Velocity-Modulation in Bilayer Graphene

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In memory of Prof. Malek Zareyan (1971-2014)

Workshop on "Quantum transport in graphene"

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Outline

- **Introduction:**
  - Bilayer Graphene: *Tight-binding model*
  - Fermi velocity modulation in graphene
- **Hamiltonian in presence of velocity modulation and gate bias:**
  - *Spectrum and band gap behavior*
- **Tunneling through velocity and electrostatic barriers:**
  - *Optical analogous of MDFs, Wave guide designed bilayer graphene*
- **Conclusion**

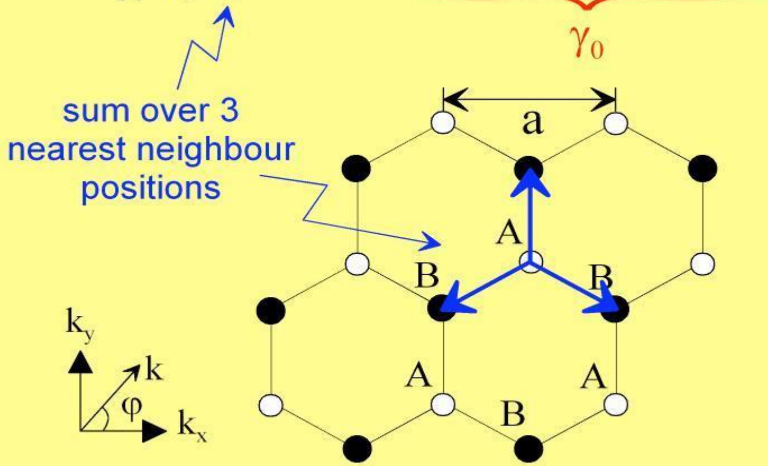
Monolayer Graphene

In the Tight Binding Model:wallace1947

Transfer integral on a hexagonal lattice

$$\mathcal{H}_{AB} = \langle \Phi_A | H | \Phi_B \rangle$$

$$\mathcal{H}_{AB} = \frac{1}{N} \sum_{\mathbf{R}_A}^{N} \sum_{\mathbf{R}_B}^{N} e^{i\mathbf{k}.(\mathbf{R}_B - \mathbf{R}_A)} \underbrace{\langle \phi_A(\mathbf{r}-\mathbf{R}_A) | H | \phi_B(\mathbf{r}-\mathbf{R}_B) \rangle}_{\gamma_0}$$



$$\mathcal{H}_{AB} = -\gamma_0 f(\mathbf{k}) \; ; \quad \mathcal{H}_{BA} = -\gamma_0 f^*(\mathbf{k})$$

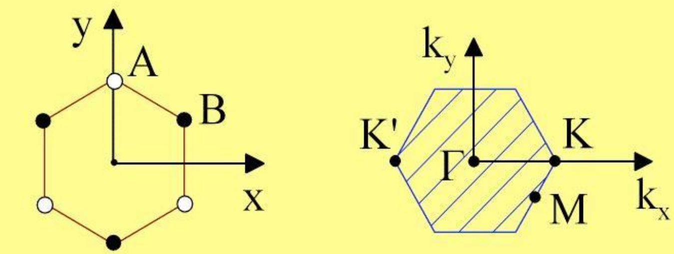
$$f(\mathbf{k}) = e^{ik_y a/\sqrt{3}} + 2e^{-ik_y a/2\sqrt{3}} \cos(k_x a/2)$$

Two valleys have opposite chirality

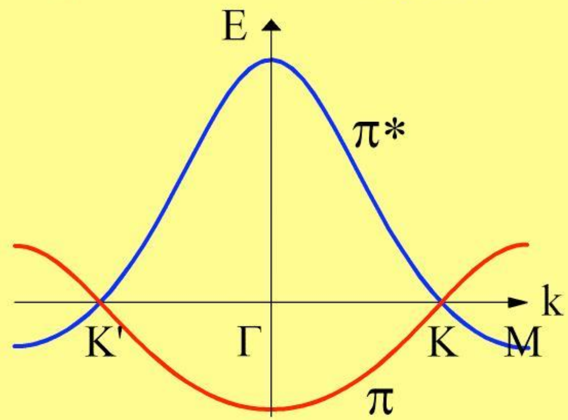
Electronic dispersion of a monolayer

Saito *et al*, "Physical Properties of Carbon Nanotubes" (Imperial College Press, London, 1998)

Symmetrical unit cell | Brillouin zone



Two non-equivalent carbon positions | Two non-equivalent K-points

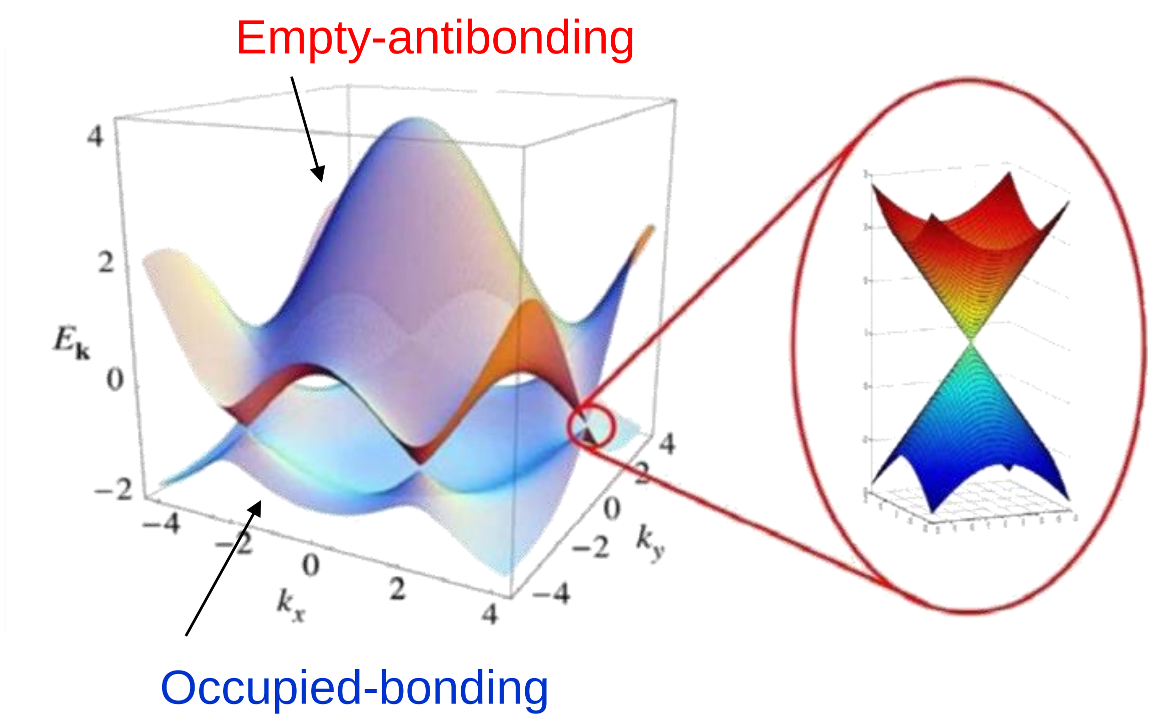


Two bands: no energy gap at the K-points

Spectrum of Monolayer Graphene

[illegible]

$$E_{\pm}(k) = \pm\hbar v_F k + O(k^2)$$



Bilayer Graphene: the Tight Binding model

E. Mccann, et al. 2013 Rep. Prog. Phys. 76 056503

$$H_b = \begin{pmatrix} \epsilon_{A1} & -\gamma_0 f(\mathbf{k}) & \gamma_4 f(\mathbf{k}) & -\gamma_3 f^*(\mathbf{k}) \\ -\gamma_0 f^*(\mathbf{k}) & \epsilon_{B1} & \gamma_1 & \gamma_4 f(\mathbf{k}) \\ \gamma_4 f^*(\mathbf{k}) & \gamma_1 & \epsilon_{A2} & -\gamma_0 f(\mathbf{k}) \\ -\gamma_3 f(\mathbf{k}) & \gamma_4 f^*(\mathbf{k}) & -\gamma_0 f^*(\mathbf{k}) & \epsilon_{B2} \end{pmatrix}$$

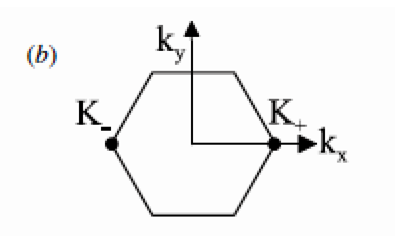
$$\gamma_0 = -\langle \varphi_{A1} | H | \varphi_{B1} \rangle = -\langle \varphi_{A2} | H | \varphi_{B2} \rangle$$

$$\gamma_1 = \langle \varphi_{A2} | H | \varphi_{B1} \rangle$$

$$\gamma_3 = \langle \varphi_{A1} | H | \varphi_{B2} \rangle$$

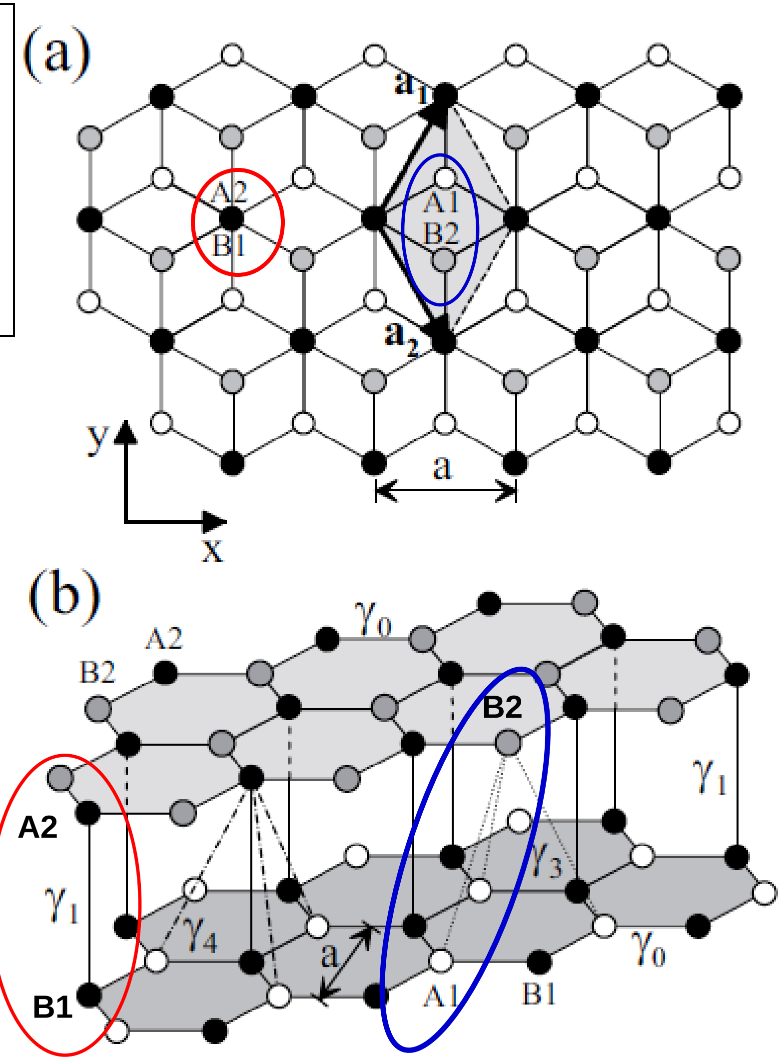
$$\gamma_4 = \langle \varphi_{A1} | H | \varphi_{A2} \rangle = \langle \varphi_{B1} | H | \varphi_{B2} \rangle$$

$$f(\mathbf{k}) = \sum_{l=1}^{3} e^{i\mathbf{k}.\delta_l}$$



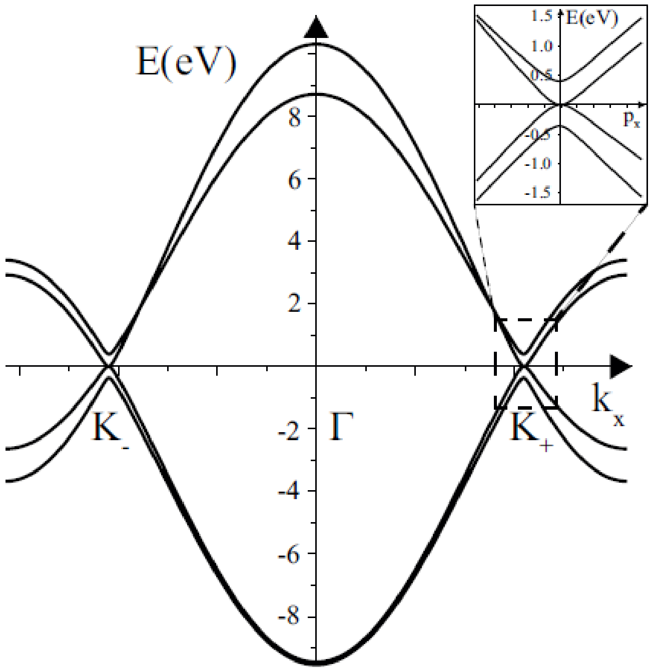
Dimer sites:B1-A2 γ_1

Non-Dimer sites:A1-B2 γ_3



Bilayer Graphene: the Tight Binding model

E. Mccann, et al. 2013 Rep. Prog. Phys. 76 056503



- A bonding and anti-bonding pair arising from the strong coupling of the dimer B1 and A2 sites.
- The '*low-energy*' bands arise from hopping between the non-dimer A1 and B2 sites.

