pp-Waves and AdS-Plane Waves in Null Aether Theory

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Outline

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- *pp*-Wave & Plane Wave Spacetimes
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- Kerr-Schild-Kundt (KSK) Class of Metrics
- KSK Metrics in NAT
- AdS-Plane Waves in NAT ➡
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    - Special Solution in $D > 3$
- Conclusion

Null Aether Theory (NAT)

- NAT is described by

$$I = \frac{1}{16\pi G} \int d^D x \sqrt{-g} \, [R - 2\Lambda - K^{\alpha\beta}{}_{\mu\nu} \nabla_\alpha v^\mu \nabla_\beta v^\nu + \lambda (v^\mu v_\mu + \varepsilon)],$$

$$K^{\mu\nu}{}_{\alpha\beta} = c_1 \, g^{\mu\nu} g_{\alpha\beta} + c_2 \, \delta^\mu_\alpha \delta^\nu_\beta + c_3 \, \delta^\mu_\beta \delta^\nu_\alpha - c_4 \, v^\mu v^\nu g_{\alpha\beta}.$$

NAT

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$$K^{\mu\nu}{}_{\alpha\beta} = c_1 g^{\mu\nu} g_{\alpha\beta} + c_2 \delta^\mu_\alpha \delta^\nu_\beta + c_3 \delta^\mu_\beta \delta^\nu_\alpha - c_4 v^\mu v^\nu g_{\alpha\beta}.$$

(*cosmological const.*) — Λ

(*aether field*) — v^μ

(*Lagrange multiplier*) — λ

(*coupling consts.*) — c_1, c_2, c_3, c_4

Null Aether Theory (NAT)

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$$K^{\mu\nu}{}_{\alpha\beta} = c_1\, g^{\mu\nu}\, g_{\alpha\beta} + c_2\,\delta^\mu_\alpha\,\delta^\nu_\beta + c_3\,\delta^\mu_\beta\,\delta^\nu_\alpha - c_4\, v^\mu\, v^\nu\, g_{\alpha\beta}.$$

- The aether field has the fixed-norm constraint

$[sign = (-, \vec{+})]$

$$v^\mu v_\mu = -\varepsilon, \quad (\varepsilon = 0, \pm 1)$$

$\binom{introduced}{by\ \lambda}$

- $\varepsilon = +1 \Longrightarrow$ Einstein-Aether theory. [Jacobson & Mattingly (2001)]

NAT

- The eqns. of motion are

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \nabla_\alpha \left[J^\alpha{}_{(\mu} v_{\nu)} - J_{(\mu}{}^\alpha v_{\nu)} + J_{(\mu\nu)} v^\alpha \right] + c_1 (\nabla_\mu v_\alpha \nabla_\nu v^\alpha - \nabla_\alpha v_\mu \nabla^\alpha v_\nu) + c_4 \dot{v}_\mu \dot{v}_\nu + \lambda v_\mu v_\nu - \frac{1}{2} L g_{\mu\nu},$$

$\left(\begin{matrix} Einstein - Aether \\ eqns. \end{matrix}\right)$

$$c_4 \dot{v}^\alpha \nabla_\mu v_\alpha + \nabla_\alpha J^\alpha{}_\mu + \lambda v_\mu = 0,$$

$\left(\begin{matrix} Aether \\ eqn. \end{matrix}\right)$

where $\dot{v}^\mu \equiv v^\alpha \nabla_\alpha v^\mu$ and $J^\mu{}_\nu \equiv K^{\mu\alpha}{}_{\nu\beta} \nabla_\alpha v^\beta$, $L \equiv J^\mu{}_\nu \nabla_\mu v^\nu$.

- From now on, $\varepsilon = 0 \Longrightarrow$ NAT.

pp-Wave Spacetimes

- *pp*-waves (plane-fronted waves with parallel rays) are defined by

$$\nabla_\mu l_\nu = 0, \quad l_\mu l^\mu = 0. \quad \longrightarrow \quad \left(\begin{matrix} covariantly\ const. \\ null\ vector\ field \end{matrix}\right)$$

which immediately implies that

$$l^\mu \nabla_\mu l_\nu = 0, \quad \longrightarrow \quad \left(geodesic\ with\ l^\mu = \frac{dx^\mu}{dv}\right)$$

$$\nabla_\mu l_\nu + \nabla_\nu l_\mu = 0, \quad \longrightarrow \quad (automatically\ a\ Killing\ vector)$$

$$\nabla_\mu l_\nu - \nabla_\nu l_\mu = 0. \quad \longrightarrow \quad (l_\mu = \partial_\mu u)$$

pp-Wave Spacetimes

- Consider Kerr-Schild class of *pp*-waves:

$$g_{\mu\nu} = \eta_{\mu\nu} + 2V(x)l_\mu l_\nu,$$

(flat background)

- In the coord. sys. $x^\mu = (u, v, x^i)$,

$$ds^2 = 2dudv + 2V(u, x^i)du^2 + dx_i dx^i.$$

(profile func.)

