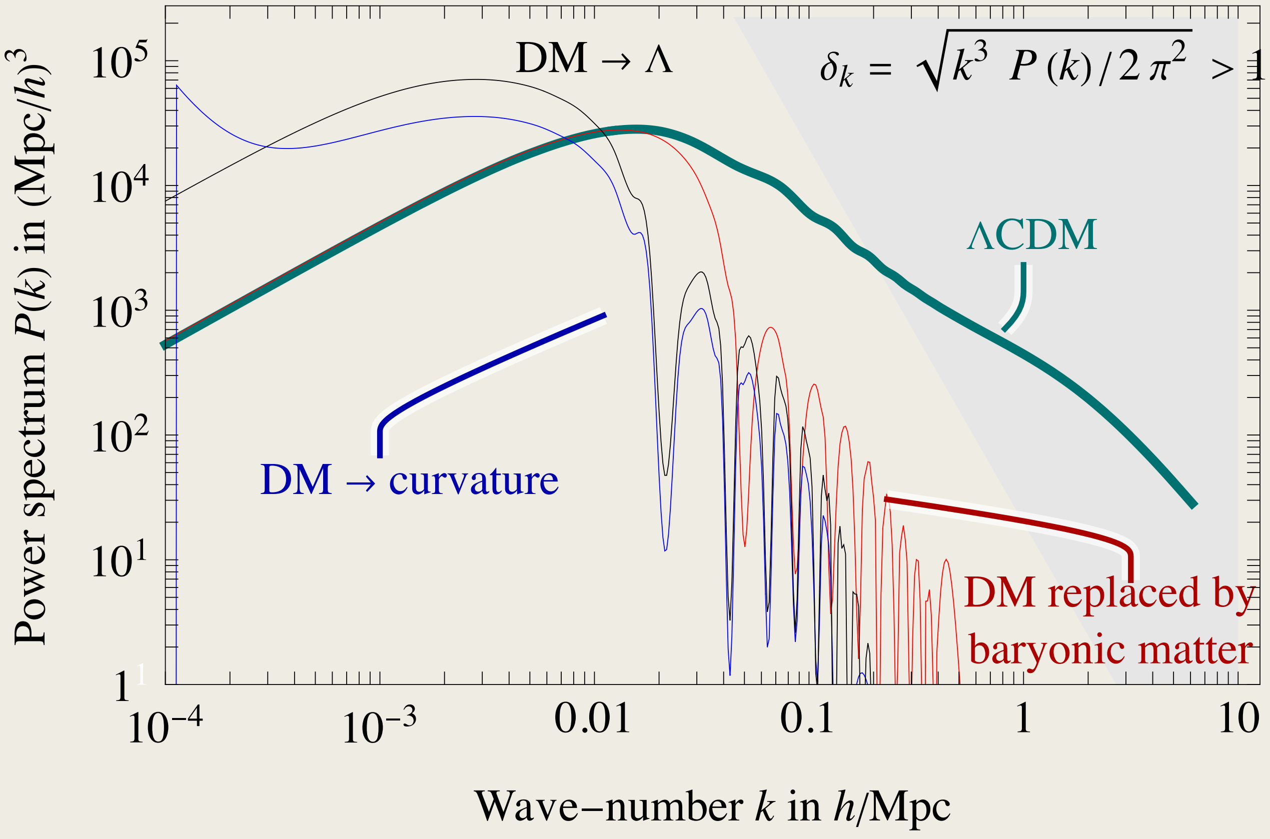
$$\rho = \rho_0(t) + \rho_1(\boldsymbol{x}, t), \qquad \wp = \wp_0 + \wp_1, \qquad \boldsymbol{v} = \boldsymbol{v}_0 + \boldsymbol{v}_1, \qquad \Phi = \Phi_0 + \Phi_1.$$