$$\varepsilon \approx p^2/2m \qquad m = \gamma_1/2v^2$$

$$\mathbf{p} = \hbar\mathbf{k} - \hbar\mathbf{K}_\xi$$

$$f(\mathbf{k}) \approx -\sqrt{3}a(\xi p_x - ip_y)/2\hbar$$

Parameter	Graphite [48]	Bilayer [57]	Bilayer [36]	Bilayer [37]	Bilayer [61]	Trilayer [63]
γ_0	3.16(5)	2.9	3.0[a]	-	3.16(3)	3.1[a]
γ_1	0.39(1)	0.30	0.40(1)	0.404(10)	0.381(3)	0.39[a]
γ_2	-0.020(2)	-	-	-	-	-0.028(4)
γ_3	0.315(15)	0.10	0.3[a]	-	0.38(6)	0.315[a]
γ_4	0.044(24)	0.12	0.15(4)	-	0.14(3)	0.041(10)

Effective 4-band Hamiltonian at low energy

$$H_b = \begin{pmatrix} \epsilon_{A1} & v\pi^\dagger & -v_4\pi^\dagger & v_3\pi \\ v\pi & \epsilon_{B1} & \gamma_1 & -v_4\pi^\dagger \\ -v_4\pi & \gamma_1 & \epsilon_{A2} & v\pi^\dagger \\ v_3\pi^\dagger & -v_4\pi & v\pi & \epsilon_{B2} \end{pmatrix}$$

Neglecting $\gamma_4 = 0$ $E = \pm\varepsilon_\alpha(\mathrm{p}),\ \alpha = 1, 2$

$$\varepsilon_\alpha^2 = \frac{\gamma_1^2}{2} + \frac{U^2}{4} + \left(v^2 + \frac{v_3^2}{2}\right)p^2 + (-1)^\alpha\sqrt{\Gamma},$$

$$\Gamma = \tfrac{1}{4}\left(\gamma_1^2 - v_3^2p^2\right)^2 + v^2p^2\left[\gamma_1^2 + U^2 + v_3^2p^2\right] + 2\xi\gamma_1 v_3 v^2 p^3 \cos 3\varphi\,,$$

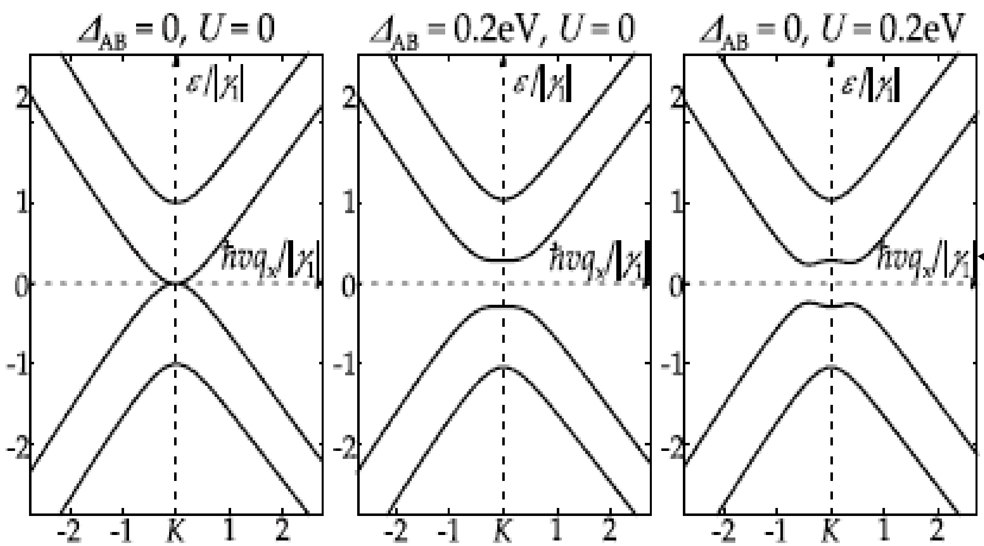
$$\pi = \xi p_x + ip_y\,, \xi = \pm 1 \ \ \&\ \ v = \sqrt{3}a\gamma_0 / 2\hbar$$

$$v_3 = \sqrt{3}a\gamma_3 / 2\hbar \ \ \&\ \ v_4 = \sqrt{3}a\gamma_4 / 2\hbar$$

$$U = \tfrac{1}{2}\left[(\epsilon_{A1} + \epsilon_{B1}) - (\epsilon_{A2} + \epsilon_{B2})\right]$$

$$\Delta' = \tfrac{1}{2}\left[(\epsilon_{B1} + \epsilon_{A2}) - (\epsilon_{A1} + \epsilon_{B2})\right]$$

$$\delta_{AB} = \tfrac{1}{2}\left[(\epsilon_{A1} + \epsilon_{A2}) - (\epsilon_{B1} + \epsilon_{B2})\right]$$



Mexican hat structure

M. Mucha-Kruczynski M, et al. 2010
Semicond. Sci. Technol, 25,033001

Effective two band model for non-dimer sites

$$\gamma_0, \gamma_1 \gg |E|, vp, |\gamma_3|, |\gamma_4|, |U|, |\Delta'|, |\delta_{AB}|$$

$$\hat{H}_2 = \hat{h}_0 + \hat{h}_w + \hat{h}_4 + \hat{h}_\Delta + \hat{h}_U + \hat{h}_{AB},$$

$$\hat{h}_0 = -\frac{1}{2m}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix},$$

→ Massive Chiral electrons with parabolic dispersion

$$\hat{h}_w = v_3 \begin{pmatrix} 0 & \pi \\ \pi^\dagger & 0 \end{pmatrix} - \frac{v_3 a}{4\sqrt{3}\hbar}\begin{pmatrix} 0 & (\pi^\dagger)^2 \\ \pi^2 & 0 \end{pmatrix},$$

Trigonal Warping

→ $|E| < \varepsilon_L$

$$\hat{h}_4 = \frac{2vv_4}{\gamma_1}\begin{pmatrix} \pi^\dagger\pi & 0 \\ 0 & \pi\pi^\dagger \end{pmatrix},$$

$\hat{h}_0 + \hat{h}_4$ → $E = \pm(p^2/2m)(1 \pm 2v_4/v)$

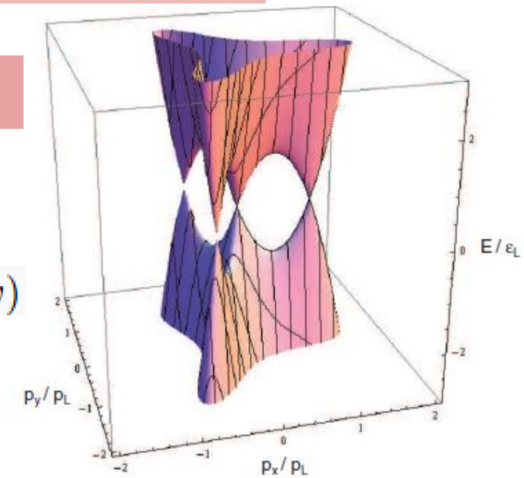
$$\hat{h}_\Delta = \frac{\Delta' v^2}{\gamma_1^2}\begin{pmatrix} \pi^\dagger\pi & 0 \\ 0 & \pi\pi^\dagger \end{pmatrix},$$

} Electron-hole asymmetry

$$\hat{h}_U = -\frac{U}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \frac{2v^2}{\gamma_1^2}\begin{pmatrix} \pi^\dagger\pi & 0 \\ 0 & -\pi\pi^\dagger \end{pmatrix}\right]$$

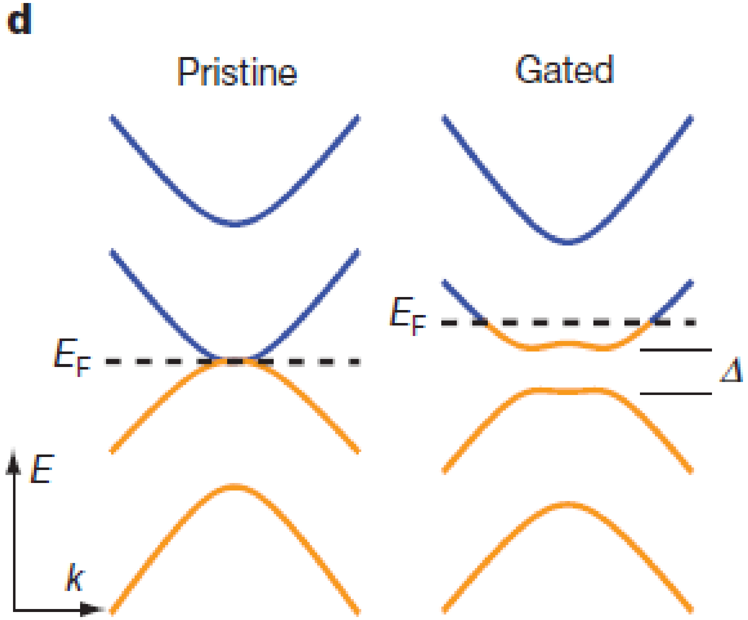
$$\hat{h}_{AB} = \frac{\delta_{AB}}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

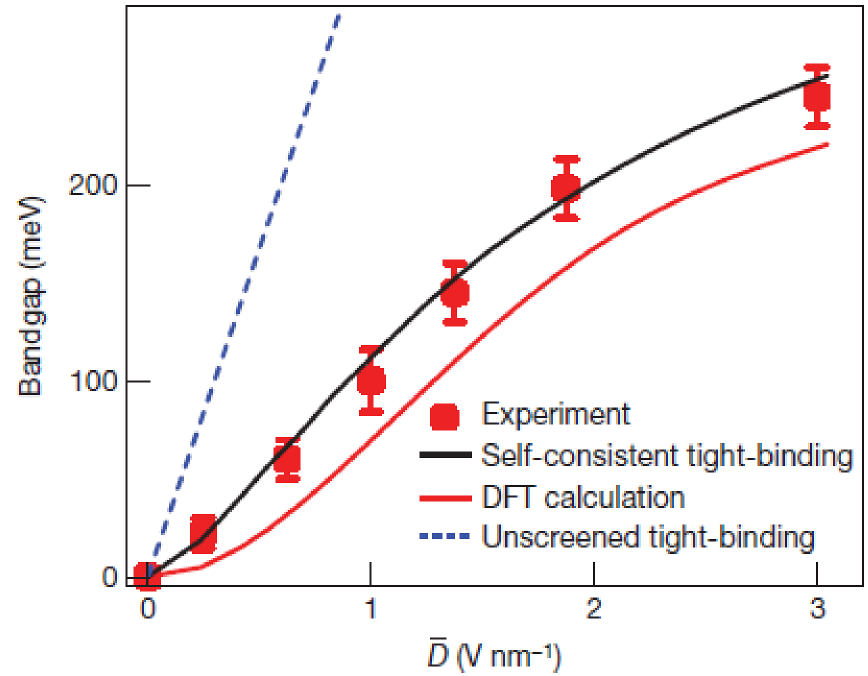
} Production of a Band Gap

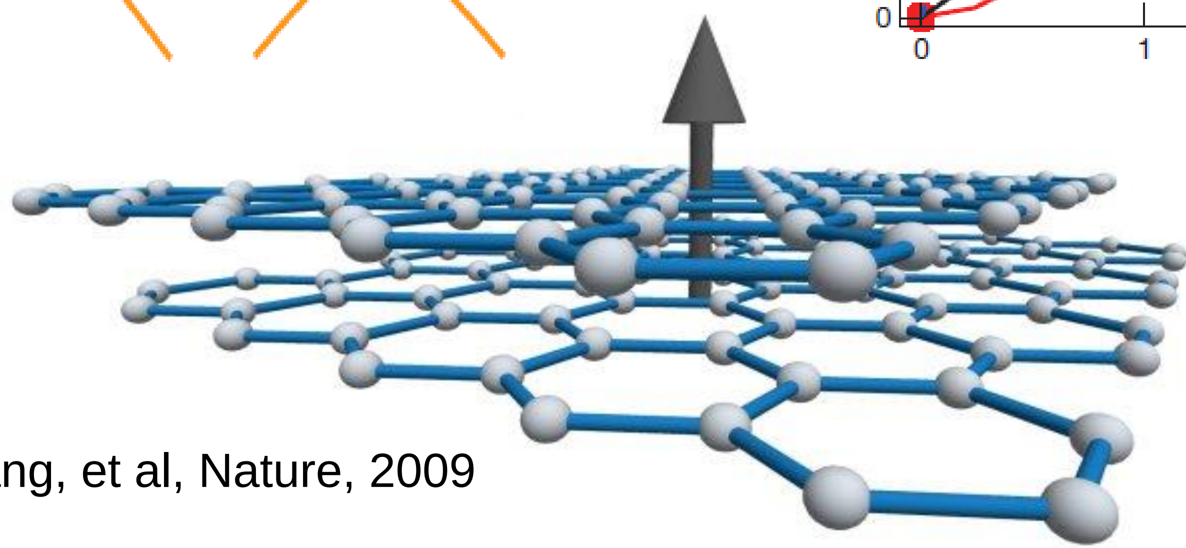


E. Mccann, et al. 2013 Rep. Prog. Phys. 76 056503

Tunable Band gap in Bilayer Graphene





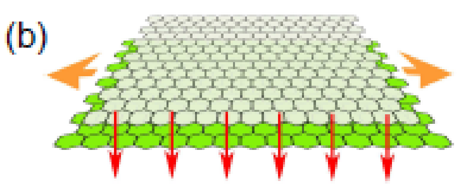


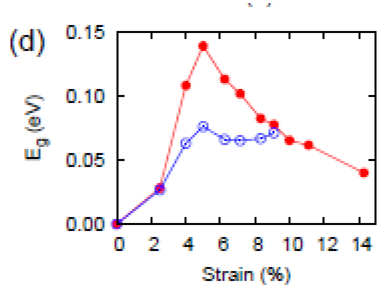
Y. Zhang, et al, Nature, 2009

Velocity Modulation in Graphene

There are several ways to engineer Fermi velocity :

