

# Interlayer superconductivity in Graphene Materials

Mir Vahid Hosseini  
with  
Dr. Malek Zareyan

# Outline

- **Superconductivity (Sc)**
  - Introduction
  - BCS Theory
- **Exotic phase of Sc**
  - Breached pairing (**Interior gap**)
  - Generality of Cooper pairing
- **Graphene**
  - Graphene properties
  - Graphene Sc
- **Interlayer Sc in Graphene materials**
  - Interlayer Sc in Bilayer Graphene
  - Interlayer Sc in Trilayer Graphene
- **Conclusions**

# Superconductivity

- Introduction



Onnes, 1911  
SC Mercury



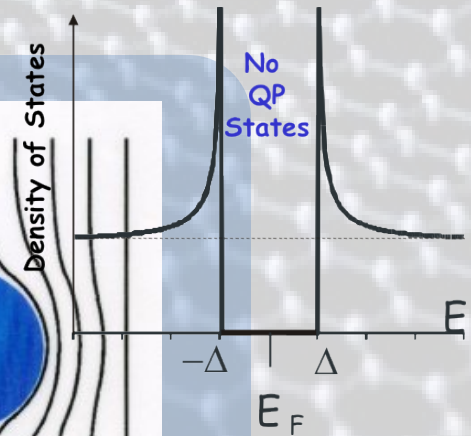
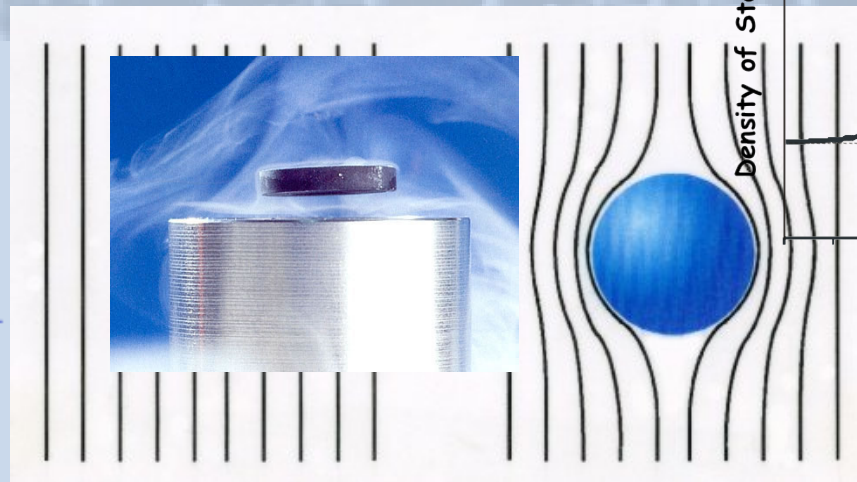
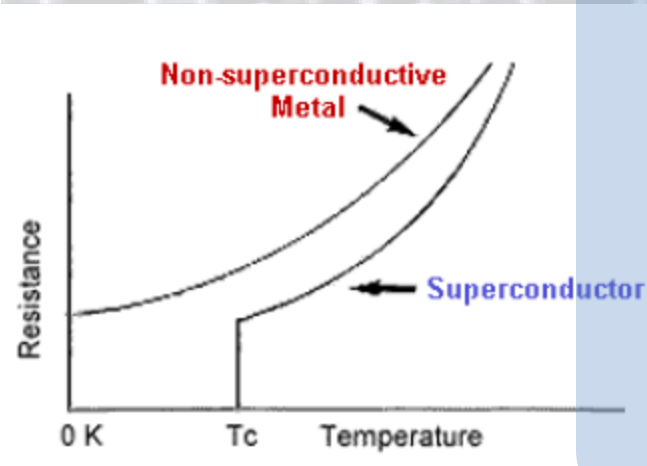
Meissner, 1933



BCS theory, 1957

- Zero resistivity

- Meissner effect

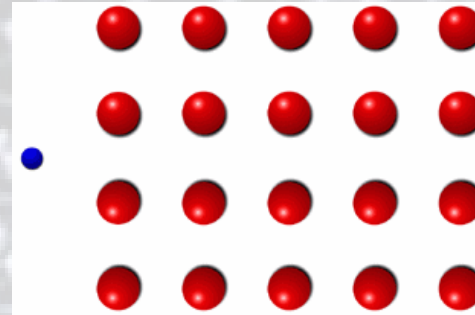
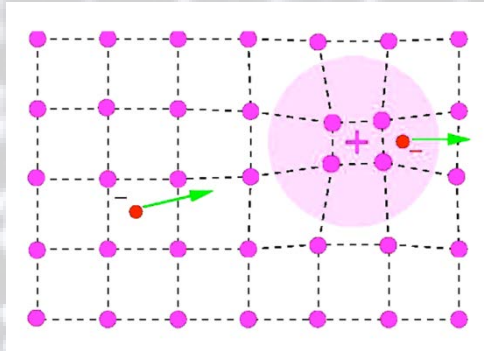


# Superconductivity

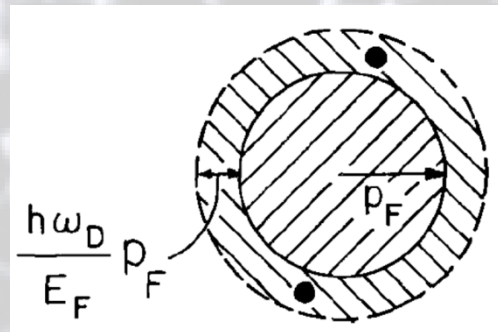
- BCS Theory

- Attractive interaction between electrons

phonons



- Cooper pair



# Superconductivity

## • BCS Theory

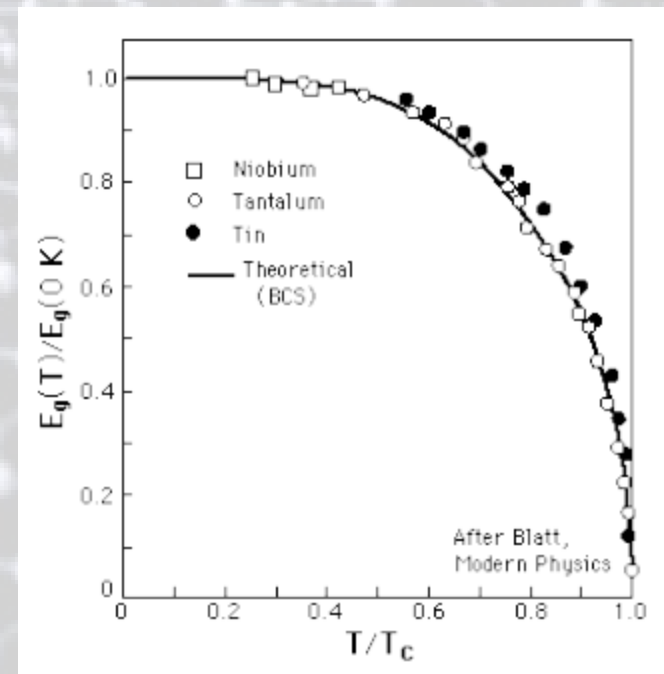
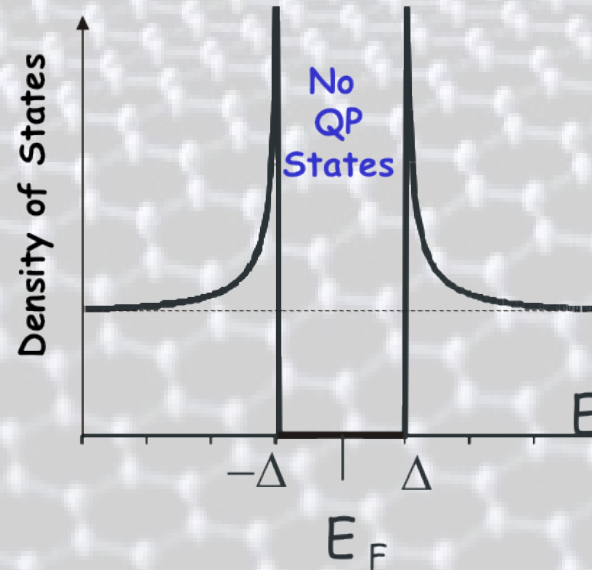
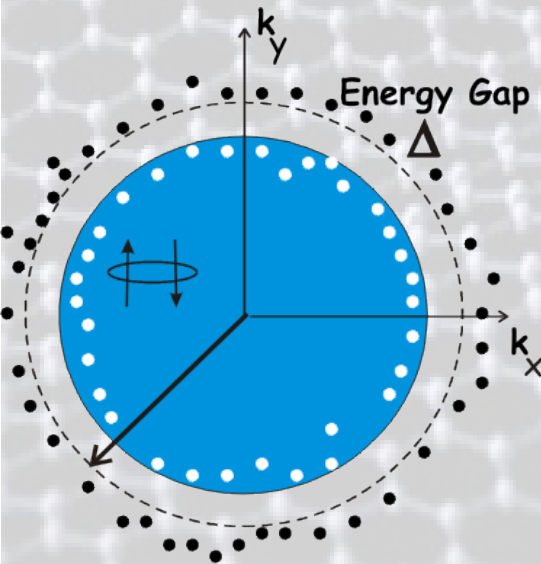
- Gap and order parameter

