Nonrelativistic, non-Newtonian gravity


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Boğaziçi University

based on arXiv:1512.03799 and work in progress
with Çağın Yunus

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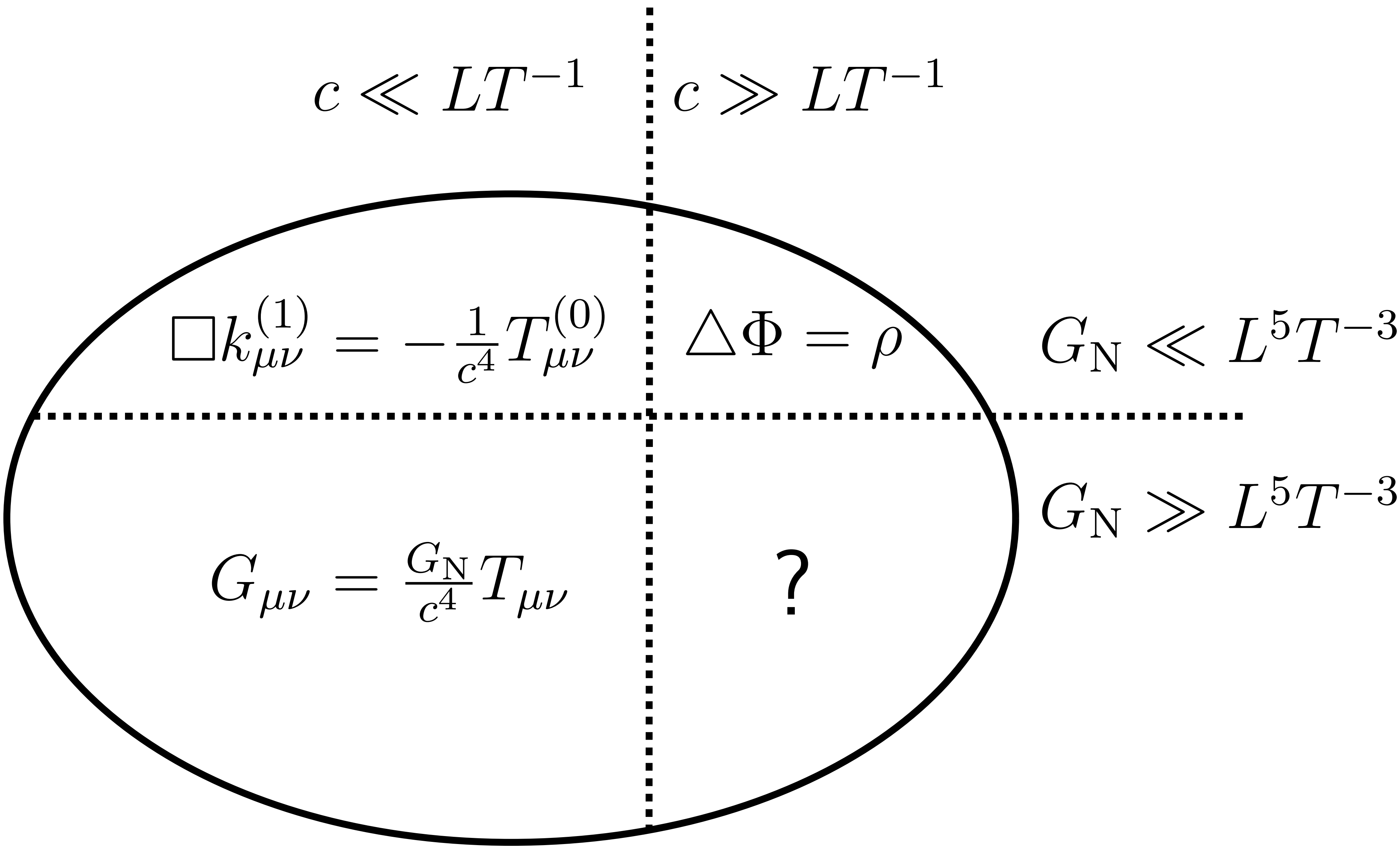
Gravity in various regimes