- e-e interaction
- Modifications in curvature of graphene sheet
- Periodic potential (Graphene superlattices)
- Appropriate doping
- Dielectric screening
- Strain





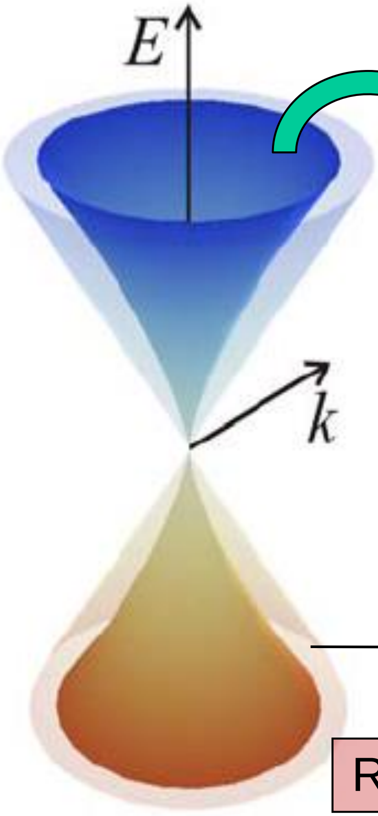
Nano Lett., **2010**, *10* (9), pp 3486–3489

$$v_{ij} = v_0 \left[\eta_{ij} + \frac{\beta}{4}(2u_{ij} + \eta_{ij} u_{kk}) \right],$$

Phys. Rev. Lett. 108, 227205 (2012)

Dirac cones reshaped by interaction effects in suspended graphene

Elias D C et al 2011 Nature Phys. 7 701



Many-body Spectrum

$$v_F(n) = v_F(n_0)\left[1 + \frac{\alpha}{8\varepsilon_G}\ln(n_0/n)\right]$$

Linear Spectrum

$$v_F \to 3\times10^6\ m/s \ \text{ at } \ n < 10^{10}$$

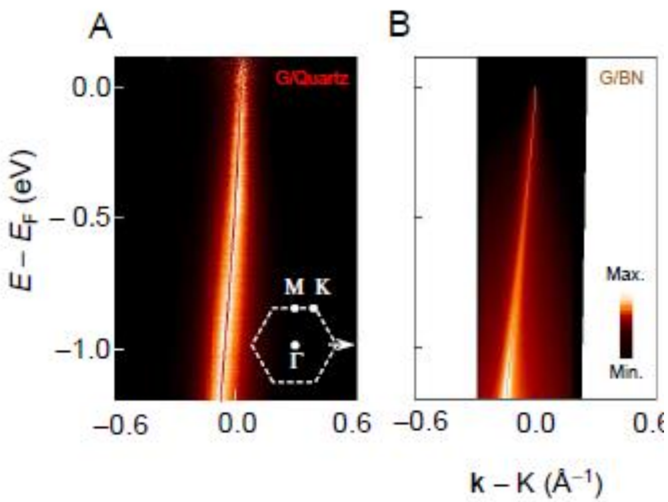
Renormalization Group Theory confirms experimental results

Non-local interband exchange leads to a renormalized Fermi velocity (G. Borghi, et al, SSC, 2009)

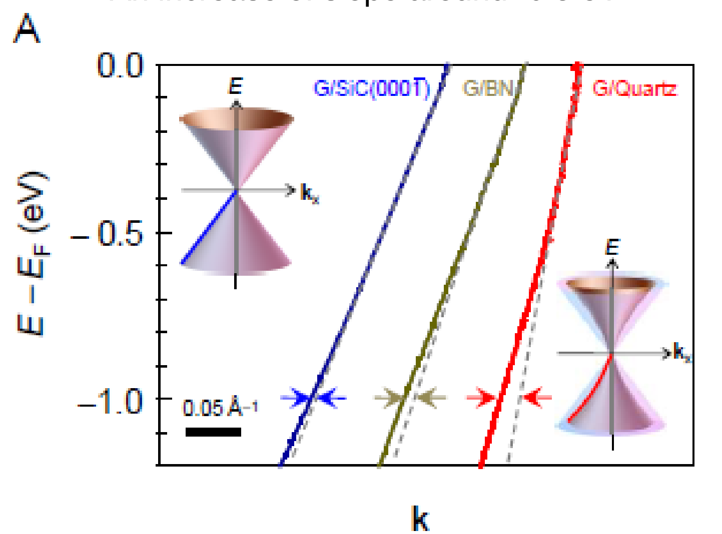
Weak interaction ⟶ $v_F \to 0.85\times10^6\ m/s$ P. E. Trevisanutto, et al, PRL, 2008

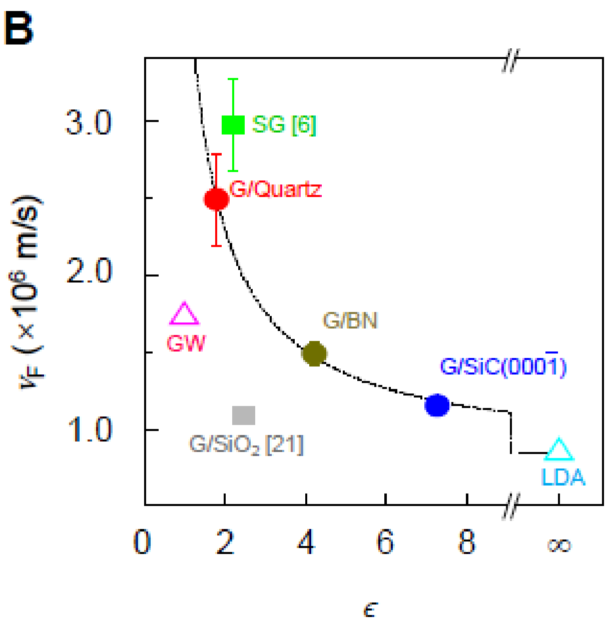
Strong interaction ⟶ $v_F \to 1.73\times10^6\ m/s$ C-H Park, et al, Nanolett. 2009

Fermi velocity engineering by substrate modification



An increase of slope around -0.5 eV

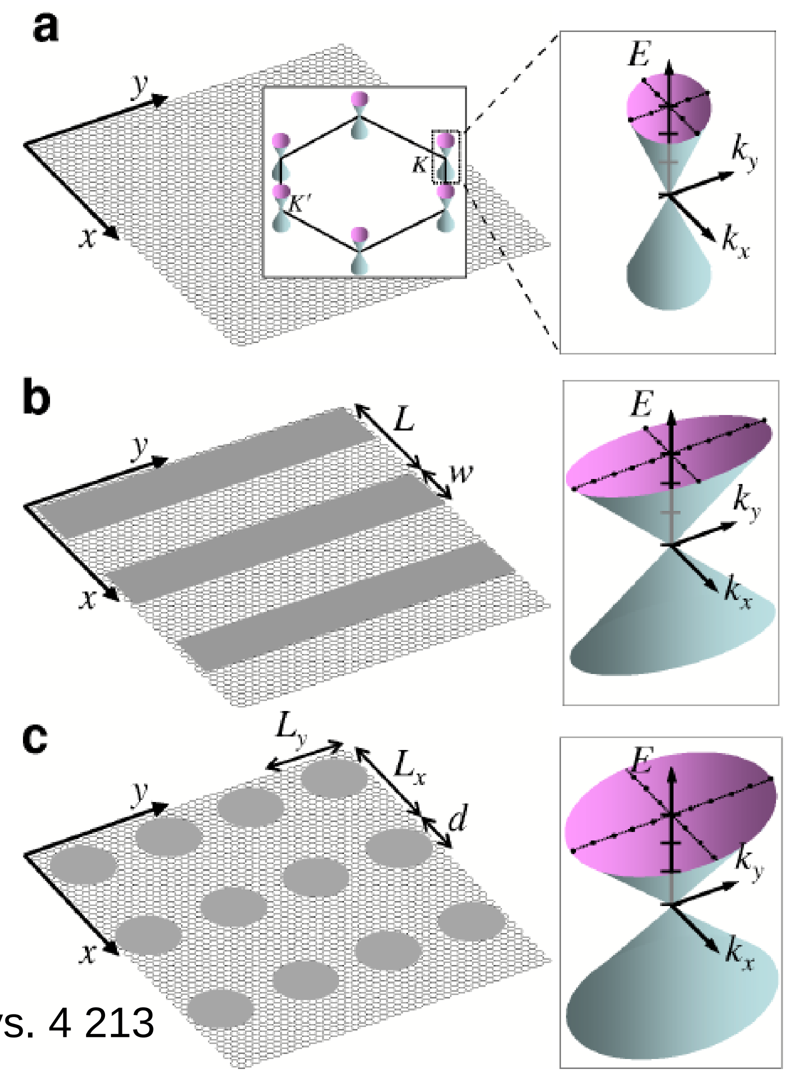




Substrate	$v_F \times 10^6$ m/s	ϵ	α
Metals (LDA)	0.85	∞	-
SiC(000$\bar{1}$)	1.15±0.02	7.26±0.02	0.35
h-BN	1.49±0.08	4.22±0.01	0.61
Quartz	2.49±0.30	1.80±0.02	1.43

C. Hwang, et al, Nature *Scientific Reports, 590 (2012)*

Anisotropic behaviors of massless Dirac fermions in graphene under periodic potential



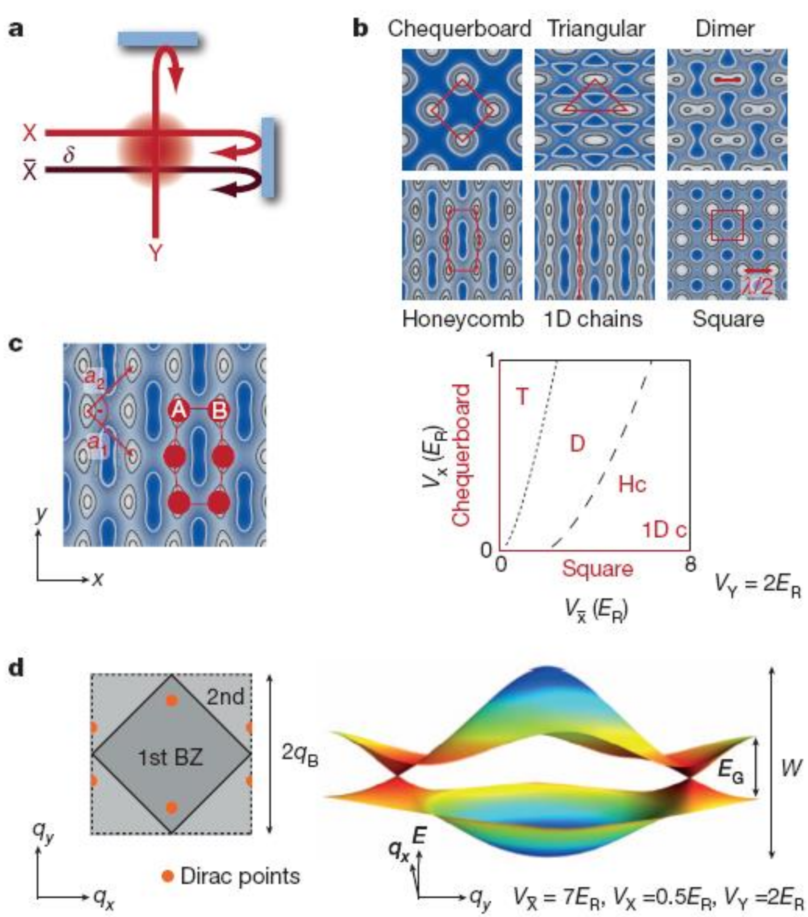
Park C H et al 2008 Nature Phys. 4 213

LETTER

doi:10.1038/nature10871

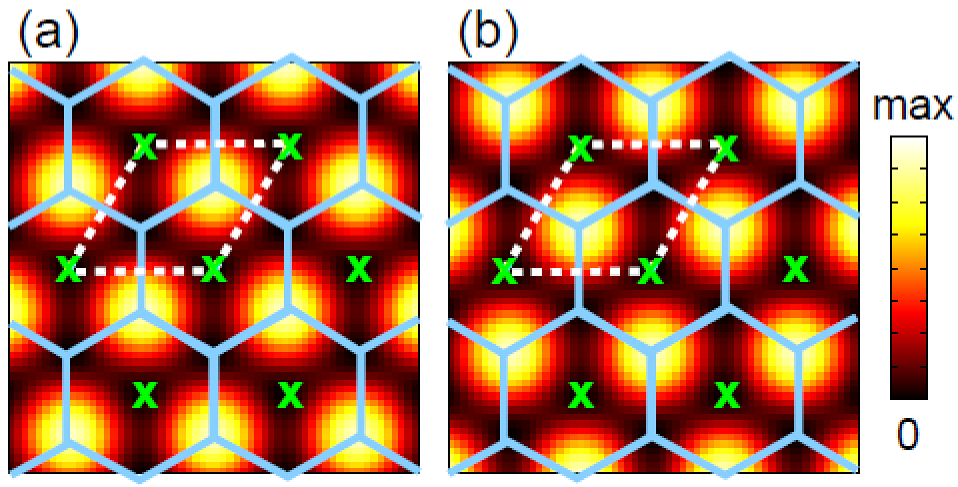
Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice

Leticia Tarruell[1], Daniel Greif[1], Thomas Uehlinger[1], Gregor Jotzu[1] & Tilman Esslinger[1]



Making massless Dirac fermions from patterned 2DEG

Park C H and Louie S G 2009 *Nano Lett.* 9 1793



$$M = \hbar\frac{v_0}{2}\left(k_x\sigma_x + k_y\sigma_y\right) .$$

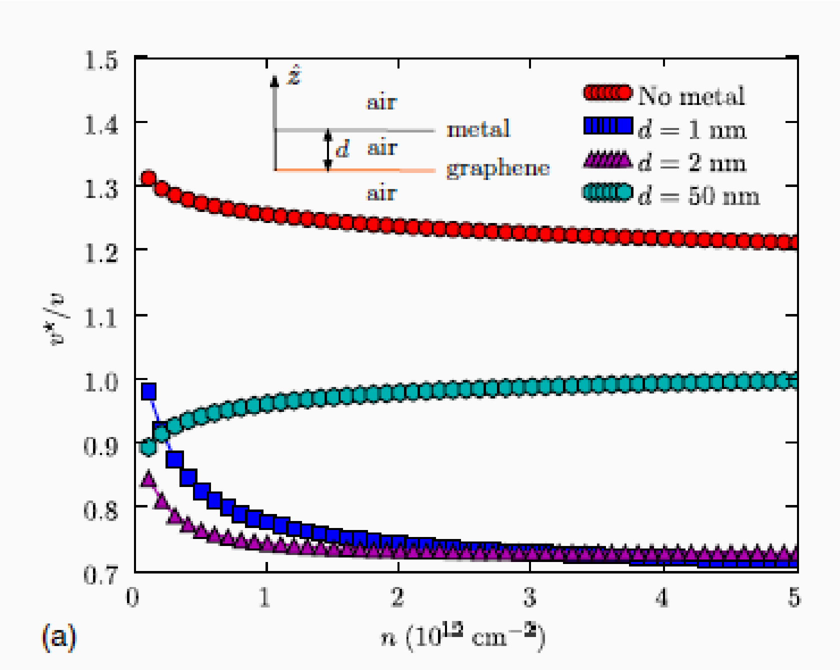
$$v_g = v_0/2 = \frac{\hbar K}{2m^*} = \frac{2\pi\hbar}{3m^* L} .$$

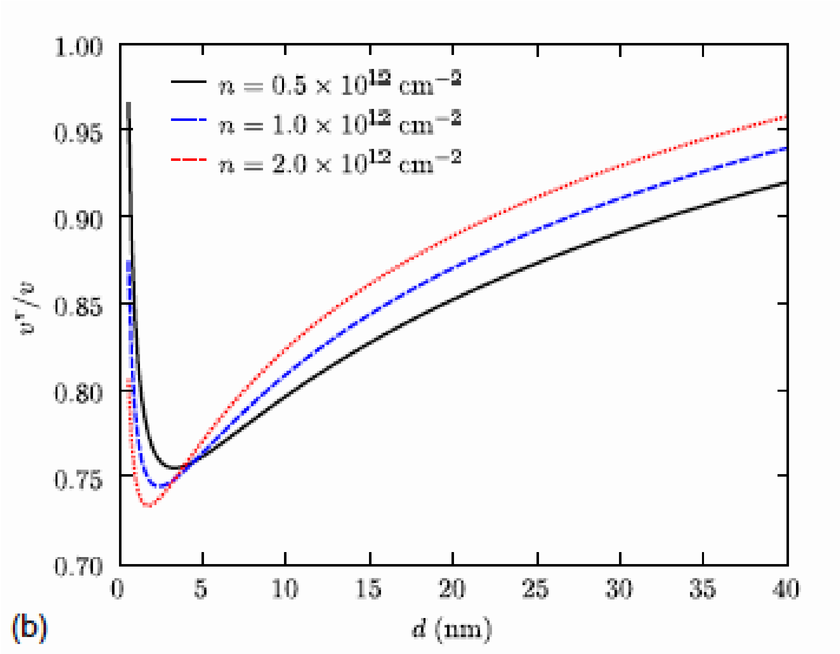
Dirac Cone has been also observed in other honeycomb crystals:

- **Acoustic surface waves (PRL2012)**
- **Photonic Crystals (nature 2012)**

Many-body renormalized Fermi velocity induced by remote metallic gates

- Consider a grounded metal plane placed close to a graphene sheet.
- Quasiparticles under the screening plane move at a speed v* that is smaller than in an isolated graphene sheet





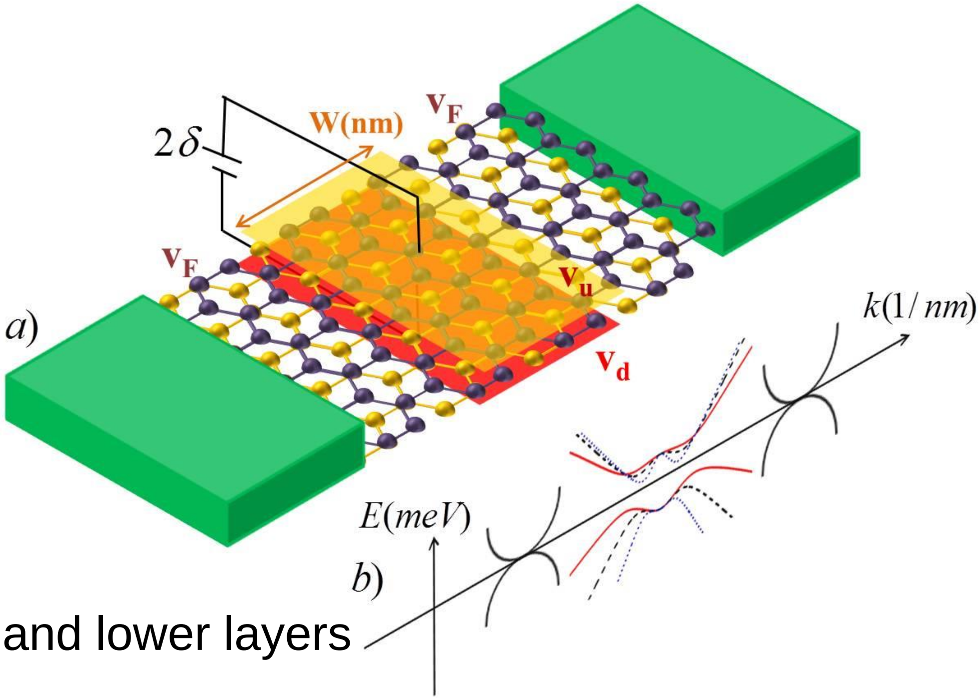
A. Rauox, et al, PRB,81, 073407 (2010)

Bilayer graphene in presence of effective velocity modulation: **Hamiltonian**

H.C, F. Adinehvand, JPCM, 26, 015302 (2014)

We consider the following dominant Four-band Hamiltonian:

$$H = \begin{pmatrix} -i\hbar v_u(\sigma\cdot\nabla)^\dagger + V_u I & F \\ F & -i\hbar v_d(\sigma\cdot\nabla) + V_d I \end{pmatrix}$$



where

$$F = \begin{pmatrix} t & 0 \\ 0 & 0 \end{pmatrix}, \qquad -i\hbar v\sigma\cdot\nabla = \begin{pmatrix} 0 & \pi^\dagger \\ \pi & 0 \end{pmatrix}$$

$\pi = -i\hbar v(\partial_x - k_y)$ t=390 meV

$$v_u = \xi_u v_F$$
$$v_d = \xi_d v_F$$

are Fermi velocity in the upper and lower layers

$$V_u = V_0 + \delta$$
$$V_d = V_0 - \delta$$

are electrostatic potential in the upper and lower layers

$\Psi = \left(\psi^u_{A_2}\ \psi^u_{B_2}\ \psi^d_{B_1}\ \psi^d_{A_1}\right)^T$ are eigen-function of Hamiltonian

Bilayer graphene in presence of effective velocity modulation: **Spectrum**

$$k(E)^2 = [a \pm \sqrt{a^2 - b}]/v_{\mathrm{u}}^2$$
$$a(E,\eta,\delta) = [\eta^2(E-\delta)^2 + (E+\delta)^2]/2$$
$$b(E,\eta,\delta) = \eta^2(E^2-\delta^2)(E^2-\delta^2-t^2)$$
$$\eta = \xi_{\mathrm{u}}/\xi_{\mathrm{d}}.$$

There are two extremum conditions: Band Gap Conditions

$$b = 0 \quad \forall \quad k = 0 \rightarrow E = \pm\delta, \pm(t+\delta)$$

$$b = a^2 \quad \forall \quad k_{c/v}(\xi_u, \xi_d) = \pm(v_u)^{-1}\sqrt{a(E_{c/v}, \eta, \delta)}$$

Results in the energy gap ➡ $E_g(\eta) = E_c(\eta) - E_v(\eta)$

At k=0, the gap is independent of the velocity ratio.

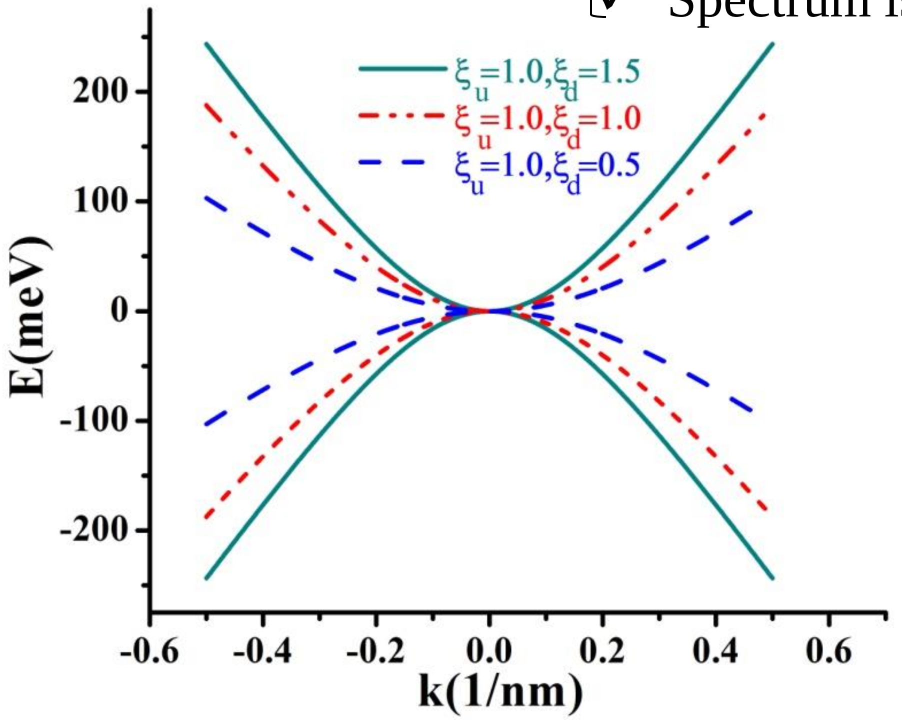
Gapless spectrum in presence of interlayer asymmetry in velocity but for $\delta = 0$

$$E = \pm\sqrt{\varphi(k) + (-1)^{\epsilon}\sqrt{\varphi^2(k) - v_u^2 v_d^2 k^4}}$$

$$\varphi(k) = ((v_u^2 + v_d^2)k^2 + t^2)/2$$

$\varepsilon = 1 \rightarrow$ for low energy band

- ✓ **Chiral symmetry** is still conserved.
- ✓ Only interlayer asymmetry in velocity is not able to break the **electron–hole symmetry**.
- ✓ Spectrum is robust against $\eta \rightarrow 1/\eta$