gap equation:

$$\Delta_{\mathbf{k}} = - \sum_{\mathbf{k}'} V_{\mathbf{k}\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{2E_{\mathbf{k}'}}$$

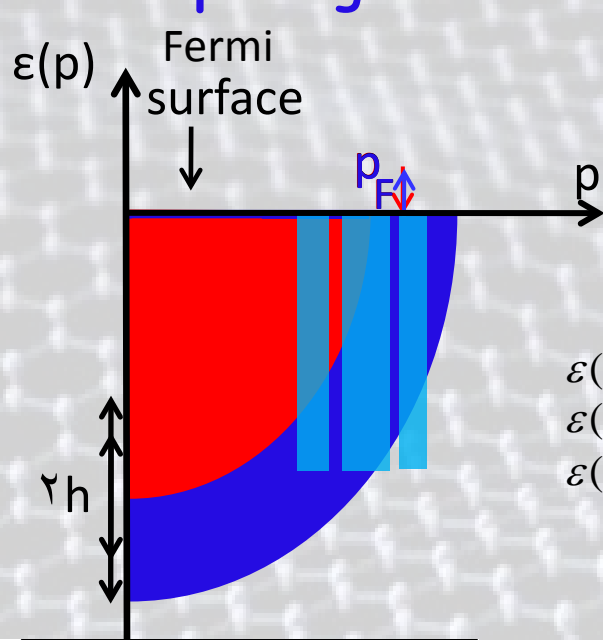
Excitation spectrum:

$$E_{\mathbf{k}'} = \sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}$$

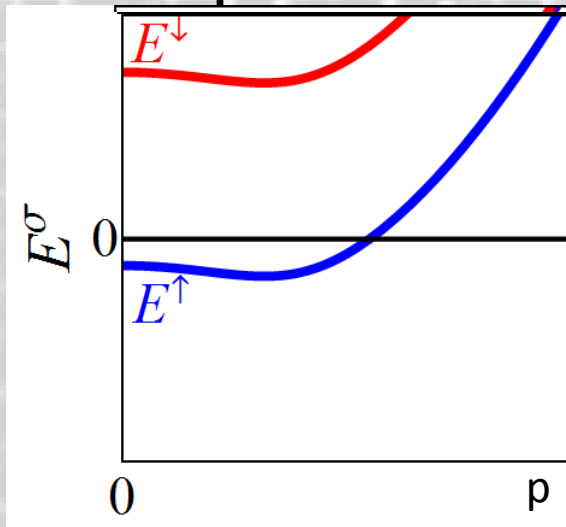
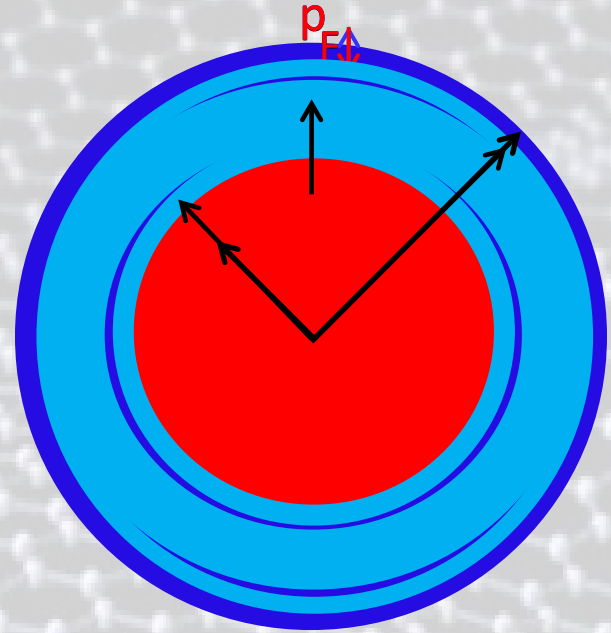


# Exotic phase of Sc

- Breached pairing



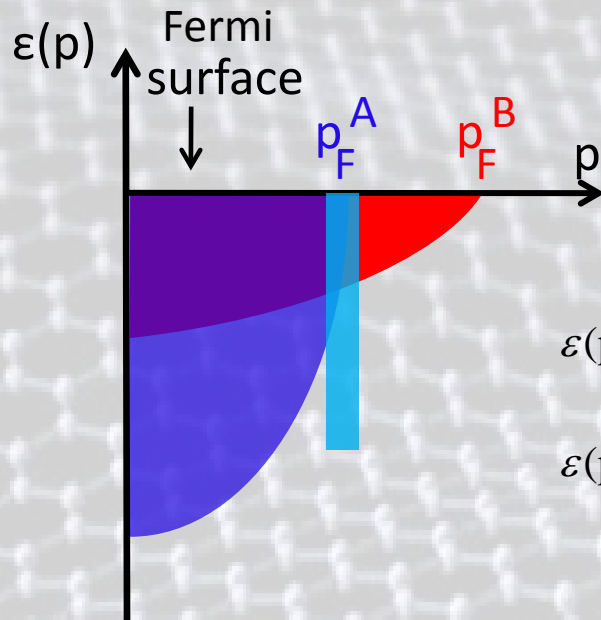
$$\begin{aligned} \epsilon(p) &= \left(\frac{p^2}{2} - \mu\right) + h \\ \epsilon(p) &= \left(\frac{p^2}{2} - \mu\right) \\ \epsilon(p) &= \left(\frac{p^2}{2} - \mu\right) - h \end{aligned}$$



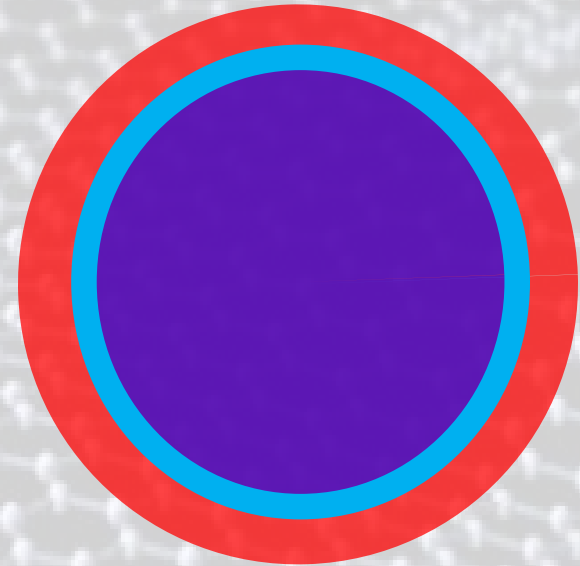
BCS pairing

# Exotic phase of Sc

- Breached pairing (**Interior gap**)



$$\epsilon(p) = \left(\frac{p^2}{2m_B} - \mu\right)$$
$$\epsilon(p) = \left(\frac{p^2}{2m_A} - \mu\right)$$