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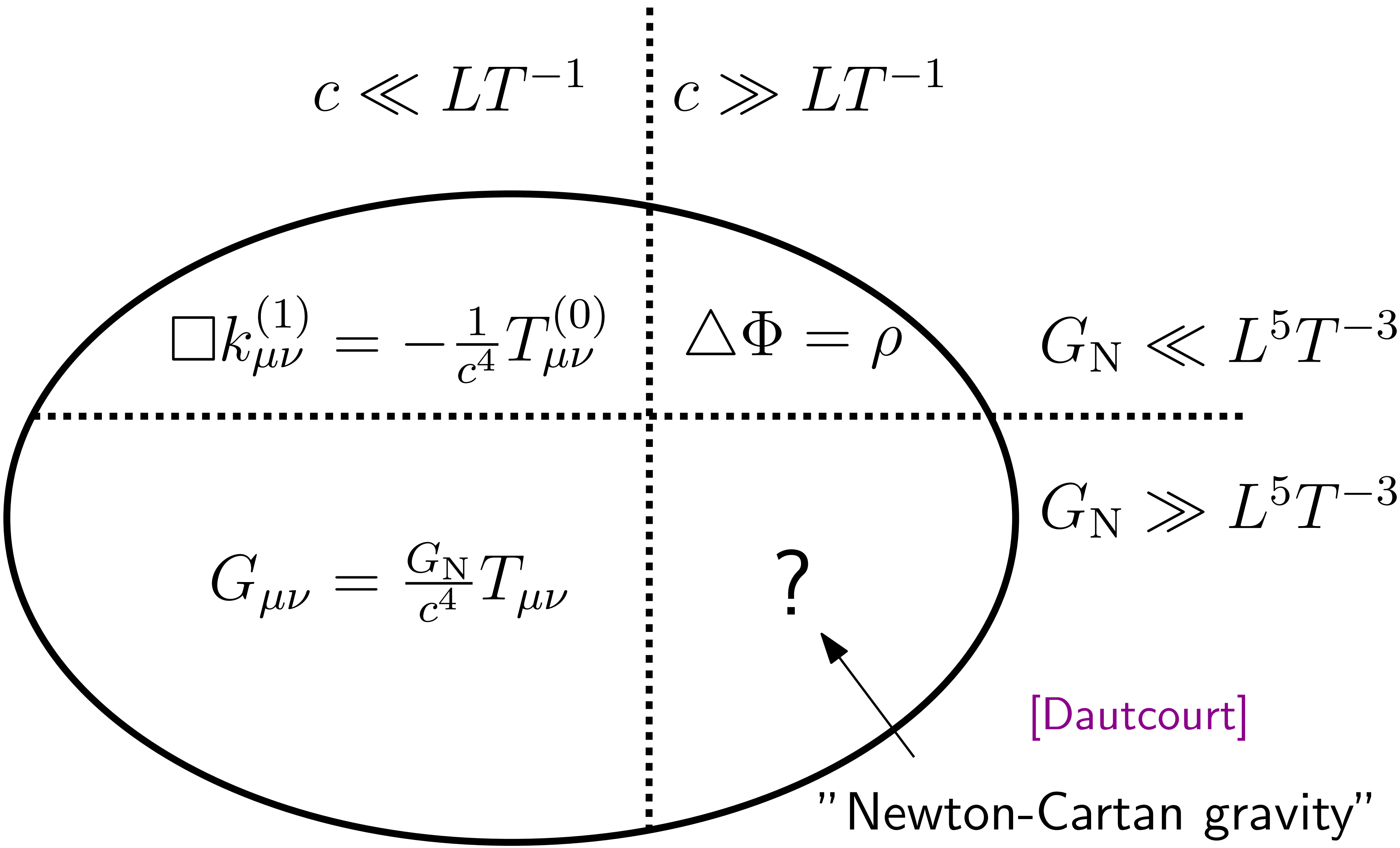

Outline

- Motivation
- From NG towards GR
- NC gravity/geometry
- From GR to NC
- NC $\supsetneq$ NG

  $\rightarrow$ dimensional reduction in NC
- From GR to TTNC
- Example

Legend

NG = Newtonian Gravity   GR = General Relativity

NC = Newton-Cartan   TT = Twistless Torsional

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- NC gravity/geometry
- From GR to NC

Review: [Cartan; Trautman; Kunzl] [Dautcourt; Tichy, Flanagan]

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New

Related work [Afshar, Bergshoeff, Hartong et al.]

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NG = Newtonian Gravity GR = General Relativity

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Motivation

- Isn't GR our best theory of gravity?

  $\rightarrow$ our most complicated theory of gravity

  all aproximation schemes are welcome

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- Why nonrelativistic?

  $\star$ c is quite large in SI units

  more recently

  $\star$ condensed matter on non-flat backgrounds

  $\star$ non-relativistic holography

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  $\star$ non-relativistic holography

- Why non-Newtonian?