Band structure in presence of interlayer asymmetry in potential but symmetry in velocity: **direct band gap**

$\eta = 1\ ,\ \delta \neq 0\ ,\ v_u = v_d = v$

$$E^2 = (vk)^2 + \delta^2 + t^2/2 + (-1)^{\epsilon} \times \sqrt{(vk)^2(4\delta^2 + t^2) + t^4/4 + k^2}.$$

$\varepsilon = 1 \rightarrow$ Mexican hat structure

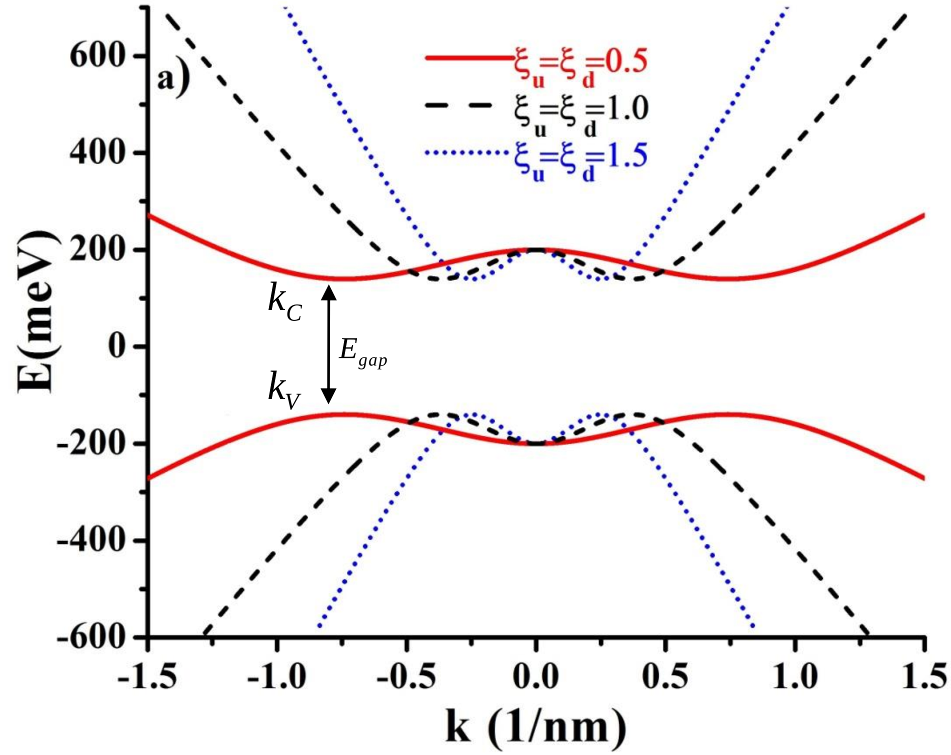
- ✓ e-h symmetry
- ✓ Symmetric conduction and valence band edges $E_c = -E_v \quad \forall\ k \neq 0$
- ✓ Momentum at the band edges $k_c = k_v = k_{gap}$
- ✓ The gap is independent of velocity

$$k_{gap} = \pm(2\delta/v_F)\sqrt{(t^2 + 2\delta^2)/(t^2 + 4\delta^2)}\ {}^{1}\!/_{\xi}$$

$$E_g = 2t\delta\Big/\sqrt{4\delta^2 + t^2}$$

$$\begin{cases} \delta << t \Rightarrow E_g \rightarrow 2\delta \\ \delta >> t \Rightarrow E_g \rightarrow t \end{cases}$$

(Band Gap Saturation)



For both limit: $k_{gap} \propto 2\delta / v$

Band structure in presence of interlayer asymmetry in potential and velocity: e-h asymmetry

$\eta \neq 1\ ,\ \delta \neq 0$

- ✓e-h asymmetry: $E_c \neq -E_v \quad \forall\, k \neq 0$
- ✓Indirect Band gap $k_c \neq k_v$
- ✓The gap depends on η instead of velocities

e-h asymmetric factor: $r = (|E_c| - |E_v|)/|E_c|$

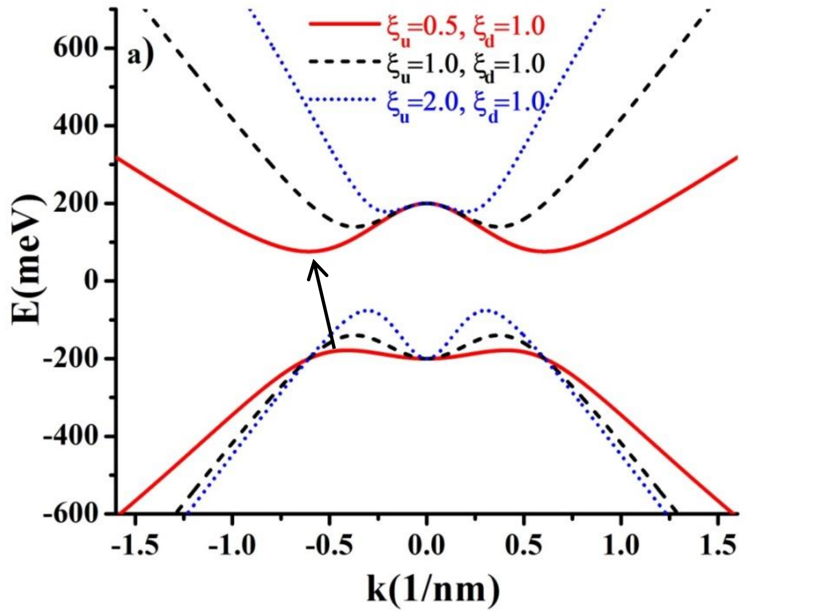
$\eta \to \eta^{-1} \Rightarrow r \to r^{-1}$

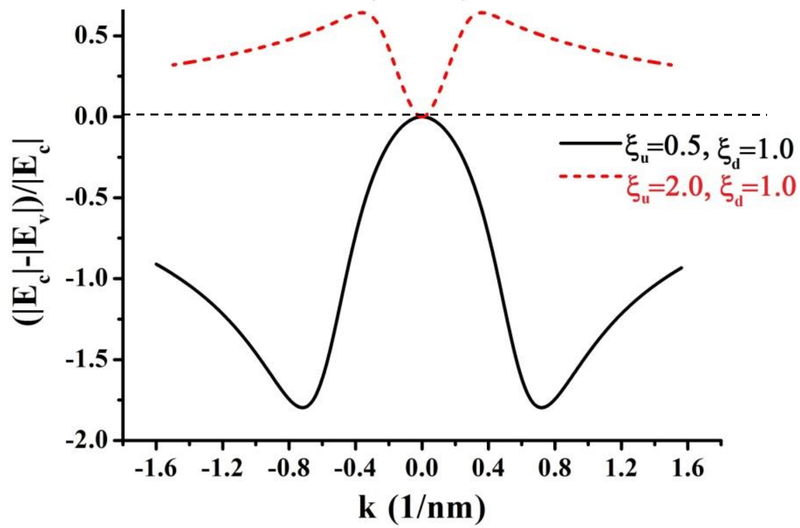
Conduction band exchange with valence band

e-h asymmetric factor caused by Full Hamiltonian:

$$r = 4\gamma_4/\gamma_0 \approx 0.1$$

the e–h asymmetry arising from the velocity engineering is a dominant factor compared with the e–h asymmetry caused by the parameter γ_4



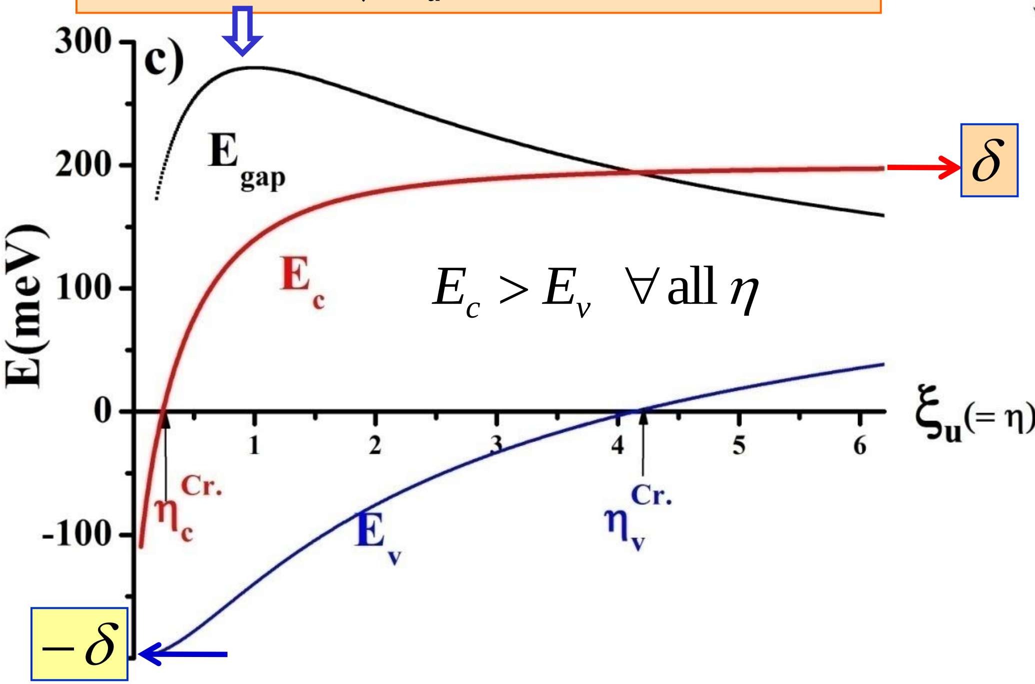


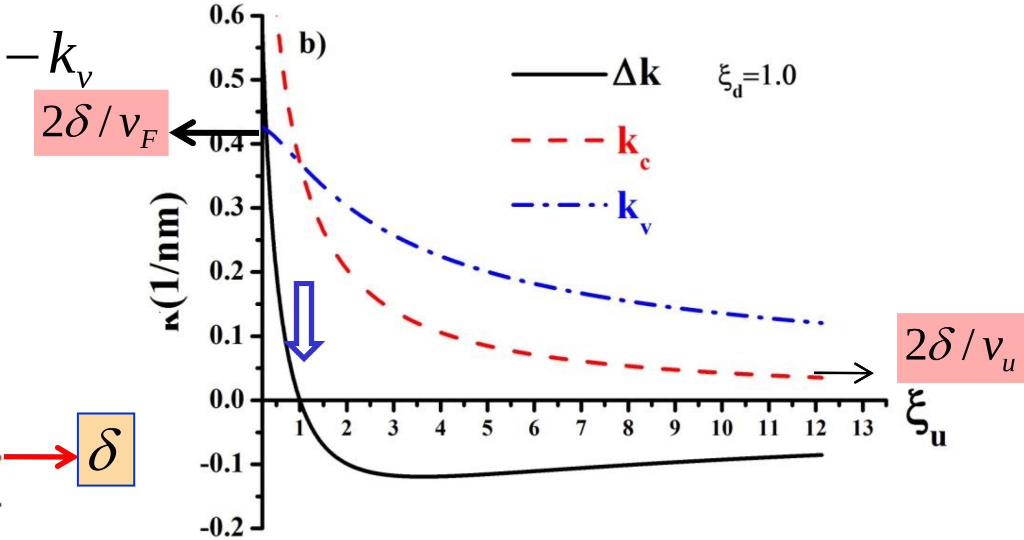
Band structure in presence of interlayer asymmetry in potential and velocity: **indirect band gap**

Indirectness of the band gap: $\Delta k = k_c - k_v$

$\eta = 1 \rightarrow \Delta k = 0 \;\therefore$ Direct Band Gap

$\eta \neq 1 \rightarrow \Delta k \propto 1/\xi_u \;\therefore$ Indirect Band Gap