- For such metrics,

$$R_{\mu\nu} = -(\nabla_\perp^2 V)l_\mu l_\nu \quad \Rightarrow \quad R = 0,$$

$$\nabla_\perp^2 \equiv \partial_i \partial^i$$

Plane Waves

- Plane waves are subclass of *pp*-waves for which

$$V(u,x^i) = \underbrace{h_{ij}(u)}x^i x^j, \quad \Rightarrow \quad R_{\mu\nu} = -2\mathrm{Tr}(h) l_\mu l_\nu,$$

$h_{ij}(u)$ → (*sym. matrix*)

- In Einstein gravity,

$$R_{\mu\nu} = 0 \quad \Rightarrow \quad \mathrm{Tr}(h) = 0$$

$\begin{bmatrix} in\ D = 4, \\ x^i = (x, y) \end{bmatrix}$

$$ds^2 = 2dudv + 2[h_{11}(u)(x^2 - y^2) + 2h_{12}(u)xy]du^2 + dx^2 + dy^2$$

pp-Waves in NAT

- *pp*-Waves constitute exact solutions to NAT.

- To show this, take the null aether field as

$$v^\mu = \underbrace{\phi(x)}l^\mu, \quad l_\mu l^\mu = 0$$

and assume that

(scalar spin−0 aether field)

$$\nabla_\mu l_\nu = 0, \quad l^\mu \partial_\mu \phi = 0.$$

so that

$$l^\mu \nabla_\mu l_\nu = 0, \quad l^\mu \nabla_\nu l_\mu = 0, \quad \dot{v}_\mu = 0,$$

pp-Waves in NAT

- Then the field eqns. of NAT become

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -c_3 \left[\nabla_\alpha \phi \nabla^\alpha \phi - \frac{\lambda}{c_1} \phi^2 \right] l_\mu l_\nu,$$

$$(c_1 \Box \phi + \lambda \phi) l_\mu = 0,$$

$(c_1 \neq 0)$

<u>*Klein-Gordon eqn.*</u>

$$\Box \phi - m^2 \phi = 0$$

$$m^2 \equiv -\frac{\lambda}{c_1} \geq 0$$

$$\Box \equiv \nabla_\mu \nabla^\mu$$

<u>*null dust*</u>

$$T_{\mu\nu} = \mathcal{E} l_\mu l_\nu$$

$$\mathcal{E} \geq 0 \Rightarrow c_3 \leq 0 \ \& \ \frac{\lambda}{c_1} \leq 0$$

pp-Waves in NAT

- For *pp*-waves spacetimes,

$$\nabla_\perp^2 V = c_3 \left[\partial_i\phi\partial^i\phi + m^2\phi^2\right],$$

$$\nabla_\perp^2\phi - m^2\phi = 0, \quad \Lambda = 0.$$

(*with the ansatz*)

$$V(u,x^i) = V_0(u,x^i) + \alpha\phi(u,x^i)^2,$$

(*they decouple*)

$$\nabla_\perp^2 V_0 = 0 \quad \text{for} \quad \alpha = \frac{c_3}{2}.$$

- Thus, *pp*-waves are solutions if the Laplace eqn. for V_0, and the Klein-Gordon eqn. for ϕ are satisfied.

Plane Waves in NAT

- Plane waves $V(u, x^i) = h_{ij}(u)x^i x^j$ can also be constructed:
  - when $c_3 = 0 \Longrightarrow V = V_0$:

$$\nabla_\perp^2 V = 0 \Rightarrow \mathrm{Tr}(h) = 0 \;\; \& \;\; \nabla_\perp^2 \phi - m^2\phi = 0$$

$$ds^2 = 2dudv + 2[h_{11}(u)(x^2 - y^2) + 2h_{12}(u)xy]du^2 + dx^2 + dy^2 \qquad (D = 4)$$