Heavy Fermion:  $m_B$

Light Fermion:  $m_A$

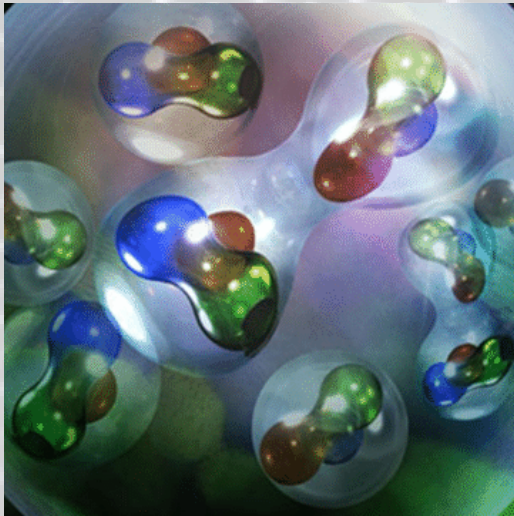
If Fermi  $m_B \gg m_A$  then  $\Rightarrow$  Interior gap

Breached Pairing (Sarma) state = a superfluid + a normal Fermi liquid at  $T=0$ ;  
has gapped and gapless quasiparticle excitations.

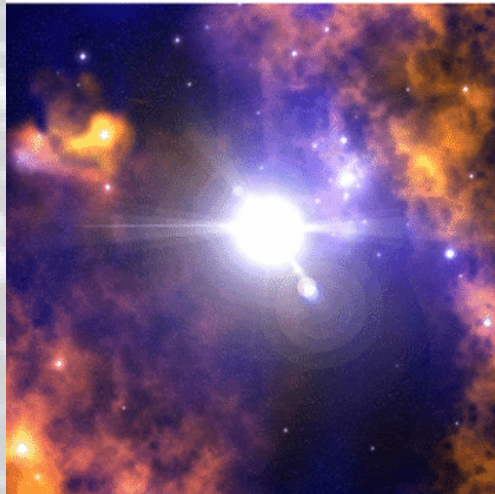
# Exotic phase of Sc

- **Generality of Cooper pairing**

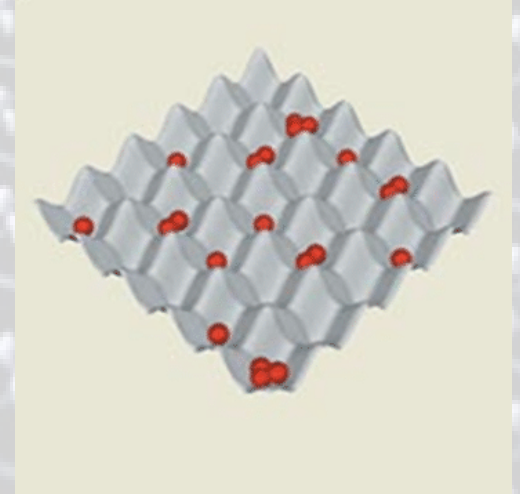
**Pairing in Nuclear matter**



**Pairing in Quark matter**



**Pairing in Cold atoms**



P. Davise, *et al.* *Nature* 469, 68–71 (06 January 2011)

Fukushima, K. & Warringa, H. *Physical Review Letters* 100, 032008 (2008).

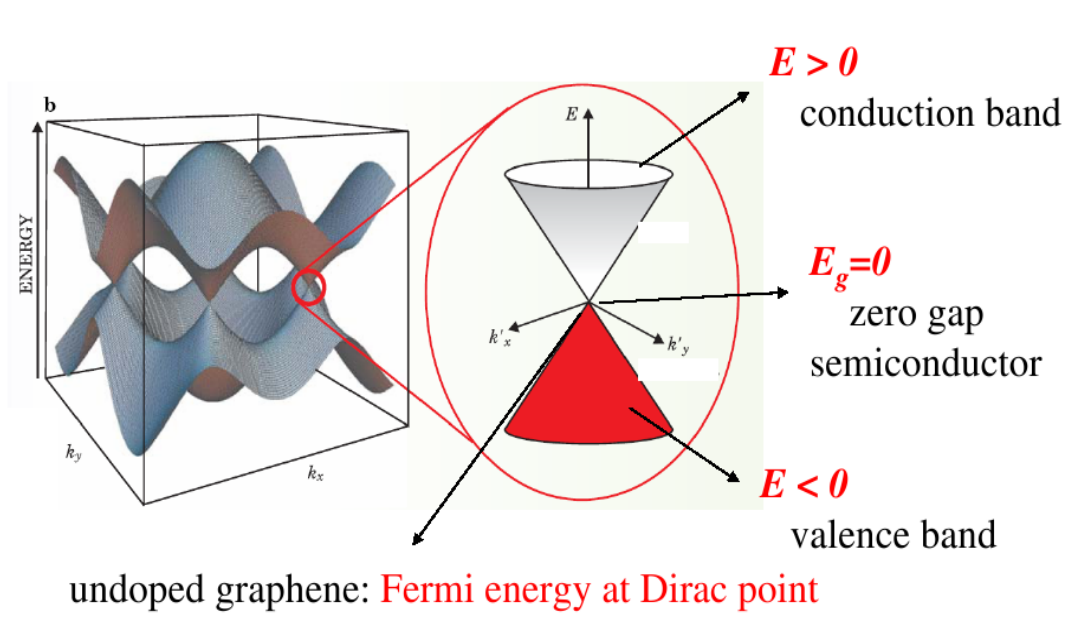
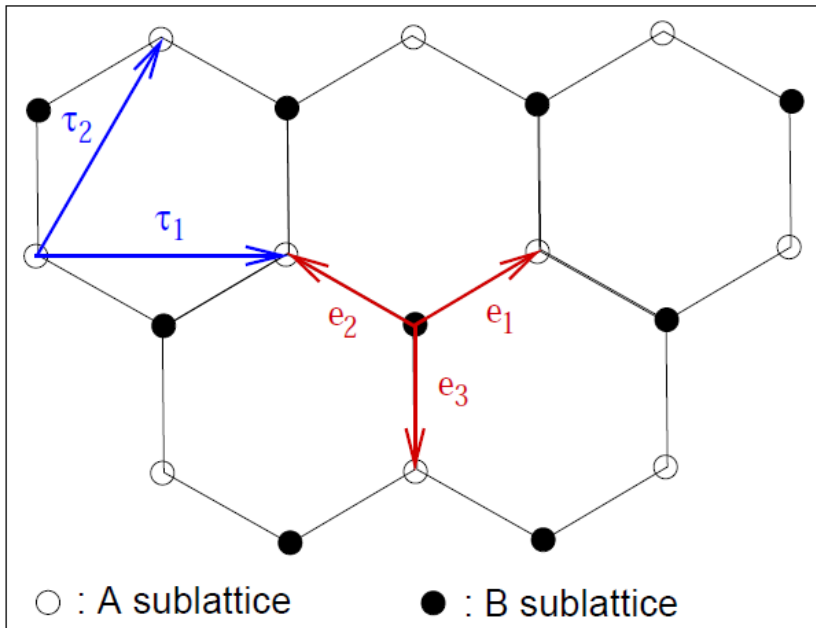