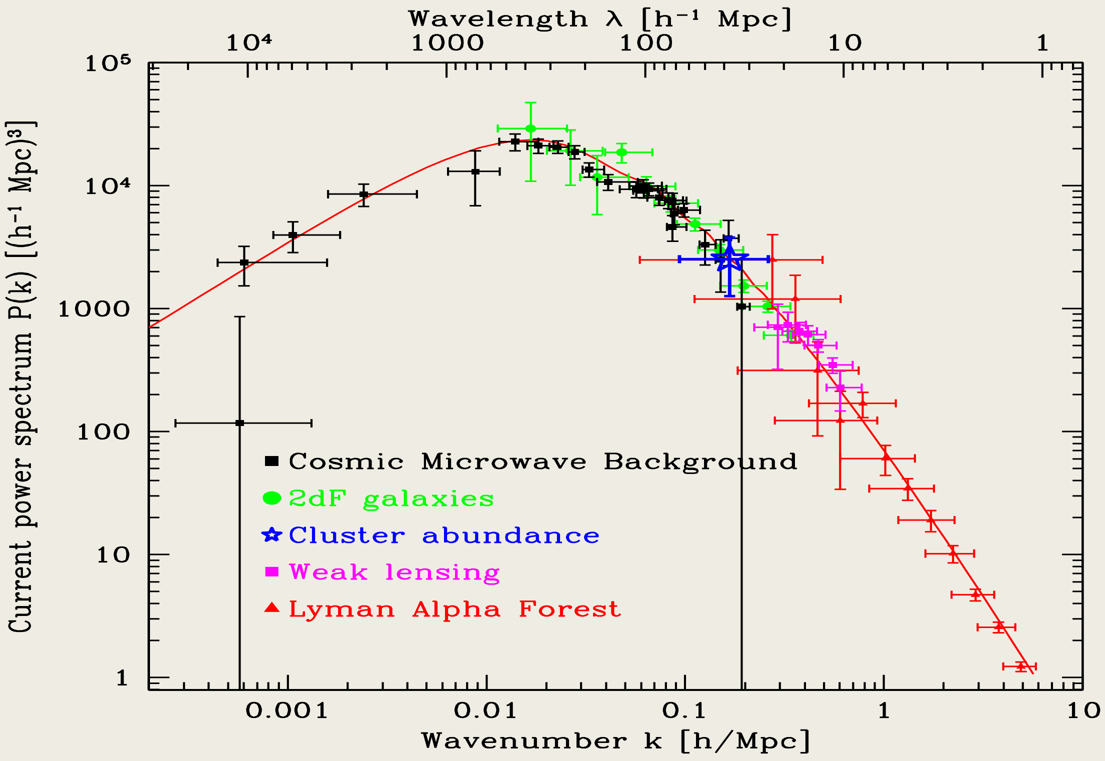
Linear regime:

$$v_s^2 = \partial \wp / \partial \rho = \wp_1 / \rho_1$$

$$\begin{cases} \dfrac{\partial \rho_1}{\partial t} + 3H\rho_1 + H(\boldsymbol{r} \cdot \boldsymbol{\nabla})\rho_1 + \rho_0 \boldsymbol{\nabla} \cdot \boldsymbol{v}_1 = 0, \\ \dfrac{\partial \boldsymbol{v}_1}{\partial t} + H\boldsymbol{v}_1 + H(\boldsymbol{r} \cdot \boldsymbol{\nabla})\boldsymbol{v}_1 + \dfrac{v_s^2}{\rho_0}\boldsymbol{\nabla}\rho_1 + \boldsymbol{\nabla}\Phi_1 = 0, & \text{(expanding Universe).} \\ \nabla^2 \Phi_1 = 4\pi G \rho_1, \end{cases}$$