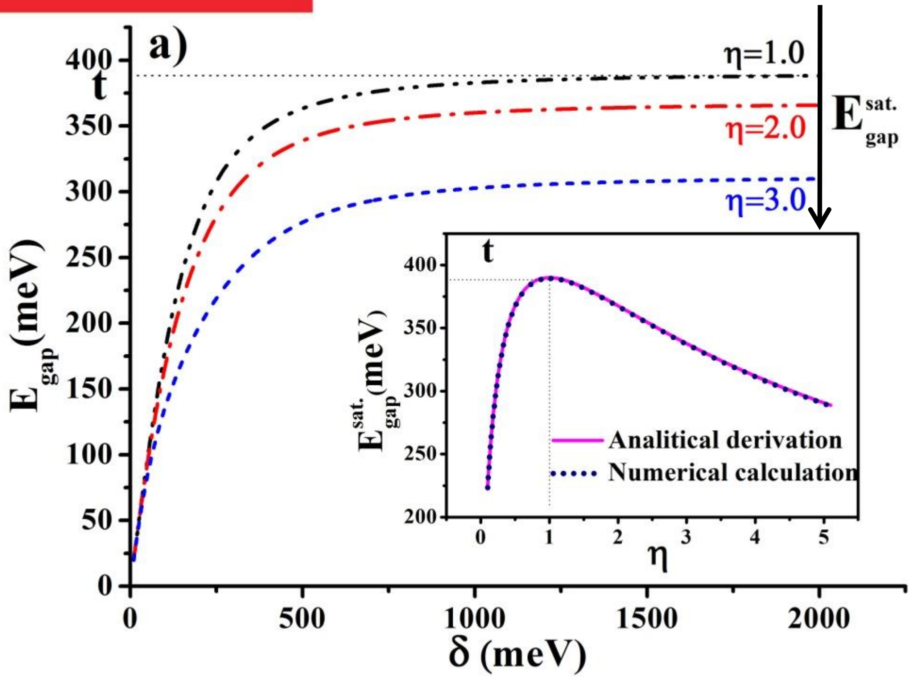
$$\eta^{cr.}_{v/c} = 1 + 2(t/\delta)^2\left[1 \pm \sqrt{1 + (\delta/t)^2}\right]$$

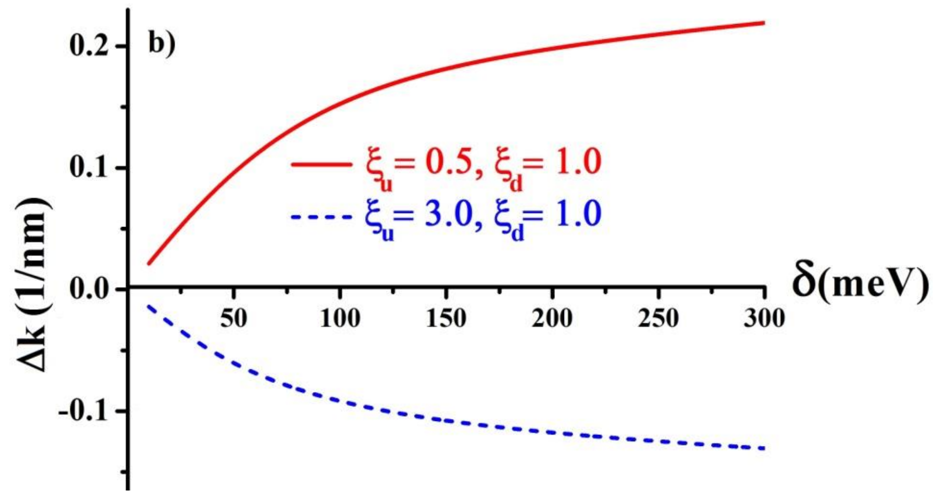
$$\Delta\eta^{cr.} = \eta_v^{cr.} - \eta_c^{cr.} \;\therefore \text{width of } E(\eta)$$

$$\left\{\begin{array}{l}\delta >> t \\ \delta << t\end{array}\right. \Rightarrow \Delta\eta^{cr.} \rightarrow 2t/\delta$$

✓**Sharp variation of $E(\eta)$ in large bias**

Band structure in presence of interlayer asymmetry in potential and velocity: Saturated indirect band gap





Indirectness of the gap can be manipulated by the gate bias.

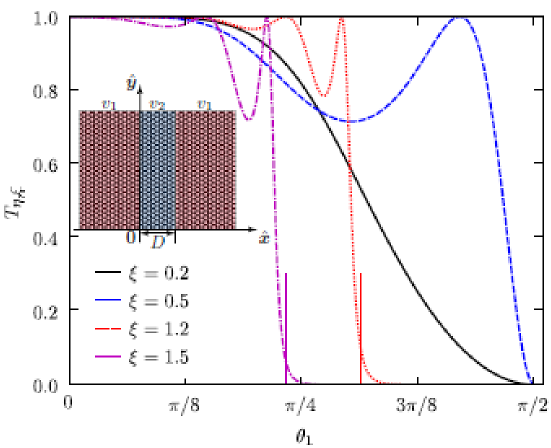
$$\lim_{\delta >> t} E_{gap}^{sat.} = \frac{2\sqrt{\eta}}{\eta + 1} t$$

the dependence of the energy gap on the velocity ratio can be manifested in transport properties through a velocity junction.

Analogy between **MDF** transport and **light** propagation

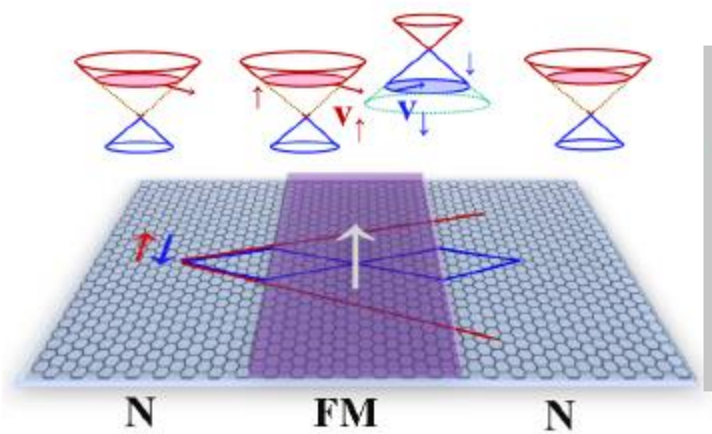
- ✓ MDF's velocity is independent of the wavelength, the same as the speed of light.
- ✓ The optical-like behaviors of electron waves in graphene such as focusing, collimation, Bragg reflection, electron wave-guides, total internal reflection.

total internal reflection



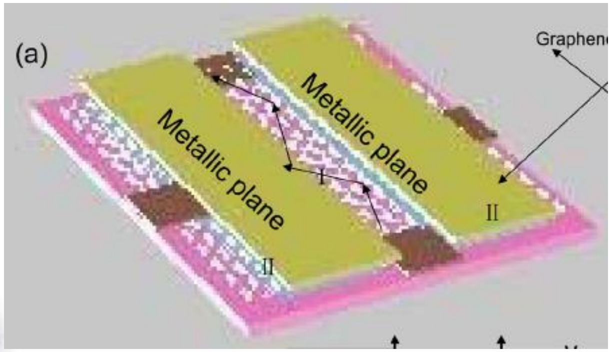
A. Rauox, et al, PRB2010

Spin lensing



A. G. Moghaddam, **M. Zareyan**, PRL2010

electron wave-guides



J. Yuan, et al, APL2011

Transport properties across non-uniform potential and velocity junctions: **transfer matrix method**

- **Current density operator**

Continuity equation: $\vec{\nabla}.\vec{j} = -\partial_t \rho$

$$j_i = \Phi_i^T \sum \Phi_i$$

$$\Sigma = \begin{pmatrix} \sigma^T & 0 \\ 0 & \sigma \end{pmatrix} \qquad \tilde{v}_i = \begin{pmatrix} \sqrt{v_u^i} & 0 \\ 0 & \sqrt{v_d^i} \end{pmatrix}$$

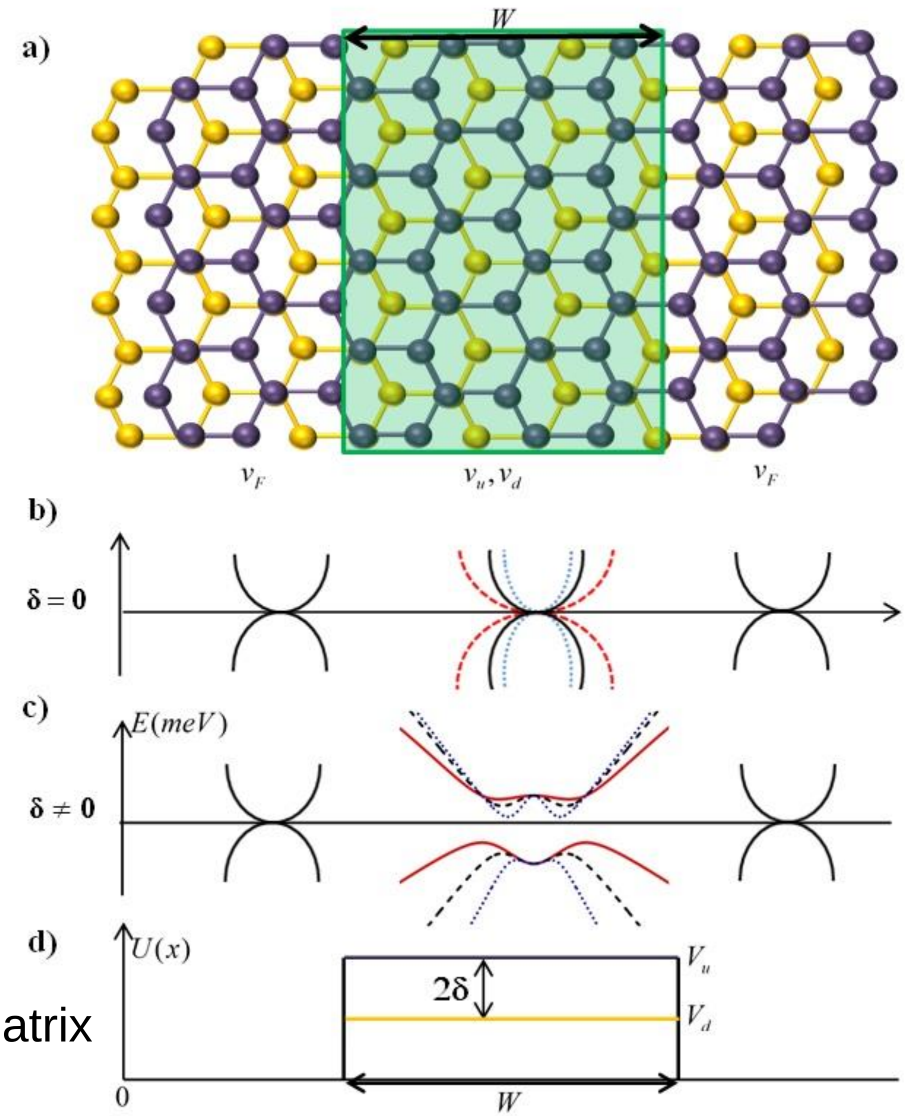
Auxiliary Spinor $\Phi_i = \tilde{v}_i \psi_i$ $\quad \psi_i = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$

- **Transfer Matrix Method**

Assuming a plane wave solution for the Hamiltonian: $\psi(x) = P(x)A$

Plane wave matrix

Coefficients matrix



Transport properties across non-uniform potential and velocity junctions: **transfer matrix method**

Conservation of current density at the interfaces → Auxiliary spinor continuity

$$j_i = A_i^T P_i^T \tilde{v}_i^T \sum \tilde{v}_i P_i A_i \quad \Longrightarrow \quad \Phi_1 = \Phi_2 \rightarrow \tilde{v}_1 \psi_1 = \tilde{v}_2 \psi_2$$

$$A_1 = M A_3$$

$$M = P_1^{-1}(0) \bar{v}_1^{-1} \bar{v}_2 P_2(0) P_2^{-1}(w) \bar{v}_2^{-1} \bar{v}_3 P_3(w)$$

← Transfer matrix

$$A_1 = \left(1\ r\ 0\ e_g\right)^{\mathrm{T}}, \qquad A_3 = \left(t\ 0\ e_{\mathrm{d}}\ 0\right)^{\mathrm{T}}$$

Transmission Probability

$$T = \frac{j_{out}^3}{j_{in}^1}$$

⟹

Conductance

$$G = 2G_0 \int_0^{\pi/2} T(E, \varphi) \cos(\varphi)\, \mathrm{d}\varphi$$

Transport properties across non-uniform potential and velocity junctions: **wave function**

$$\psi(x) = P(x)A$$

$$P(x) = \begin{pmatrix} e^{i\alpha_+ x} & e^{-i\alpha_+ x} & e^{i\alpha_- x} & e^{-i\alpha_- x} \\ f_+^+ e^{i\alpha_+ x} & f_+^- e^{-i\alpha_+ x} & f_-^+ e^{i\alpha_- x} & f_-^- e^{-i\alpha_- x} \\ s_+ e^{i\alpha_+ x} & s_+ e^{-i\alpha_+ x} & s_- e^{i\alpha_- x} & s_- e^{-i\alpha_- x} \\ g_+^+ s_+ e^{i\alpha_+ x} & g_+^- s_+ e^{-i\alpha_+ x} & g_-^+ s_- e^{i\alpha_- x} & g_-^- s_- e^{-i\alpha_- x} \end{pmatrix}$$

$$f_+^\pm = v_u \frac{\pm\alpha_+ - ik_y}{\varepsilon - \delta}, \qquad f_-^\pm = v_u \frac{\pm\alpha_- - ik_y}{\varepsilon - \delta}$$