  - when $c_3 \neq 0$, but $V_0 = t_{ij}(u)x^i x^j$: $V = V_0 + \frac{c_3}{2}\phi^2,$

$$\nabla_\perp^2 V_0 = 0 \Rightarrow \mathrm{Tr}(t) = 0,$$

$$\underbrace{[h^k{}_k(h_{ij} - t_{ij}) - (h_{ki} - t_{ki})(h^k{}_j - t^k{}_j)]}_{\text{zero}}\, x^i x^j - \underbrace{m^2}_{\text{zero}}[(h_{ij} - t_{ij})x^i x^j]^2 = 0.$$

Kerr-Schild-Kundt (KSK) Class of Metrics

- KSK class of metrics are defined by [Gürses, Şişman, Tekin (2012)]

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + 2V l_\mu l_\nu, \longrightarrow (generalized\ KS\ form)$$

$$\bigoplus$$

$$l_\mu l^\mu = 0, \quad \nabla_\mu l_\nu = \frac{1}{2}(l_\mu \xi_\nu + l_\nu \xi_\mu),$$
$$l_\mu \xi^\mu = 0, \quad l^\mu \partial_\mu V = 0.$$

Kerr-Schild-Kundt (KSK) Class of Metrics

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$$g_{\mu\nu} = \bar{g}_{\mu\nu} + 2V l_\mu l_\nu,$$ → $(generalized\ KS\ form)$

$\begin{pmatrix} max.sym. \\ background \end{pmatrix}$

$$\bigoplus$$

$$l_\mu l^\mu = 0, \quad \nabla_\mu l_\nu = \frac{1}{2}(l_\mu \xi_\nu + l_\nu \xi_\mu),$$
$$l_\mu \xi^\mu = 0, \quad l^\mu \partial_\mu V = 0.$$

$$l^\mu \nabla_\mu l_\nu = 0,$$

$$\bar{R}_{\mu\alpha\nu\beta} = K(\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta} - \bar{g}_{\mu\beta}\bar{g}_{\nu\alpha})$$
$$K = \frac{\bar{R}}{D(D-1)} = const.$$

$K > 0$	dS
$K = 0$	M
$K < 0$	AdS

$$\theta \equiv \nabla_\mu l^\mu = 0,$$
$$\sigma^2 \equiv \nabla^\mu l^\nu \nabla_{(\mu} l_{\nu)} = 0,$$
$$\omega^2 \equiv \nabla^\mu l^\nu \nabla_{[\mu} l_{\nu]} = 0.$$

$(Kundt\ class)$

KSK Metrics in NAT

- Again assume that

$$v^\mu = \underbrace{\phi(x)}l^\mu, \quad l_\mu l^\mu = 0$$

$\left(\begin{matrix} scalar\ spin{-}0 \\ aether\ field \end{matrix}\right)$

but with

$$\nabla_\mu l_\nu = \frac{1}{2}(l_\mu \xi_\nu + l_\nu \xi_\mu), \quad l_\mu \xi^\mu = 0, \quad l^\mu \partial_\mu \phi = 0,$$

so that

$$l^\mu \nabla_\mu l_\nu = 0, \quad l^\mu \nabla_\nu l_\mu = 0, \quad \nabla_\mu l^\mu = 0, \quad \dot{v}_\mu = 0,$$

- Then the field eqns. of NAT become

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \Big[-c_3 \nabla_\alpha \phi \nabla^\alpha \phi + (c_1 - c_3)\phi \Box \phi - 2c_3 \phi \xi^\alpha \partial_\alpha \phi + \left(\lambda - \frac{c_1 + c_3}{4} \xi_\alpha \xi^\alpha \right) \phi^2 \Big] l_\mu l_\nu - (c_1 + c_3)\phi^2 R_{\mu\alpha\nu\beta} l^\alpha l^\beta,$$

$$[c_1(\Box\phi + \xi^\alpha \partial_\alpha \phi) + \lambda\phi] l_\mu + (c_1 + c_3)\phi R_{\mu\nu} l^\nu = 0,$$