## Exotic phase of Sc

**Is there new exotic phase of Cooper pairing?**

# Graphene

## • Graphene Band structure



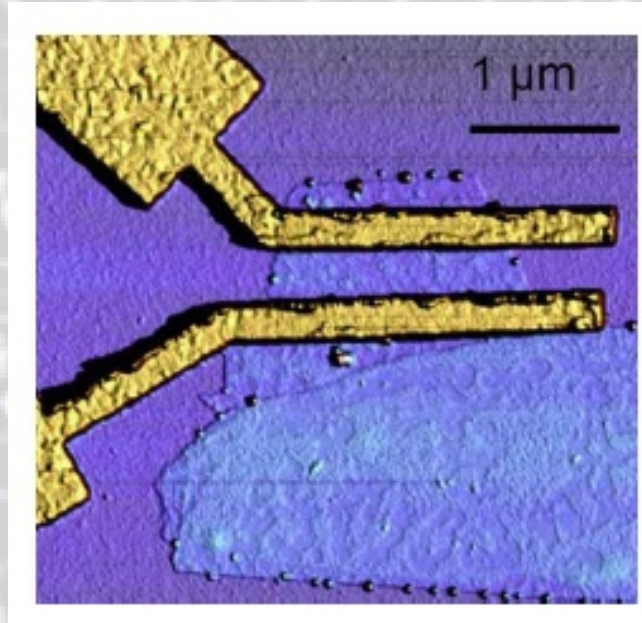
Band-structure calculation in the tight-binding model

$$H_0 = -t \sum_{i \in A} \sum_{j=1}^3 \left( b_{\mathbf{R}_i + \mathbf{e}_j}^\dagger a_{\mathbf{R}_i} + \text{H.c.} \right)$$

$$\gamma_{\mathbf{k}} = \sum_{j=1}^3 \exp[i\mathbf{k} \cdot \delta_j],$$

# Graphene

- Graphene Superconductivity

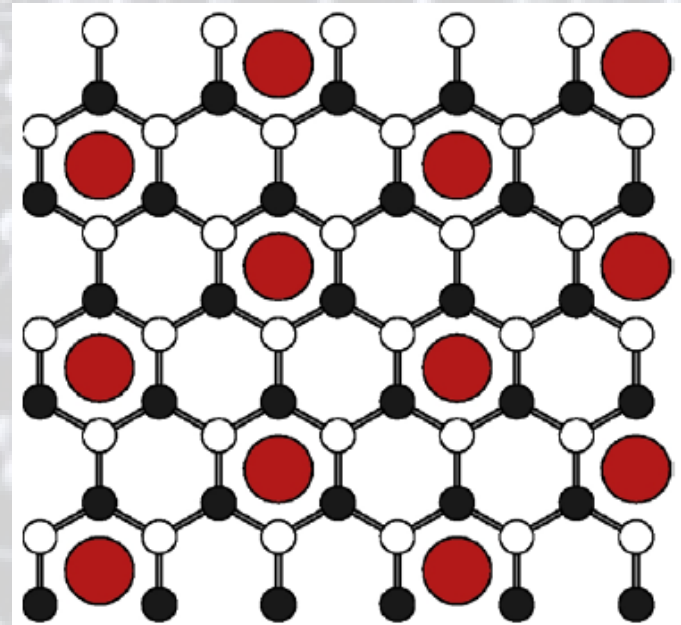
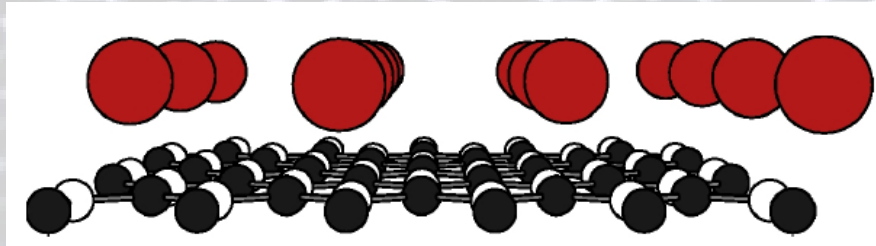


H. B. Heersche, P. JarilloHerrero, J. O. Oostinga, L. M. K. Vandersypen, and A. F. Morpurgo, Nature (London) **446**, 56 (2007).

# Graphene

- Graphene Superconductivity

phonon and plasmon mechanisms in graphene coated with a layer of a metal

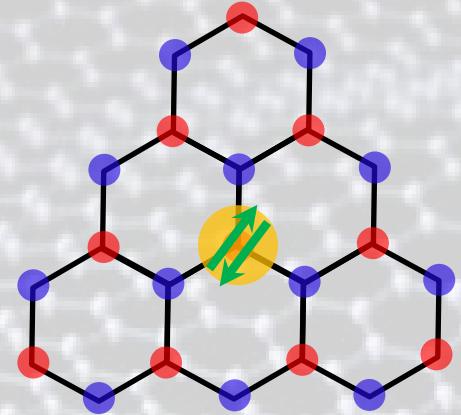


# Graphene

## • Graphene Superconductivity

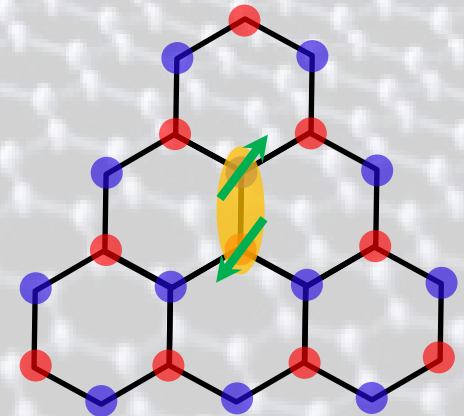
$$\Delta_0 = -g_0 \langle a_{i\downarrow} a_{i\uparrow} \rangle = -g_0 \langle b_{i\downarrow} b_{i\uparrow} \rangle$$
$$\Psi = \psi_{S=0} \otimes \psi_{L=0} \otimes \psi_{A-A}$$

} S-wave



$$\Delta_{1,ij} = -g_1 \langle a_{i\downarrow} b_{i\uparrow} - a_{i\uparrow} b_{i\downarrow} \rangle$$
$$\Psi = \psi_{S=0} \otimes \psi_{L=1} \otimes \psi_{A-B}$$

} P+ip-wave

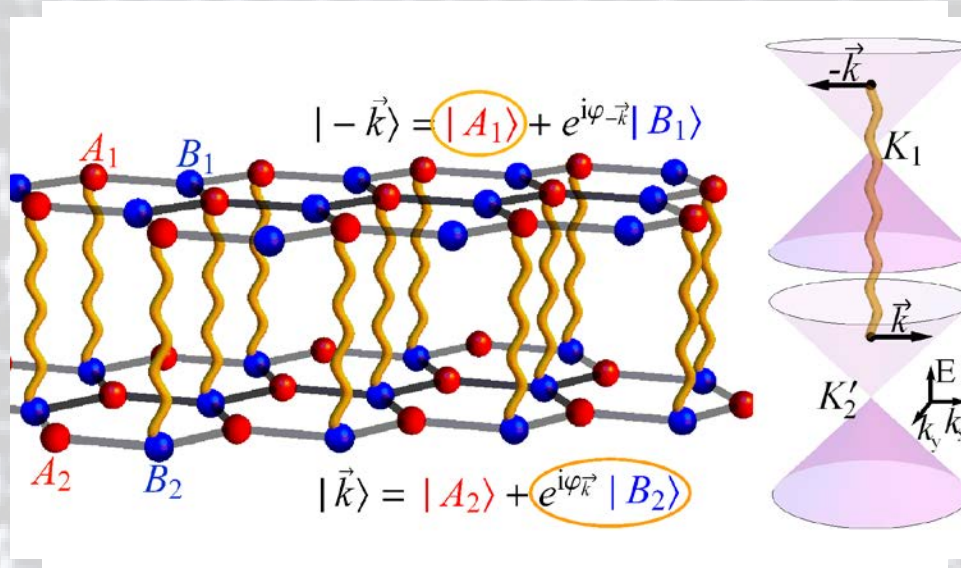


B. Uchoa and A. H. Castro Neto, Phys. Rev. Lett. **98**, 146801 (2007).

M.V.Hosseini and M. Zareyan, Appl. Phys. Lett. 101, 252602 (2012).