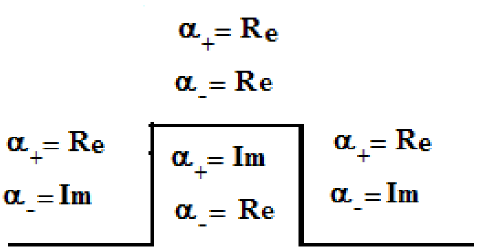
$$g_+^\pm = v_d \frac{\pm\alpha_+ + ik_y}{\varepsilon + \delta} \qquad g_-^\pm = v_d \frac{\pm\alpha_- + ik_y}{\varepsilon + \delta}$$

$$s_\pm = \frac{(\varepsilon - \delta)^2 - v_u^2[(\alpha_\pm)^2 + k_y^2]}{t(\varepsilon - \delta)}, \qquad \varepsilon_i = E - V_i$$

$$\alpha_\pm = \sqrt{a(\varepsilon, \eta, \delta) - v_u^2 k_y^2 \pm \sqrt{a(\varepsilon, \eta, \delta)^2 - b(\varepsilon, \eta, \delta)}}/v_u.$$

$$A = \begin{pmatrix} U_{A_2} & U_{B_2} & D_{B_1} & D_{A_1} \end{pmatrix}^\top$$

$$0 < \varepsilon_1 < t$$

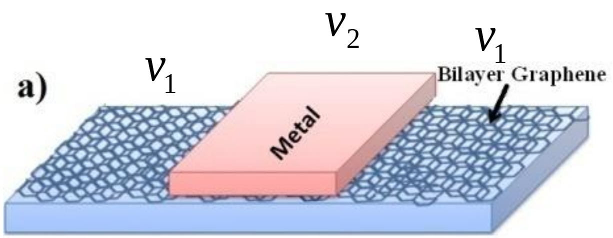


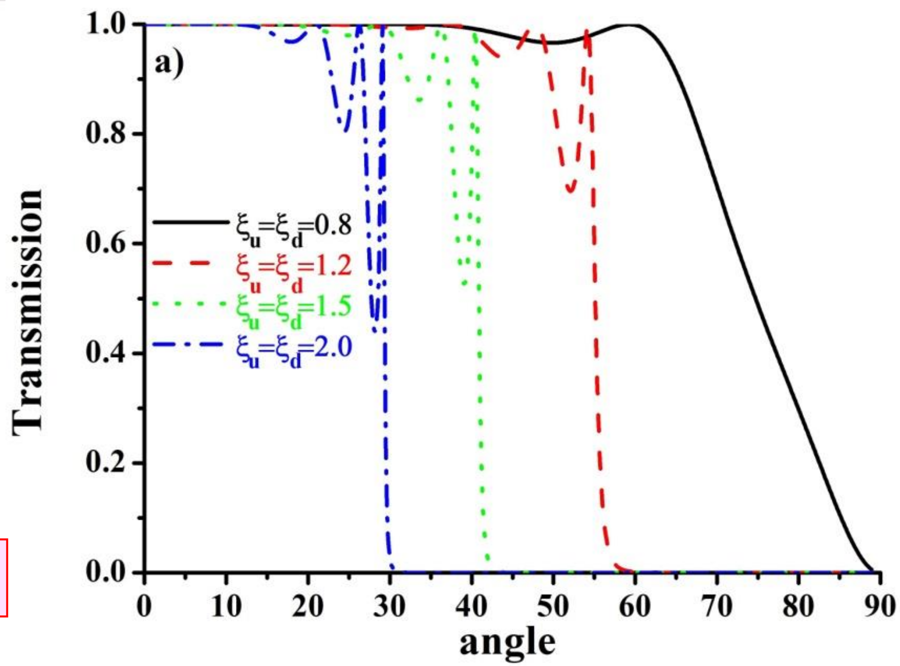
Transport across a single pure velocity barrier

Total internal reflection

$$\delta = 0$$

$$\xi = \xi_u = \xi_d = v_2 / v_F \ \& \ v_1 = v_F$$





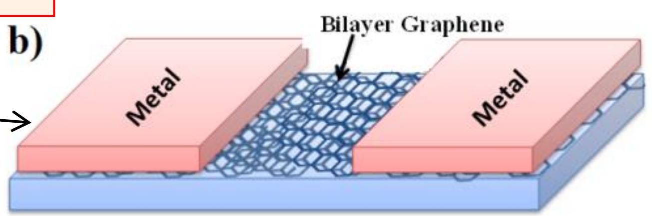
$$\text{For } \theta_1 = 0 \rightarrow T = 1 / (Cos^2(\alpha_1 w) + Sin^2(\alpha_1 w)) = 1$$

The wavevector inside the barrier

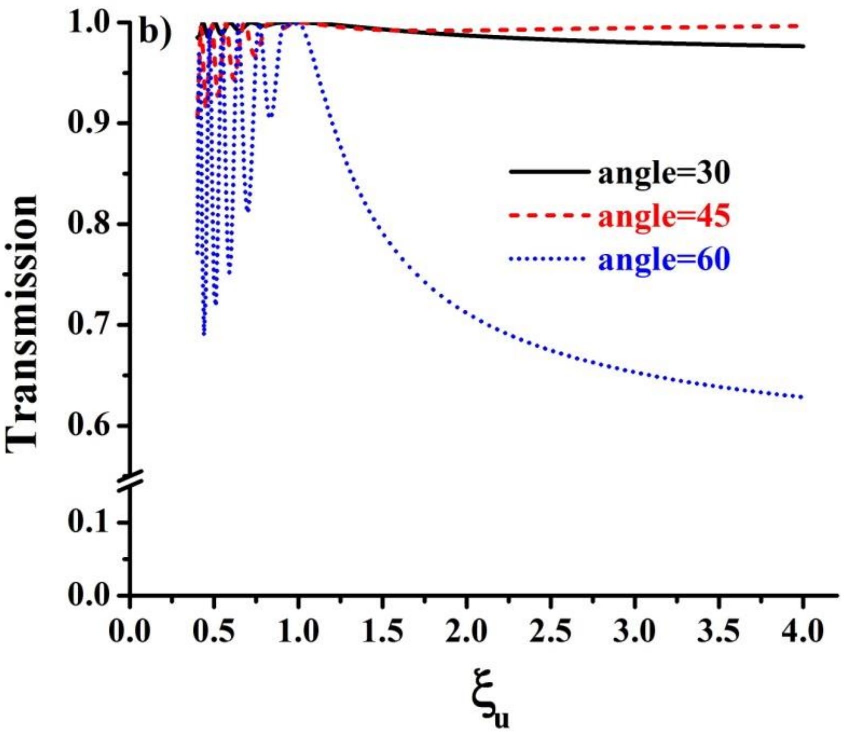
$$\alpha_2 = k\sqrt{1/\xi^2 - \sin^2\theta} \ \text{ is } \begin{cases} \text{Real for } \xi < 1 \Rightarrow \text{Propagating Modes} \\ \xi > 1 \Rightarrow \begin{cases} \theta_1 < \theta_{cr.} \Rightarrow \text{Propagating Modes} \\ \theta_1 > \theta_{cr.} \Rightarrow \text{Evanescent Modes} \end{cases} \end{cases}$$

$$\theta_{cr.} = \arcsin(1/\xi)$$

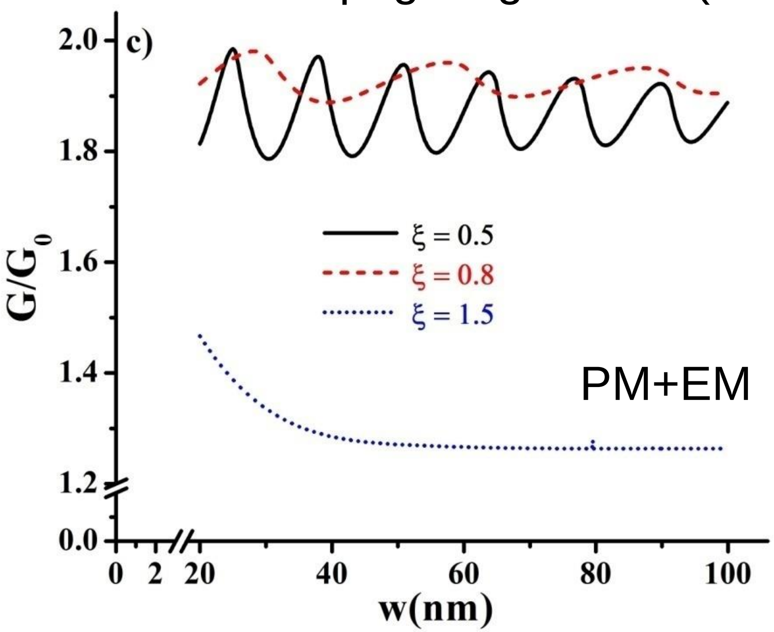
Waveguide



Transport across a single pure velocity barrier



Propagating Modes (PM)