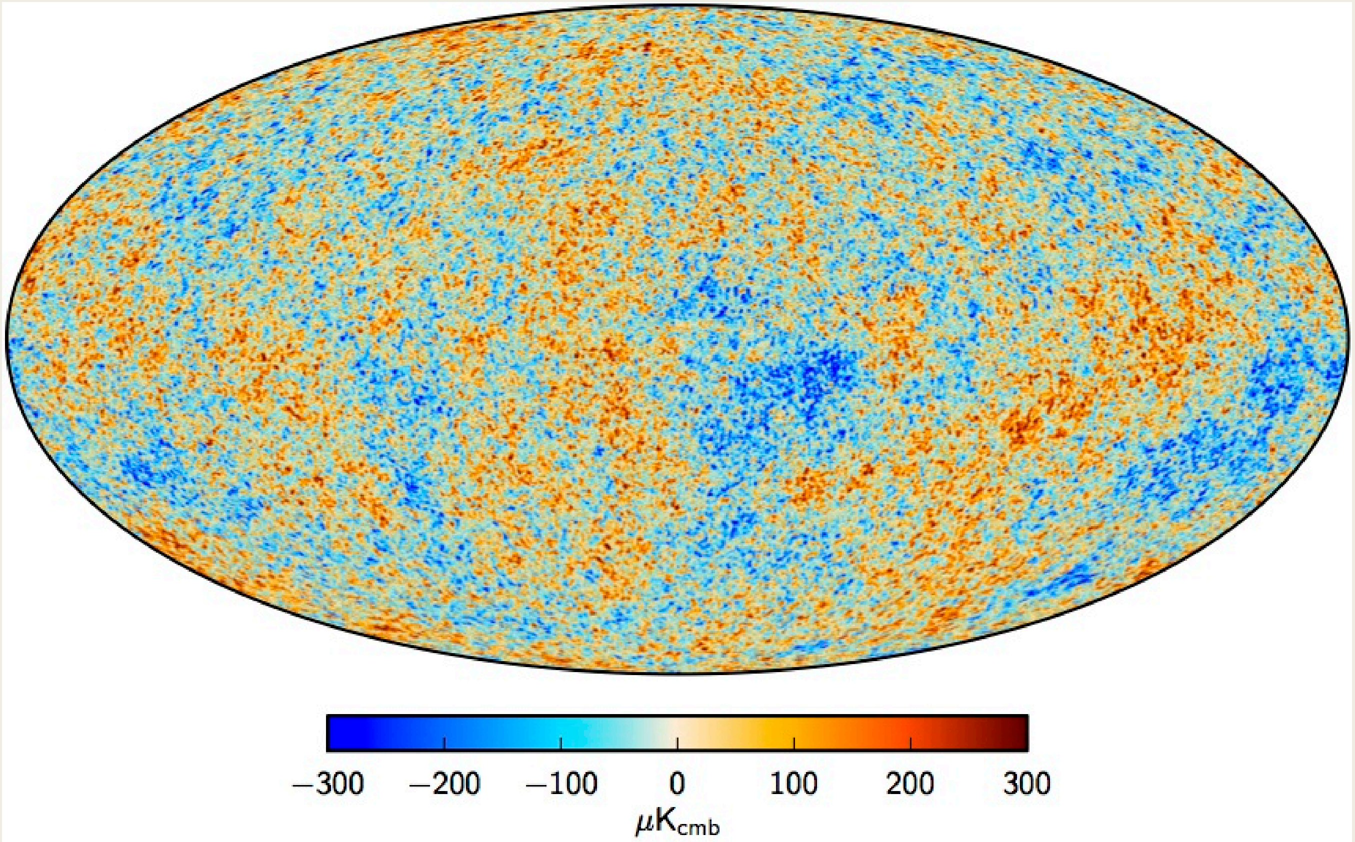
Matter power spectrum

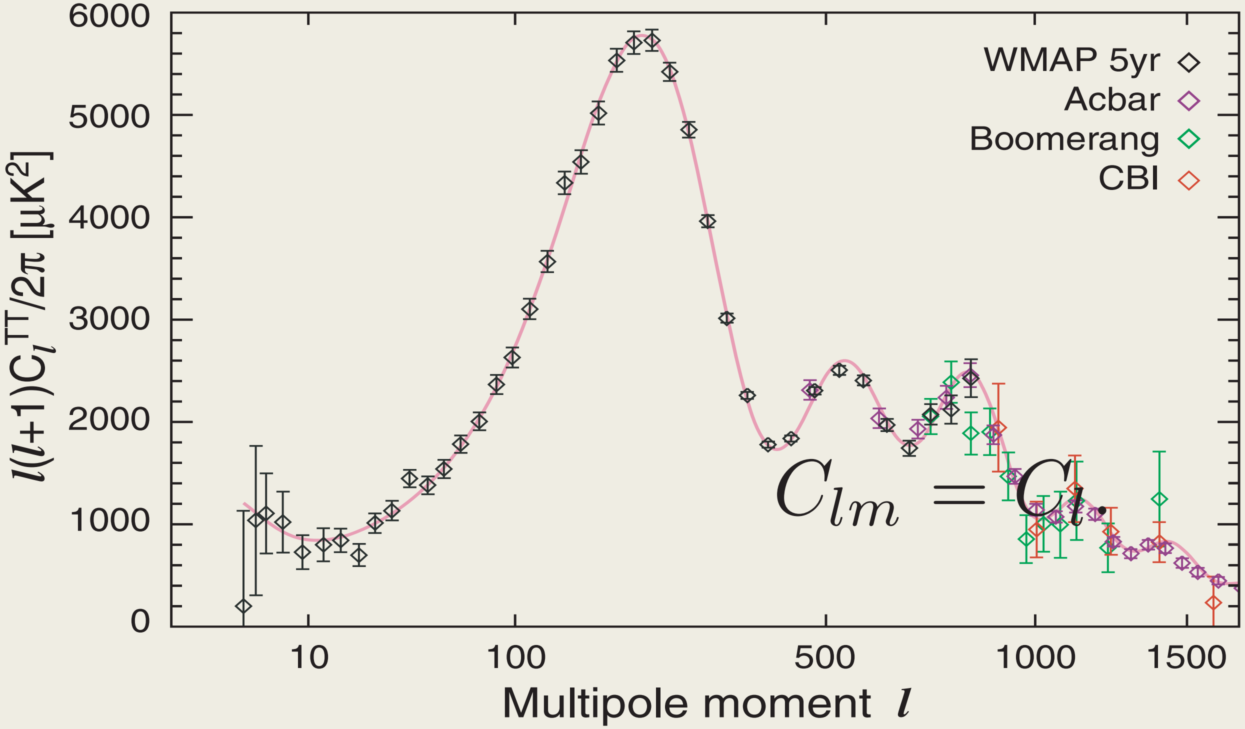




Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
10⁵
10⁴
1000
100
10
1
Current power spectrum P(k) [(h⁻¹ Mpc)³]
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]
Cosmic Microwave Background
2dF galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest

CMB

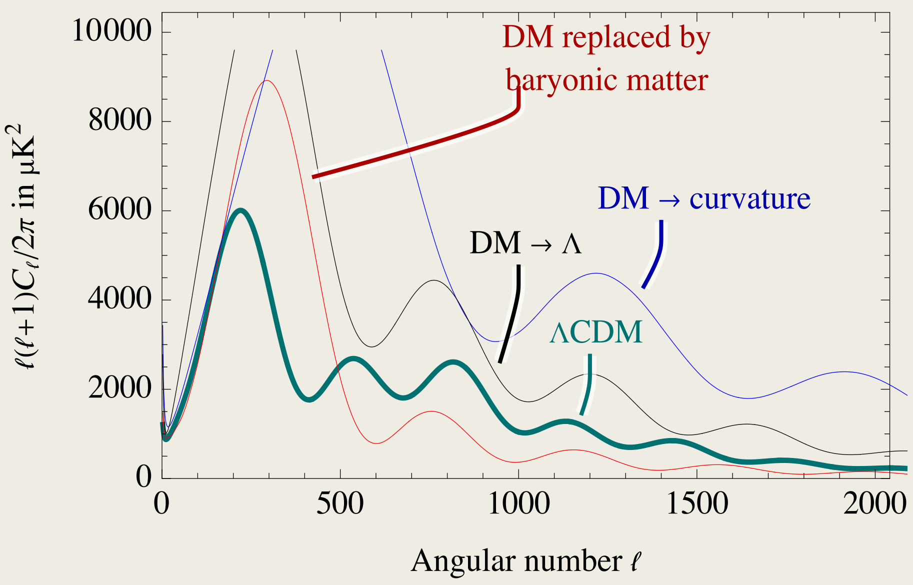


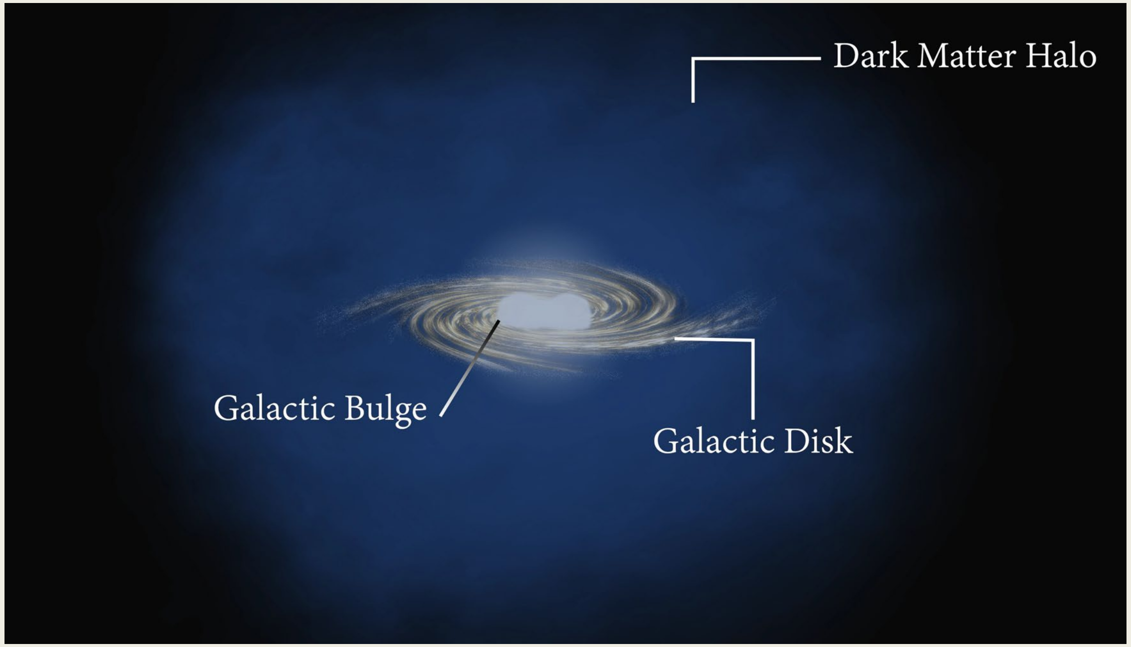


$$\delta T(\mathbf{n}) = T(\mathbf{n}) - T_0 - \delta T_{\rm dipole} \qquad \delta T(\mathbf{n}) = \sum a_{l,m} Y_{lm}(\mathbf{n}), \qquad \langle a_{l,m} a^*_{l',m'} \rangle = C_{lm} \delta_{ll'} \delta_{mm'},$$

$$C_{lm} = C_l. \qquad \langle \delta T(\mathbf{n_1}) \delta T(\mathbf{n_2}) \rangle = \sum_l \frac{2l+1}{4\pi} C_l P_l(\cos\theta),$$