# Interlayer Sc in Graphene materials

- Interlayer Sc in Bilayer Graphene



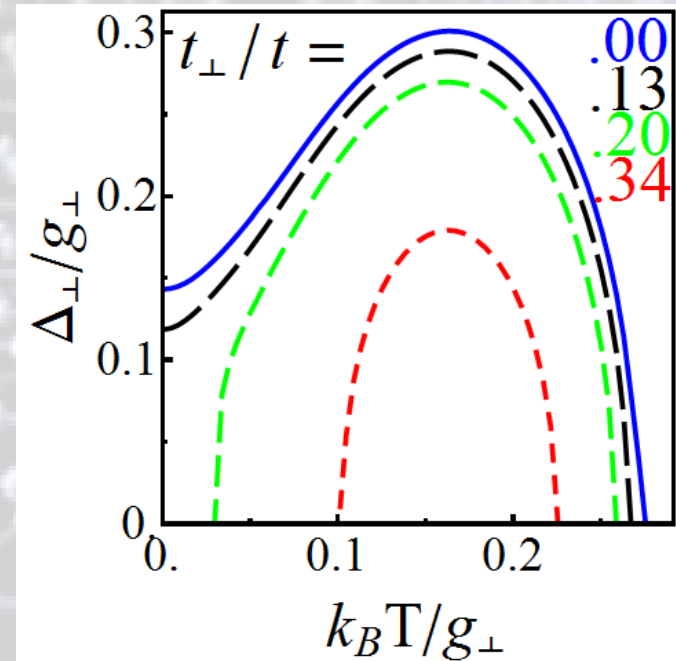
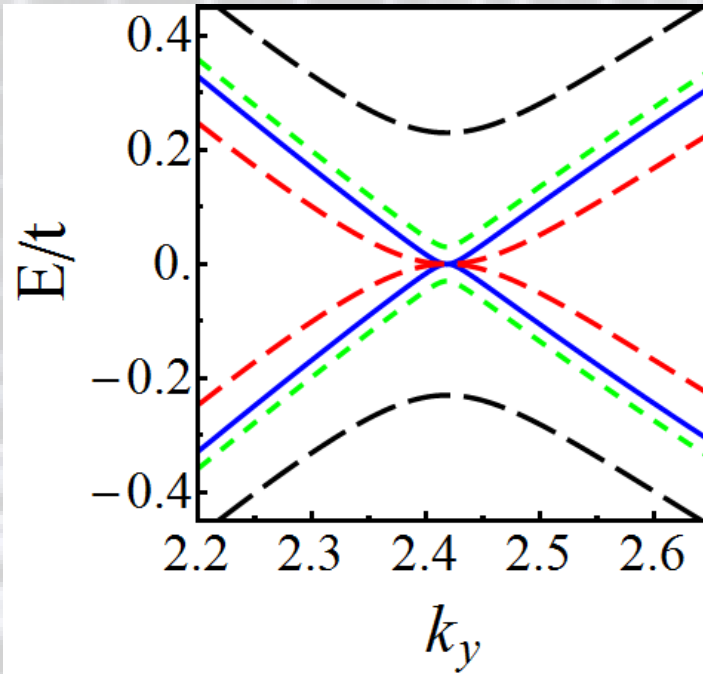
$$\Delta_{i,\perp} = -g_{\perp} \langle a_{i\downarrow} b_{i\uparrow} + a_{i\uparrow} b_{i\downarrow} \rangle$$

$$\Psi = \psi_{S=1} \otimes \psi_{L=0} \otimes \psi_{A-B}$$

S-wave Spin-Triplet !!!

# Interlayer Sc in Graphene materials

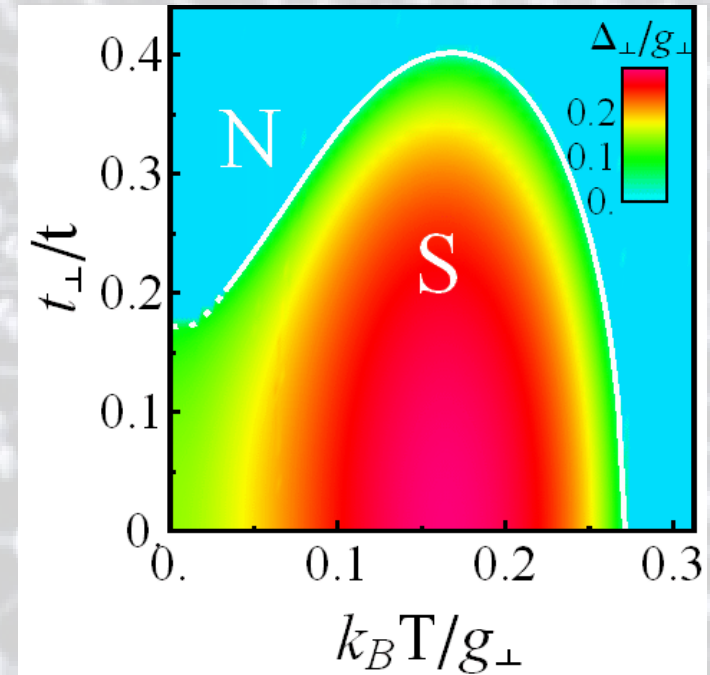
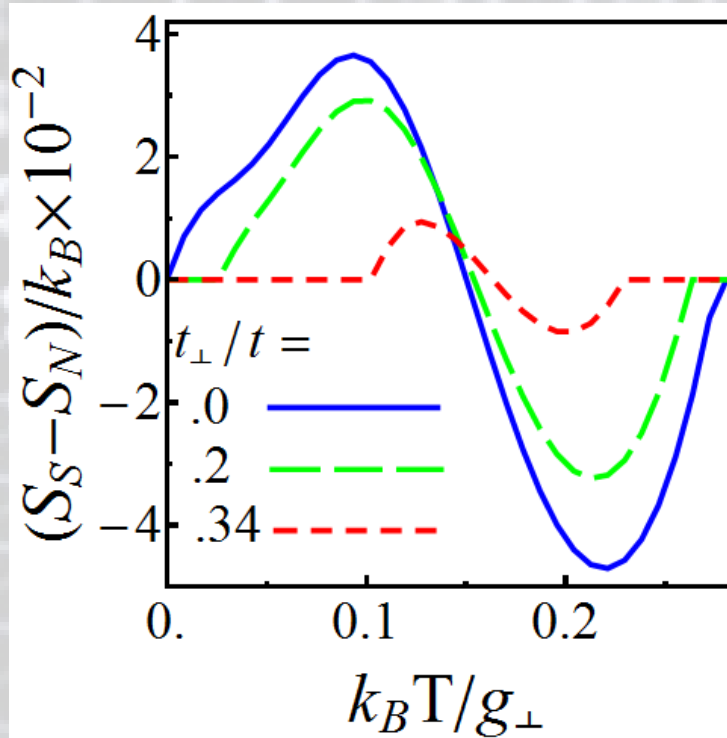
- Interlayer Sc in Bilayer Graphene



$$\alpha E_{\mathbf{k}}^{sl} = \frac{\alpha}{\gamma} \left[ \sqrt{(\Delta_{\perp} + st_{\perp})^2 + \gamma \epsilon_{\mathbf{k}}^2} + l(\Delta_{\perp} + st_{\perp}) \right],$$