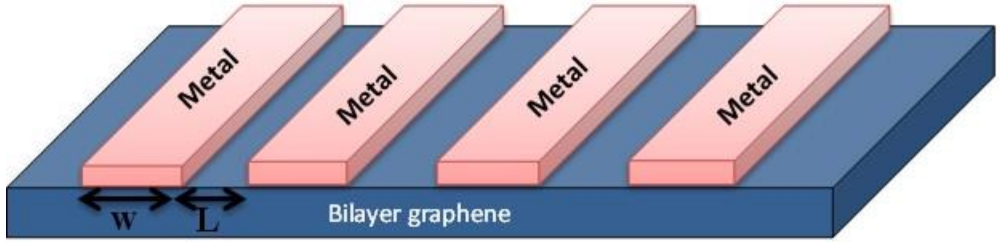


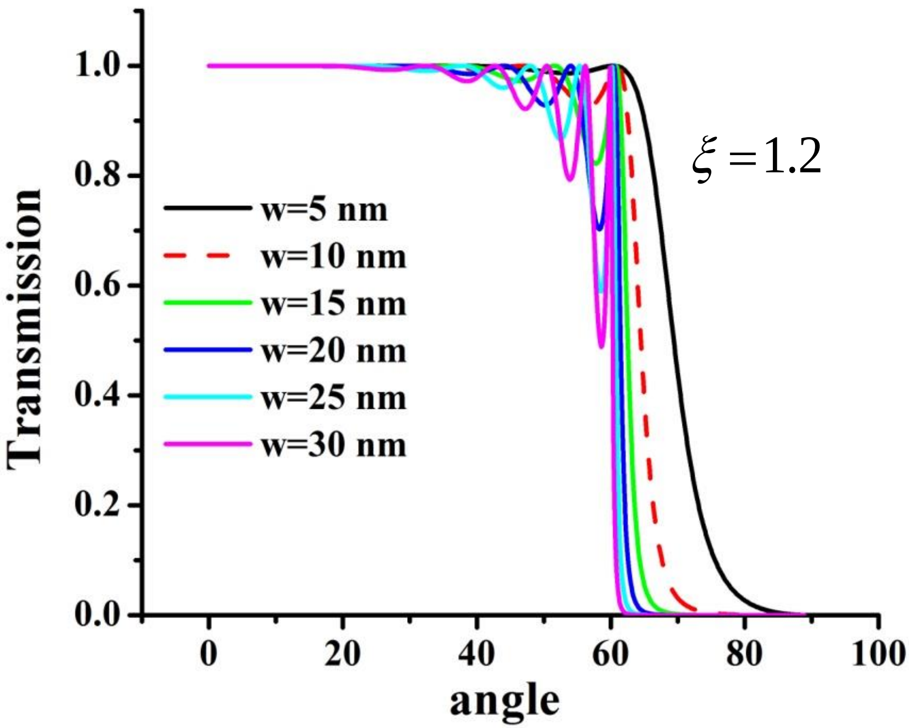
PM+EM

Resonance Condition

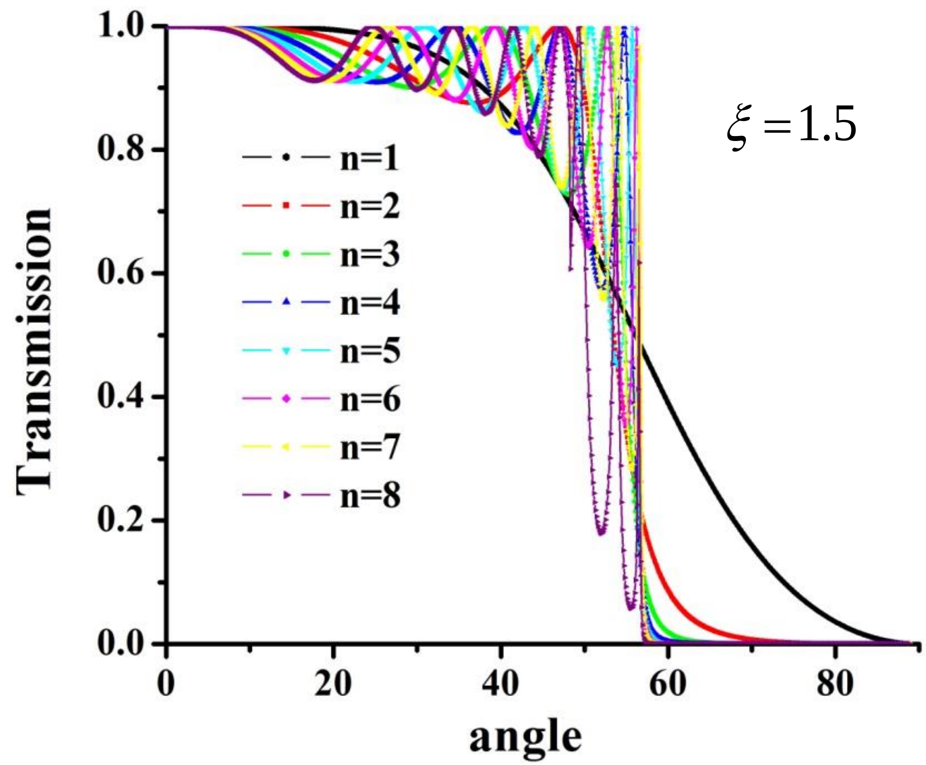
$$\alpha_2 w = n\pi \ \forall \xi < 1$$

Transport across multiple structure of pure velocity barriers





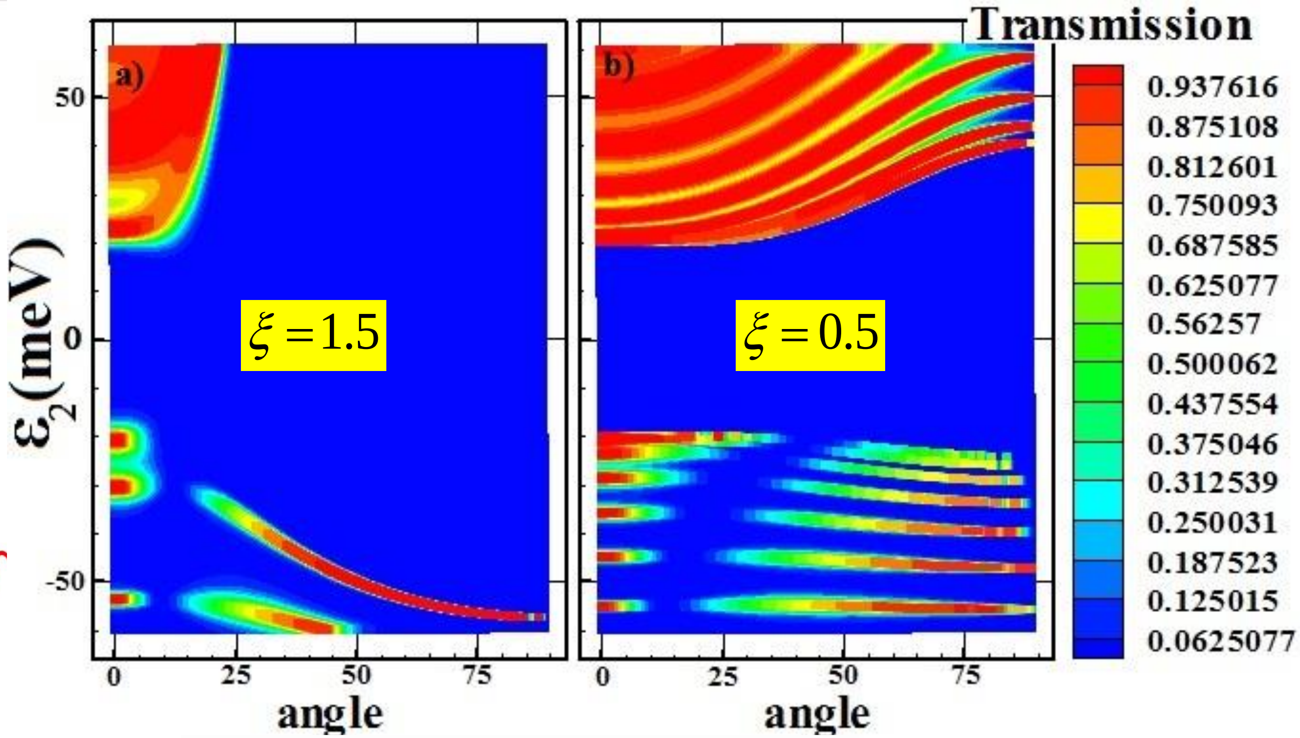
n=7

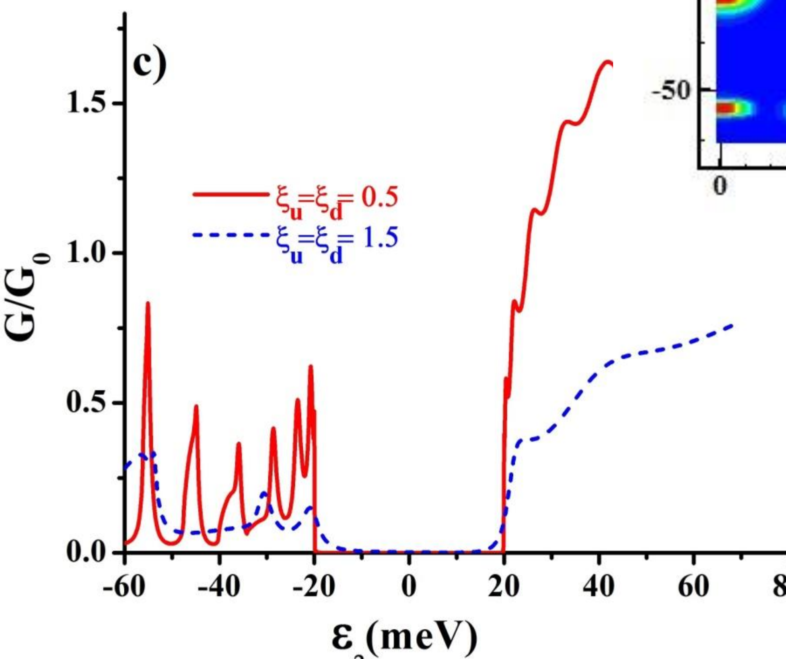


W=10 nm

Transport across velocity barrier in presence of a gate bias $\delta \neq 0$

$\eta = 1$





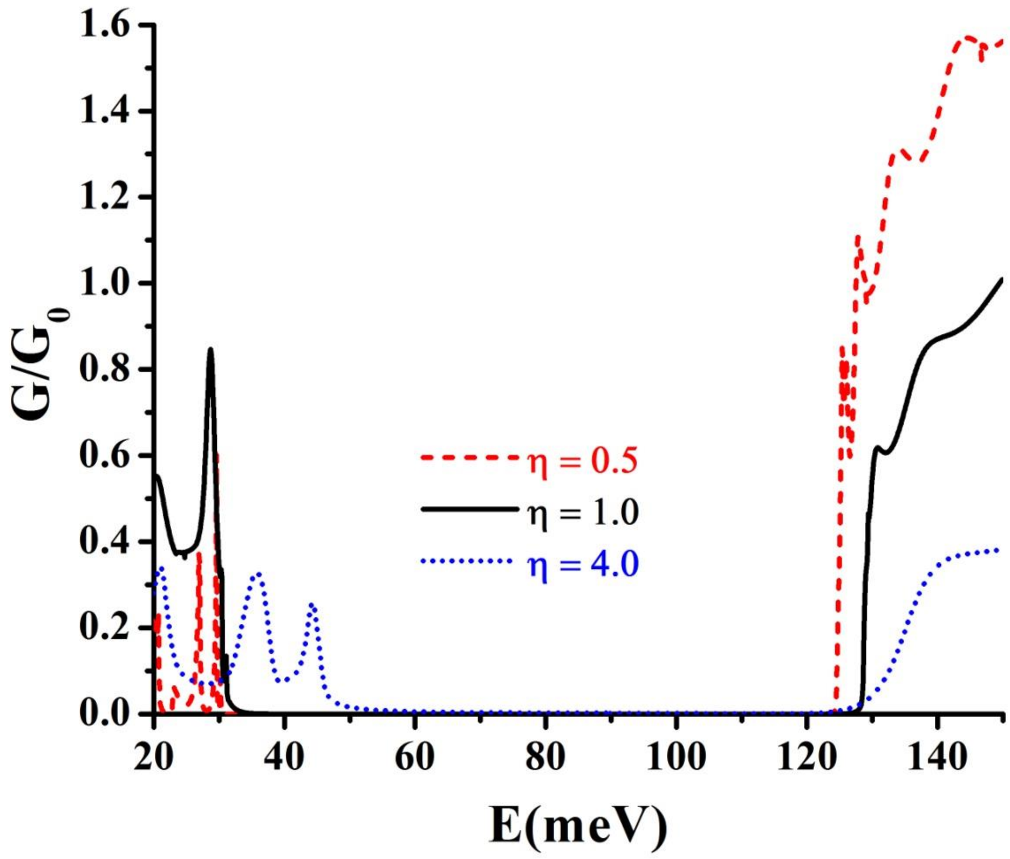
$$\alpha_2 = k\sqrt{\frac{\mu}{\lambda \xi^2} - \sin^2\theta_1} \Longrightarrow \theta_{\text{cr.}}(E) = \arcsin\left(\sqrt{\frac{\mu}{\lambda}}\frac{1}{\xi}\right)$$

$$\mu = \varepsilon_2^2 + \delta^2 + \sqrt{4\varepsilon_2^2\delta^2 - t^2(\delta^2 - \varepsilon_2^2)}$$
$$\lambda = \varepsilon_1^2 + t\varepsilon_1.$$

Transport gap depends on the velocity ratio

$\eta \neq 1$

$\delta \neq 0$



Conclusion

- The chiral symmetry is conserved for pure velocity modulation $\delta = 0$.
- In the broken-symmetry BLG, e-h symmetry preserved whenever the same velocity is modulated in both layers $\eta = 1$. In this case, the band gap is direct.
- In the broken-symmetry BLG and non-equal velocities in two layers $\eta \neq 1$ result in a transition of the direct to indirect band gap. The electron–hole symmetry fails. Indirectness increases with the gate bias.
- In analogy with optics, we propose a total internal reflection angle.
- The transport gap which is induced by application of the gate bias in the barrier region depends on the velocity ratio.

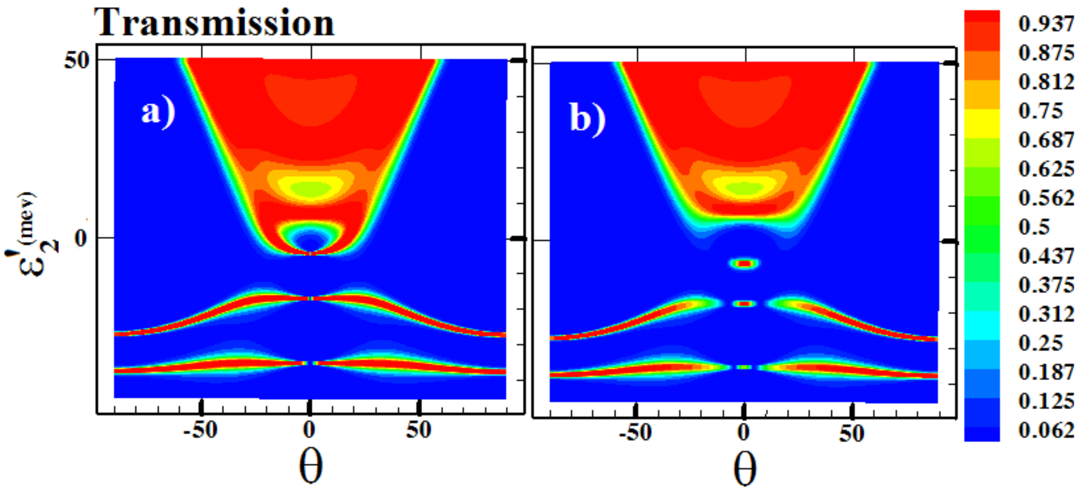
- Collaboration: Fatemeh Adinehvand (PhD student)
- With special thanks:

Dr. Reza Asgari

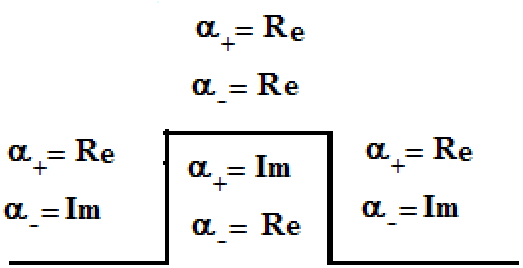
Thank you for your attention

Transmission through a barrier on bilayer graphene $\eta = 1 \& \xi = 1$

$\delta = 0$



$\delta = 10\,\text{meV}$



$\delta = 0$