- For the KSK metrics, we have

$$G_{\mu\nu} = -\frac{(D-1)(D-2)}{2} K \bar{g}_{\mu\nu} - \rho l_\mu l_\nu,$$

$$\rho \equiv \bar{\Box} V + 2\xi^\alpha \partial_\alpha V + \left[\frac{1}{2} \xi_\alpha \xi^\alpha + (D+1)(D-2)K \right] V,$$

$$\bar{\Box} \equiv \bar{\nabla}_\mu \bar{\nabla}^\mu$$

- Then we obtain

$$\Lambda = \frac{(D-1)(D-2)}{2}K,$$

$$\bar{\Box}V + 2\xi^\alpha\partial_\alpha V + \left[\frac{1}{2}\xi_\alpha\xi^\alpha + 2(D-2)K\right]V$$
$$= c_3\left[\bar{\nabla}_\alpha\phi\bar{\nabla}^\alpha\phi - \frac{\lambda}{c_1}\phi^2\right] + (c_1+c_3)\phi\xi^\alpha\partial_\alpha\phi$$
$$+ \frac{c_1+c_3}{c_1}\left\{[c_1(D-2) - c_3(D-1)]K + \frac{c_1}{4}\xi_\alpha\xi^\alpha\right\}\phi^2,$$

$$c_1(\bar{\Box}\phi + \xi^\alpha\partial_\alpha\phi) + [\lambda + (c_1+c_3)(D-1)K]\phi = 0,$$

$$(\bar{\Box} + \xi^\alpha\partial_\alpha)\phi - m^2\phi = 0,$$

$$m^2 \equiv -\frac{1}{c_1}[\lambda + (c_1+c_3)(D-1)K]$$

(*For $K = 0$ and $\xi^\mu = 0$, we recover the pp−wave case!*)

- Let us assume the ansatz

$$V(x) = V_0(x) + \alpha\phi(x)^2,$$

- There are two possible choices for α:

$$\bar{\Box}V_0 + 2\xi^\alpha\partial_\alpha V_0 + \left[\frac{1}{2}\xi_\alpha\xi^\alpha + 2(D-2)K\right]V_0 = c_1\left\{\phi\xi^\alpha\partial_\alpha\phi + \left[(D-2)K + \frac{1}{4}\xi_\alpha\xi^\alpha\right]\phi^2\right\}, \quad \left(for\ \alpha = \frac{c_3}{2}\right)$$

$$\bar{\Box}V_0 + 2\xi^\alpha\partial_\alpha V_0 + \left[\frac{1}{2}\xi_\alpha\xi^\alpha + 2(D-2)K\right]V_0 = -c_1\left[\bar{\nabla}_\alpha\phi\bar{\nabla}^\alpha\phi + m^2\phi^2\right]. \quad \left(for\ \alpha = \frac{c_1 + c_3}{2}\right)$$

- Thus, exact wave solutions propagating in nonflat backgrounds can be constructed in NAT.

AdS-Plane Waves in NAT

- Now assume that the background is AdS; i.e.,

$$d\bar{s}^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = \frac{\ell^2}{z^2}(2dudv + dx_i dx^i + dz^2),$$

$$K \equiv -\frac{1}{\ell^2} = -\frac{2|\Lambda|}{(D-1)(D-2)},$$

($z = 0$ *represents the AdS boundary*)

- Then taking $l_\mu = \delta^u_\mu$, one can show that

$$l^\mu = g^{\mu\nu} l_\nu = \bar{g}^{\mu\nu} l_\nu = \frac{z^2}{\ell^2}\delta^\mu_v \quad \Rightarrow \quad l^\alpha \partial_\alpha V = \frac{z^2}{\ell^2}\frac{\partial V}{\partial v} = 0 \;\;\&\;\; l^\alpha \partial_\alpha \phi = \frac{z^2}{\ell^2}\frac{\partial \phi}{\partial v} = 0,$$

$$\nabla_\mu l_\nu = \bar{\nabla}_\mu l_\nu = \frac{1}{z}(l_\mu \delta^z_\nu + l_\nu \delta^z_\mu),$$

$$\left.\begin{aligned} \xi_\mu &= \frac{2}{z}\delta^z_\mu, \\ \xi^\mu &= \frac{2z}{\ell^2}\delta^\mu_z, \end{aligned}\right\} \Rightarrow \quad \xi_\mu \xi^\mu = \frac{4}{\ell^2},$$

- Therefore, AdS-plane waves can be constructed as follows:

$$ds^2 = [\bar{g}_{\mu\nu} + 2V(u, x^i, z) l_\mu l_\nu] dx^\mu dx^\nu = d\bar{s}^2 + 2V(u, x^i, z) du^2,$$

$$V(u, x^i, z) = V_0(u, x^i, z) + \alpha\, \phi(u, x^i, z)^2$$

$$z^2 \hat{\partial}^2 \phi + (4 - D) z\, \partial_z \phi - m^2 \ell^2 \phi = 0,$$

$$z^2 \hat{\partial}^2 V_0 + (6 - D) z\, \partial_z V_0 + 2(3 - D) V_0 = c_1 [2 z \phi \partial_z \phi + (3 - D) \phi^2], \quad \left(for\ \alpha = \frac{c_3}{2} \right)$$