# Interlayer Sc in Graphene materials

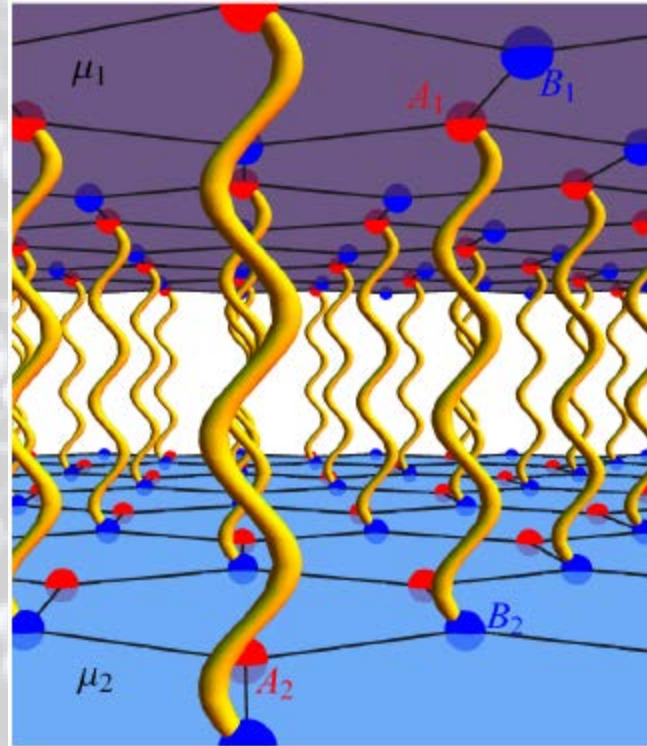
- Interlayer Sc in Bilayer Graphene





# Interlayer Sc in Graphene materials

- Interlayer Sc in Bilayer Graphene



$$h = (\mu_1 - \mu_2)/2$$

# Interlayer Sc in Graphene materials

## • Interlayer Sc in Bilayer Graphene

(a)

$$h < \mu \sqrt{\frac{\Delta_{\perp}^{\gamma} - t_{\perp}^{\gamma}}{\Delta_{\perp}^{\gamma} - t_{\perp}^{\gamma} + \Upsilon \mu^{\gamma}}},$$

(b)

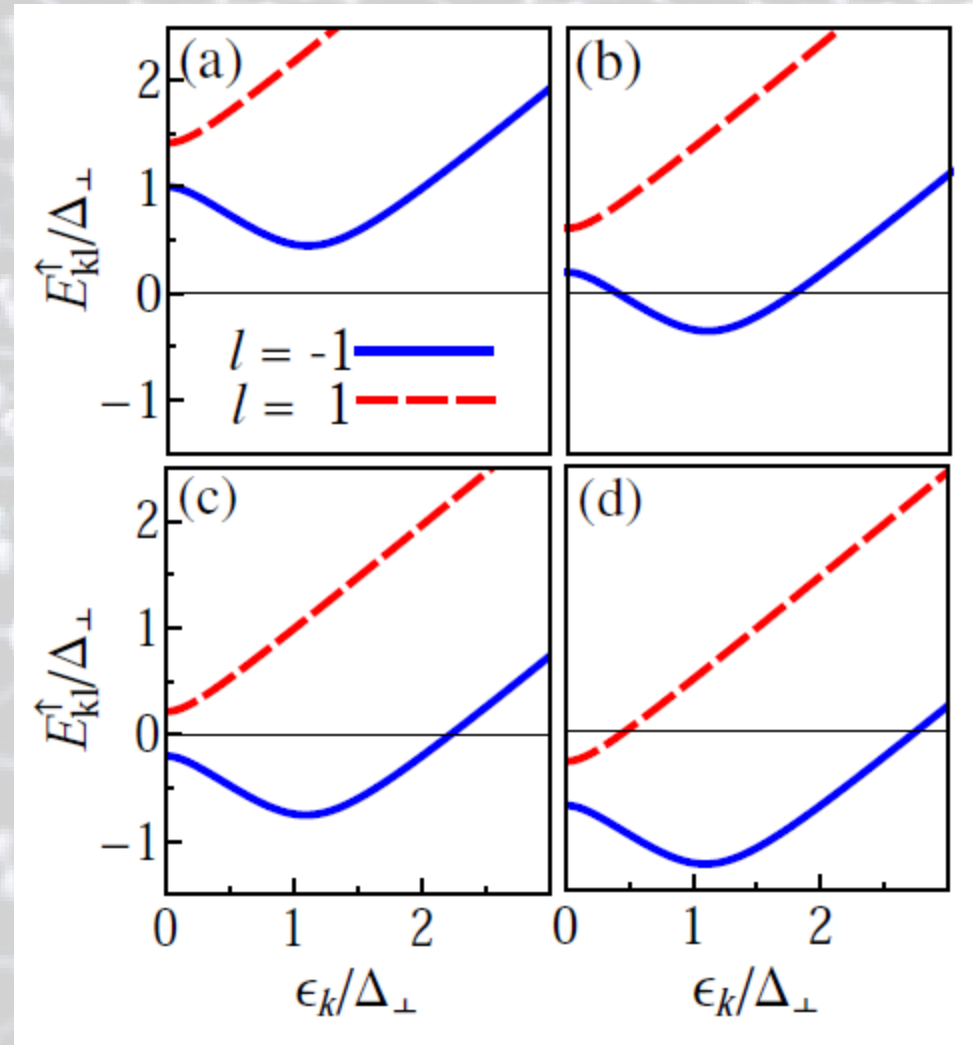
$$\mu \sqrt{\frac{\Delta_{\perp}^{\gamma} - t_{\perp}^{\gamma}}{\Delta_{\perp}^{\gamma} - t_{\perp}^{\gamma} + \Upsilon \mu^{\gamma}}} < h < \mu,$$

(c)

$$\mu < h < \sqrt{\Delta_{\perp}^{\gamma} - t_{\perp}^{\gamma} + \mu^{\gamma}},$$