CMB power spectrum

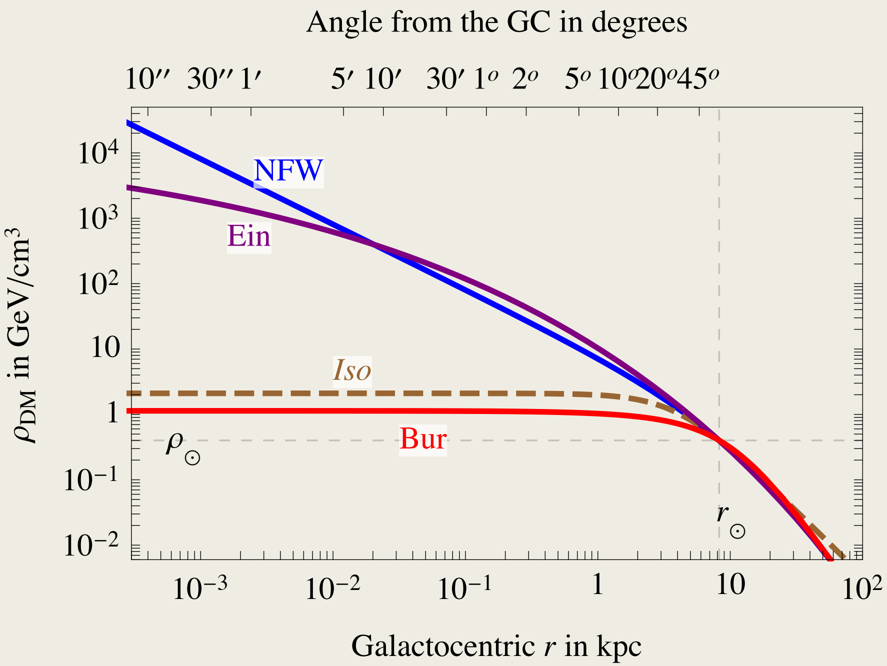



Dark Matter Halo
Galactic Bulge
Galactic Disk

DM profiles

DM halo	Functional form
NFW	$\rho_{\mathrm{NFW}}(r) = \rho_s \frac{r_s}{r} \left(1 + \frac{r}{r_s}\right)^{-2}$
Generalized NFW	$\rho_{\mathrm{gNFW}}(r) = \rho_s \left(\frac{r_s}{r}\right)^{\gamma} \left(1 + \frac{r}{r_s}\right)^{\gamma - 3}$
Einasto	$\rho_{\mathrm{Ein}}(r) = \rho_s \exp\left\{-\frac{2}{\alpha_{\mathrm{Ein}}}\left[\left(\frac{r}{r_s}\right)^{\alpha_{\mathrm{Ein}}} - 1\right]\right\}$
Cored Isothermal	$\rho_{\mathrm{Iso}}(r) = \frac{\rho_s}{1 + (r/r_s)^2}$
Burkert	$\rho_{\mathrm{Bur}}(r) = \frac{\rho_s}{(1 + r/r_s)(1 + (r/r_s)^2)}.$

Table 2.1: *Plausible spherical density profiles $\rho(r)$ for DM halos in galaxies.*



DM halo	r_s in kpc	ρ_s in GeV/cm^3
NFW	14.59	0.554
Einasto	13.76	0.150
Burkert	10.66	1.134
Isothermal	*4.00*	*2.100*

$$\rho_\odot = \rho(r_\odot) = 0.40\ \mathrm{GeV/cm^3} \approx 0.0106\, M_\odot/\mathrm{pc}^3 .$$

$$f(v) = N\, e^{-v^2/v_0^2}\ \Theta(v_{\rm esc} - v) .$$

$$v_{\rm esc} \approx (544 \pm 35)\,\mathrm{km/\,s} .$$

$$220\,\mathrm{km/s} < v_0 < 270\,\mathrm{km/s} .$$

Beyond the dark spherical (and isotropic) cow limit

Non-sphericity of DM halos

Rotating DM halos

Dark disk?

Anisotropic DM velocity distribution

DM streams

DM around black holes