(or)

$$z^2 \hat{\partial}^2 V_0 + (6 - D) z\, \partial_z V_0 + 2(3 - D) V_0 = -c_1 [z^2 (\hat{\partial}\phi)^2 + m^2 \ell^2 \phi^2], \quad \left(for\ \alpha = \frac{c_1 + c_3}{2} \right)$$

where $\hat{\partial}^2 \equiv \partial_i \partial^i + \partial_z^2$ and $(\hat{\partial}\phi)^2 \equiv \partial_i \phi \partial^i \phi + (\partial_z \phi)^2$.

AdS-Plane Waves in 3D

- The field eqns. can be solved exactly in 3D because

$$x^\mu = (u, v, z) \quad \Rightarrow \quad V_0 = V_0(u, z) \;\; \& \;\; \phi = \phi(u, z)$$

- The solution of the aether eqn. is

$$z^2 \partial_z^2 \phi + z \partial_z \phi - m^2 \ell^2 \phi = 0$$

$$\phi(u, z) = a_1(u) z^{m\ell} + a_2(u) z^{-m\ell} \quad (for\ m \neq 0)$$

$$\phi(u, z) = a_1(u) + a_2(u) \ln z \quad (for\ m = 0)$$

where

$$m^2 \equiv -\frac{1}{c_1}\left[\lambda - \frac{2(c_1 + c_3)}{\ell^2}\right]$$

($a_{1,2}$ *are arbitrary funcs.*)

- And the Einstein-Aether eqns. become

$$z^2\partial_z^2 V_0 + 3z\partial_z V_0 = E_1(u) z^{2m\ell} + E_2(u) z^{-2m\ell},$$

$$\left.\begin{aligned} E_1(u) &\equiv 2c_1 m\ell\, a_1(u)^2, \\ E_2(u) &\equiv -2c_1 m\ell\, a_2(u)^2, \end{aligned}\right\} \text{ for } \alpha = \frac{c_3}{2},$$

$$\left.\begin{aligned} E_1(u) &\equiv -2c_1 m^2\ell^2\, a_1(u)^2, \\ E_2(u) &\equiv -2c_1 m^2\ell^2\, a_2(u)^2, \end{aligned}\right\} \text{ for } \alpha = \frac{c_1 + c_3}{2}.$$

$$V_0(u,z) = b_1(u) + b_2(u) z^{-2} + \frac{1}{4m\ell}\left[\frac{E_1(u)}{m\ell + 1}\, z^{2m\ell} + \frac{E_2(u)}{m\ell - 1}\, z^{-2m\ell}\right],$$

when $m\ell \pm 1 \neq 0$.

(*can be absorbed into AdS*)

AdS-Plane Waves in 3D

- When $m\ell + 1 = 0$,

$$V_0(u,z) = b_1(u) + b_2(u) z^{-2} - \frac{E_1(u)}{2}\, z^{-2} \ln z + \frac{E_2(u)}{8}\, z^2,$$

- When $m\ell - 1 = 0$,

$$V_0(u,z) = b_1(u) + b_2(u) z^{-2} + \frac{E_1(u)}{8}\, z^2 - \frac{E_2(u)}{2}\, z^{-2} \ln z.$$

AdS-Plane Waves in 3D

- When $m = 0$,

$$z^2\partial_z^2 V_0 + 3z\partial_z V_0 = E_1(u) + E_2(u)\ln z,$$

$$\left.\begin{array}{l} E_1(u) \equiv 2c_1 a_1(u) a_2(u), \\ E_2(u) \equiv 2c_1 a_2(u)^2, \end{array}\right\} \text{ for } \alpha = \frac{c_3}{2},$$

$$\left.\begin{array}{l} E_1(u) \equiv -c_1 a_2(u)^2, \\ E_2(u) \equiv 0, \end{array}\right\} \text{ for } \alpha = \frac{c_1 + c_3}{2}.$$

$$V_0(u,z) = b_1(u) + b_2(u) z^{-2} + \frac{E_1(u)}{2}\ln z + \frac{E_2(u)}{4}\ln z(\ln z - 1).$$