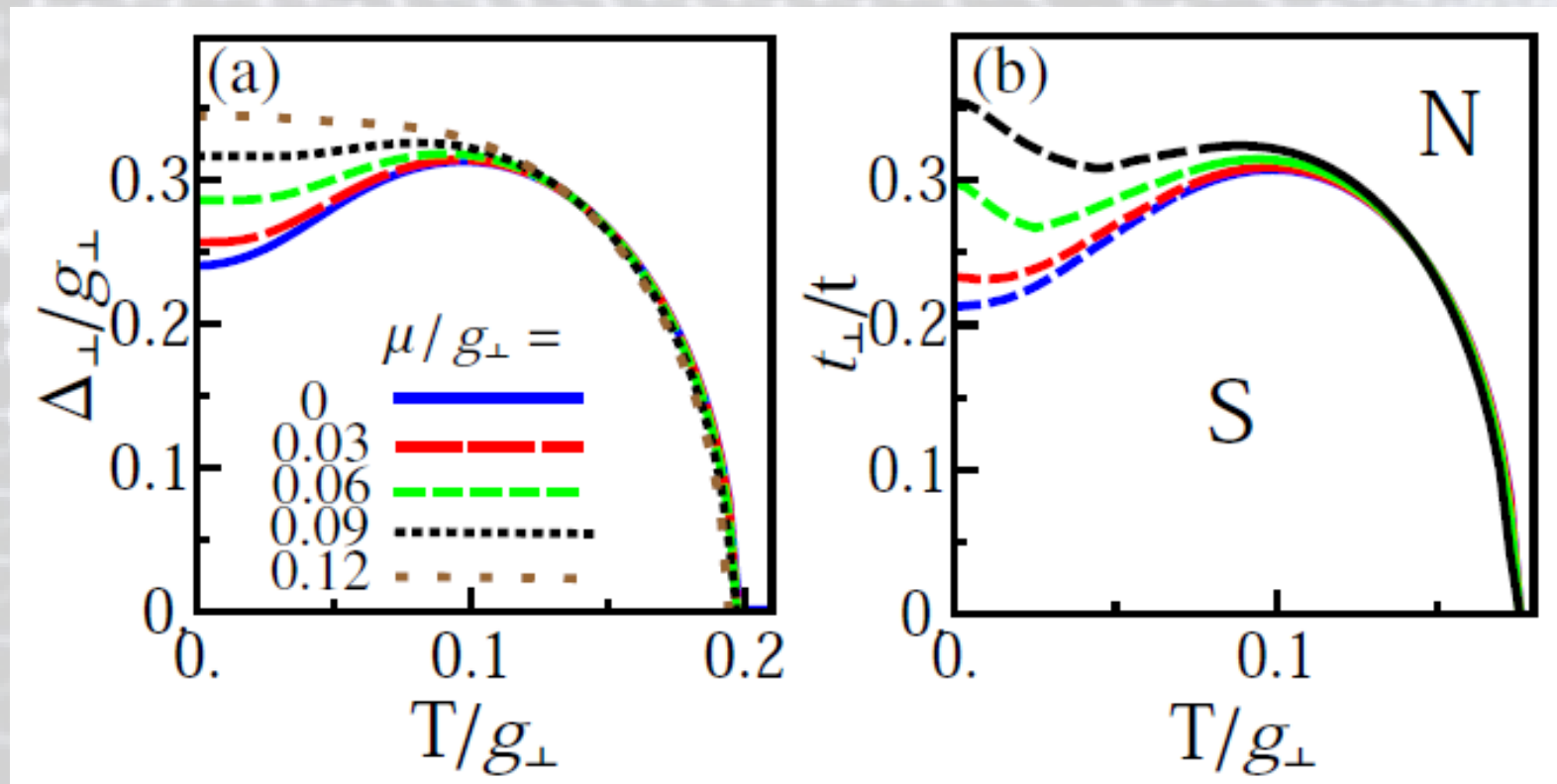
(d)

$$h > \sqrt{\Delta_{\perp}^{\gamma} - t_{\perp}^{\gamma} + \mu^{\gamma}},$$



# Interlayer Sc in Graphene materials

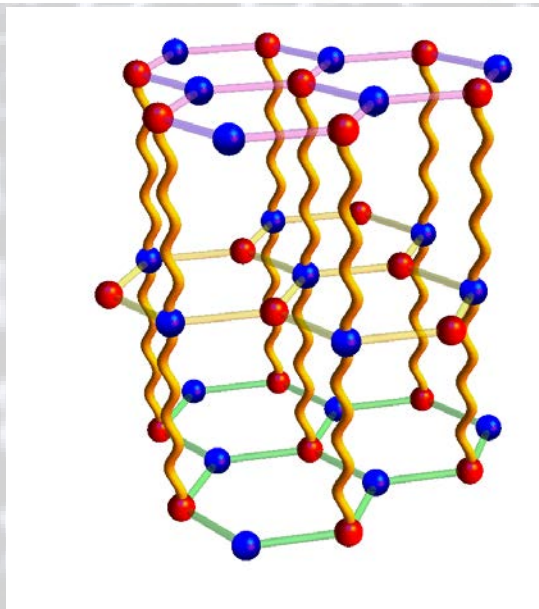
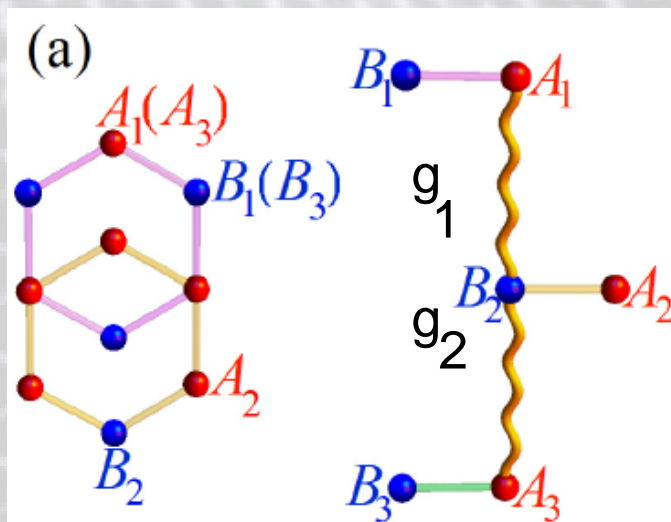
- Interlayer Sc in Bilayer Graphene



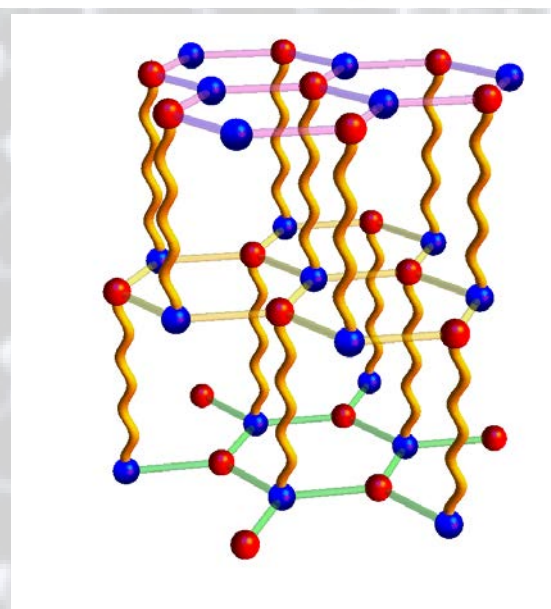
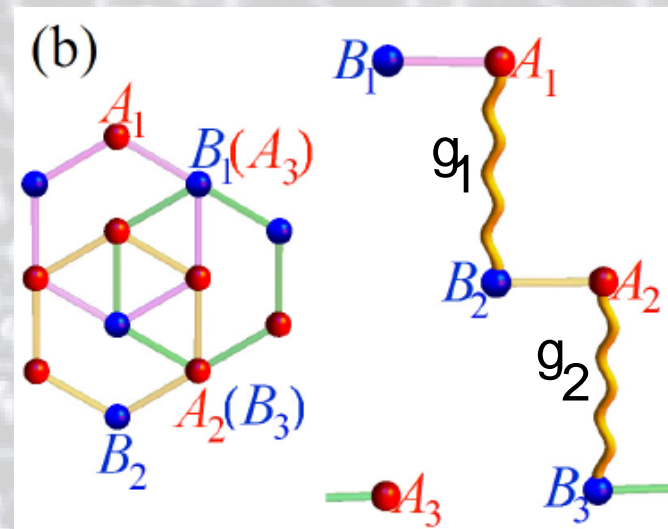
# Interlayer Sc in Graphene materials

- Interlayer Sc in Trilayer Graphene

ABA Trilayer

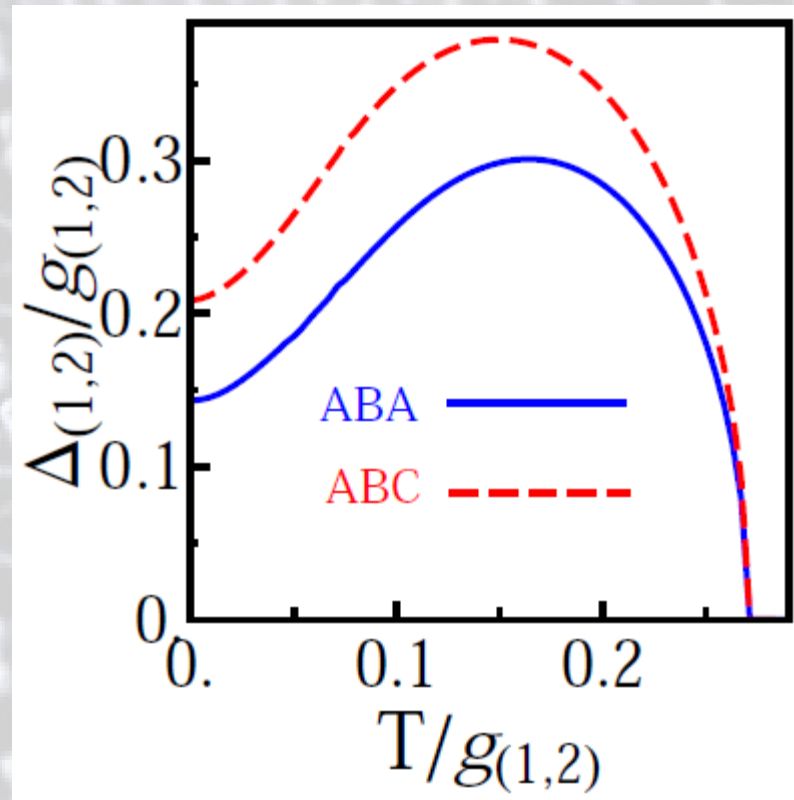


ABC Trilayer



# Interlayer Sc in Graphene materials

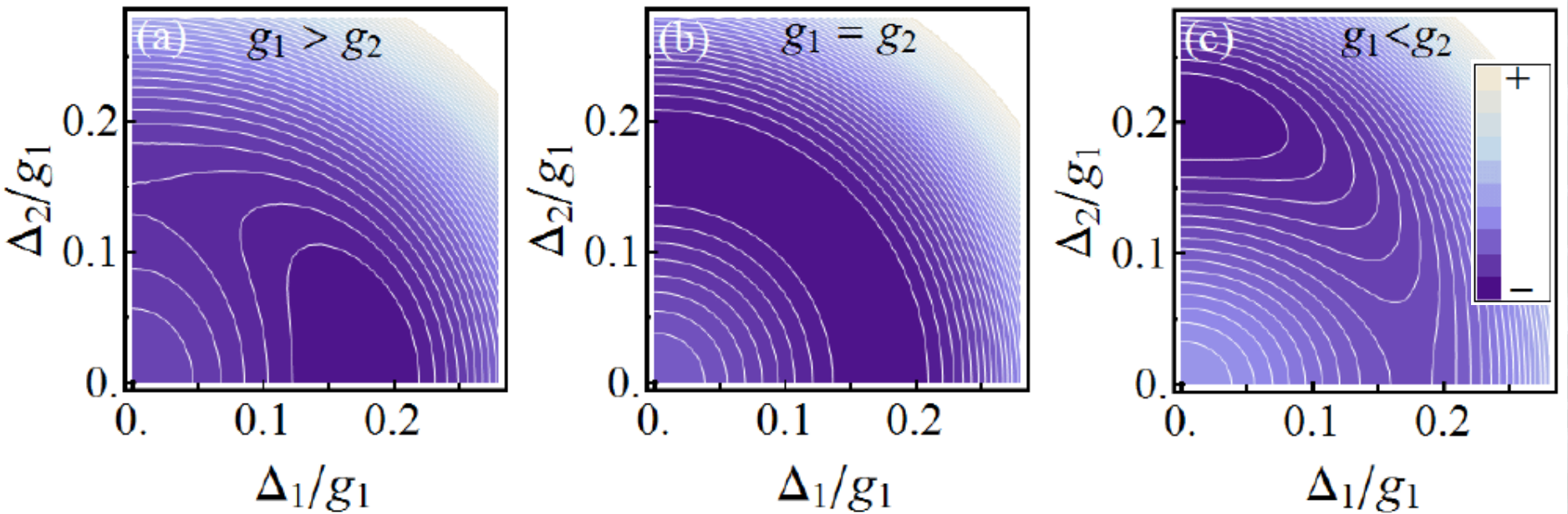
- Interlayer Sc in Trilayer Graphene



# Interlayer Sc in Graphene materials

- Interlayer Sc in Trilayer Graphene

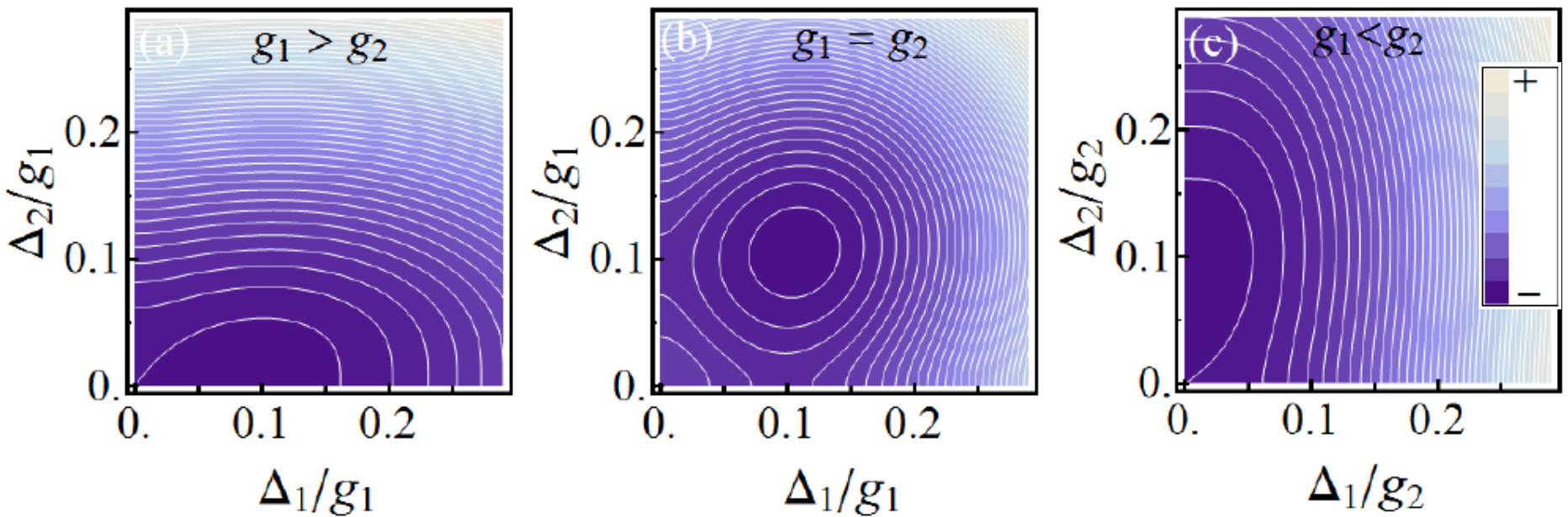
## ABA Trilayer



# Interlayer Sc in Graphene materials

- Interlayer Sc in Trilayer Graphene

## ABC Trilayer



# Conclusions

- (1) We theoretically demonstrate the formation of a new type of unconventional superconductivity in graphene materials.
- (2) This superconductivity is based on an interlayer pairing of chiral electrons in bilayer graphene with exotic s-wave spin-triplet condensate order and anomalous thermodynamic properties.
- (3) These include the possibility of a temperature-induced condensation causing an increase of the pairing gap with increasing temperature and an entropy of the stable superconducting state which can be higher than its value in the normal state.
- (4) At finite mean value chemical potentials of layers removes gapless property of pairing gap and in the high value of mean chemical potential, pairing gap exhibits BCS-like temperature dependence behavior.
- (5) While interlayer superconductivity between layers of ABA stacking displays first order phase transition, phase transition of ABC stacking is of second order.



Thank you for your attention