AdS-Plane Waves in 3D

- For a valid behavior as $z \to 0$,

$$-1 < m\ell < 1 \quad \Rightarrow \quad 0 < m < \sqrt{|\Lambda|},$$

$$\Rightarrow \quad \begin{aligned} &(c_1 + 2c_3)|\Lambda| < \lambda < 2(c_1 + c_3)|\Lambda| \quad \text{if } c_1 > 0, \\ &2(c_1 + c_3)|\Lambda| < \lambda < (c_1 + 2c_3)|\Lambda| \quad \text{if } c_1 < 0. \end{aligned}$$

- Thus, the AdS-plane wave solution is

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{\ell^2}{z^2}(2dudv + dz^2) + 2V(u,z)du^2,$$

$$V(u,z) = V_0(u,z) + \alpha\phi(u,z)^2$$

AdS-Plane Waves in Higher D>3

- In dimensions D>3, generic solution is not possible!

- However, solutions can be obtained if we assume the wave is homogeneous along the transverse coords.:

$$x^\mu = (u, v, x^i, z) \quad \Rightarrow \quad V_0 = V_0(u, z) \;\; \& \;\; \phi = \phi(u, z)$$

- In this case,

$$z^2 \partial_z^2 \phi + (4 - D) z \partial_z \phi - m^2 \ell^2 \phi = 0 \quad \Rightarrow \quad \phi(u, z) = a_1(u) z^{r_+} + a_2(u) z^{r_-}$$

where

$$r_\pm = \frac{1}{2} \left[D - 3 \pm \sqrt{(D-3)^2 + 4m^2\ell^2} \right] .$$

- And

$$z^2\partial_z^2 V_0 + (6-D)z\partial_z V_0 + 2(3-D)V_0 = E_1(u)z^{2r_+} + E_2(u)z^{2r_-},$$

$$\left.\begin{array}{l} E_1(u) \equiv c_1(2r_+ + 3 - D)\, a_1(u)^2, \\ E_2(u) \equiv c_1(2r_- + 3 - D)\, a_2(u)^2, \end{array}\right\} \text{ for } \alpha = \frac{c_3}{2},$$

$$\left.\begin{array}{l} E_1(u) \equiv -c_1(r_+^2 + m^2\ell^2)\, a_1(u)^2, \\ E_2(u) \equiv -c_1(r_-^2 + m^2\ell^2)\, a_2(u)^2, \end{array}\right\} \text{ for } \alpha = \frac{c_1 + c_3}{2}.$$

whose general solution is

$$V_0(u,z) = b_1(u)z^{D-3} + b_2(u)z^{-2} + \frac{E_1(u)}{d_+}\, z^{2r_+} + \frac{E_2(u)}{d_-}\, z^{2r_-},$$

$$\begin{array}{l} d_+ \equiv 4r_+^2 + 2(5-D)r_+ + 2(3-D) \neq 0, \\ d_- \equiv 4r_-^2 + 2(5-D)r_- + 2(3-D) \neq 0. \end{array}$$

- On the other hand, when $d_+ = 0$,

$$V_0(u,z) = b_1(u) z^{D-3} + b_2(u) z^{-2} + \frac{E_1(u)}{4r_+ + 5 - D} z^{2r_+} \ln z + \frac{E_2(u)}{d_-} z^{2r_-},$$

or, when $d_- = 0$,

$$V_0(u,z) = b_1(u) z^{D-3} + b_2(u) z^{-2} + \frac{E_1(u)}{d_+} z^{2r_+} + \frac{E_2(u)}{4r_- + 5 - D} z^{2r_-} \ln z.$$

- All these mean that

$$r_- > -1 \quad \Rightarrow \quad m < \sqrt{\frac{2|\Lambda|}{D-1}} \quad \Rightarrow \quad m < 10^{-42}\,\text{GeV}$$

[$D = 4$ & $|\Lambda| < 10^{-52}\ \text{m}^{-2} \approx 10^{-84}\ (\text{GeV})^2$]

AdS-Plane Waves in Higher D>3

- Then the solution is

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \frac{\ell^2}{z^2}(2dudv + dx_i dx^i + dz^2) + 2V(u,z)du^2,$$

$$V(u,z) = V_0(u,z) + \alpha\phi(u,z)^2$$

$$\left(\begin{array}{c} exact\ plane\ wave\ propagating \\ in\ D{-}dim.\ AdS\ background \\ in\ NAT \end{array}\right)$$

Conclusion

- We constructed exact plane wave solutions in NAT.
- These are important in that they are exact solutions.
- Waves propagating in dS backgrounds can also be constructed! (In progress)