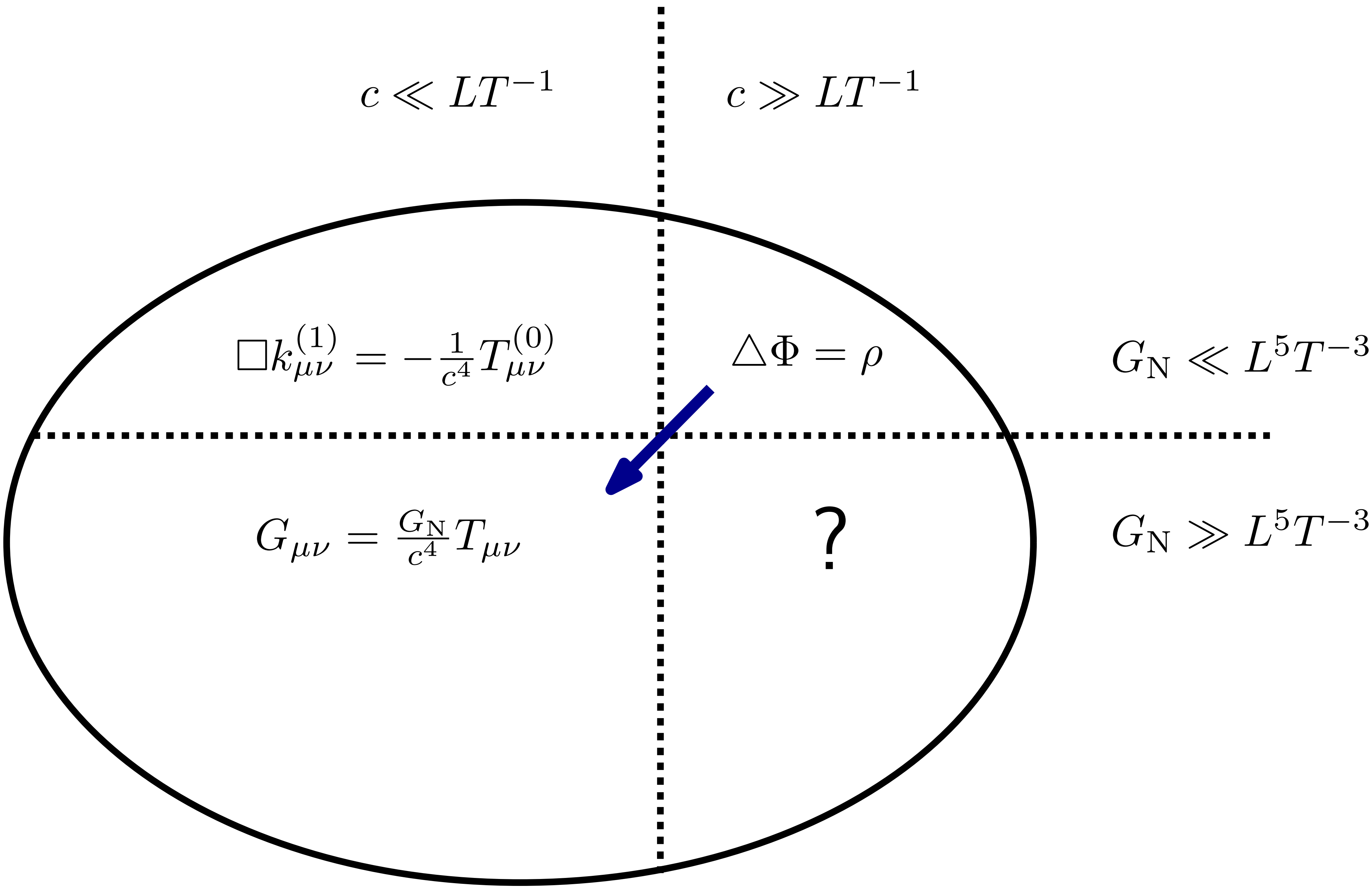
  $\star$ analytic description of strong gravity!

  isn't this an empty set? actually, no...

  is this a physically relevant regime?

  wasn't this done 50 years ago? please let me know!

From NG towards GR



From NG towards GR

Geometrize Newtonian gravity

$\vec{a} = -\vec{\nabla}\Phi \quad \Leftrightarrow \quad$ geodesic equation

$$\downarrow$$

$$\boxed{\Gamma^i_{00} = \partial_i \Phi}$$

$\triangle\Phi = \rho \quad \Leftrightarrow \quad$ curvature condition

$$\downarrow$$

$$\boxed{R_{00}(\Gamma) = \rho}$$

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↓

$$R_{00}(\Gamma) = \rho$$

$\Rightarrow$ covariantize this: Newton-Cartan geometry/gravity

Newton-Cartan geometry/gravity

- Geometry

  fields: $\tau_\mu\,, \quad h^{\mu\nu}\,, \quad \Gamma^\lambda_{\mu\nu}$

  constraints:

$$\tau_\mu h^{\mu\nu} = 0\,, \quad h^{[\mu\nu]} = 0\,, \quad \Gamma^\lambda_{[\mu\nu]} = 0$$

$$\nabla_\mu \tau_\nu = 0 \qquad \nabla_\mu h^{\nu\lambda} = 0$$

$$h^{\lambda(\mu} R^{\nu)}{}_{\lambda\rho\sigma}(\Gamma) = 0$$

Newton-Cartan geometry/gravity

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- Gravity

  EOM: $R_{\mu\nu}(\Gamma) = \tau_\mu \tau_\nu\, \rho$

Newton-Cartan geometry/gravity

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- Gravity

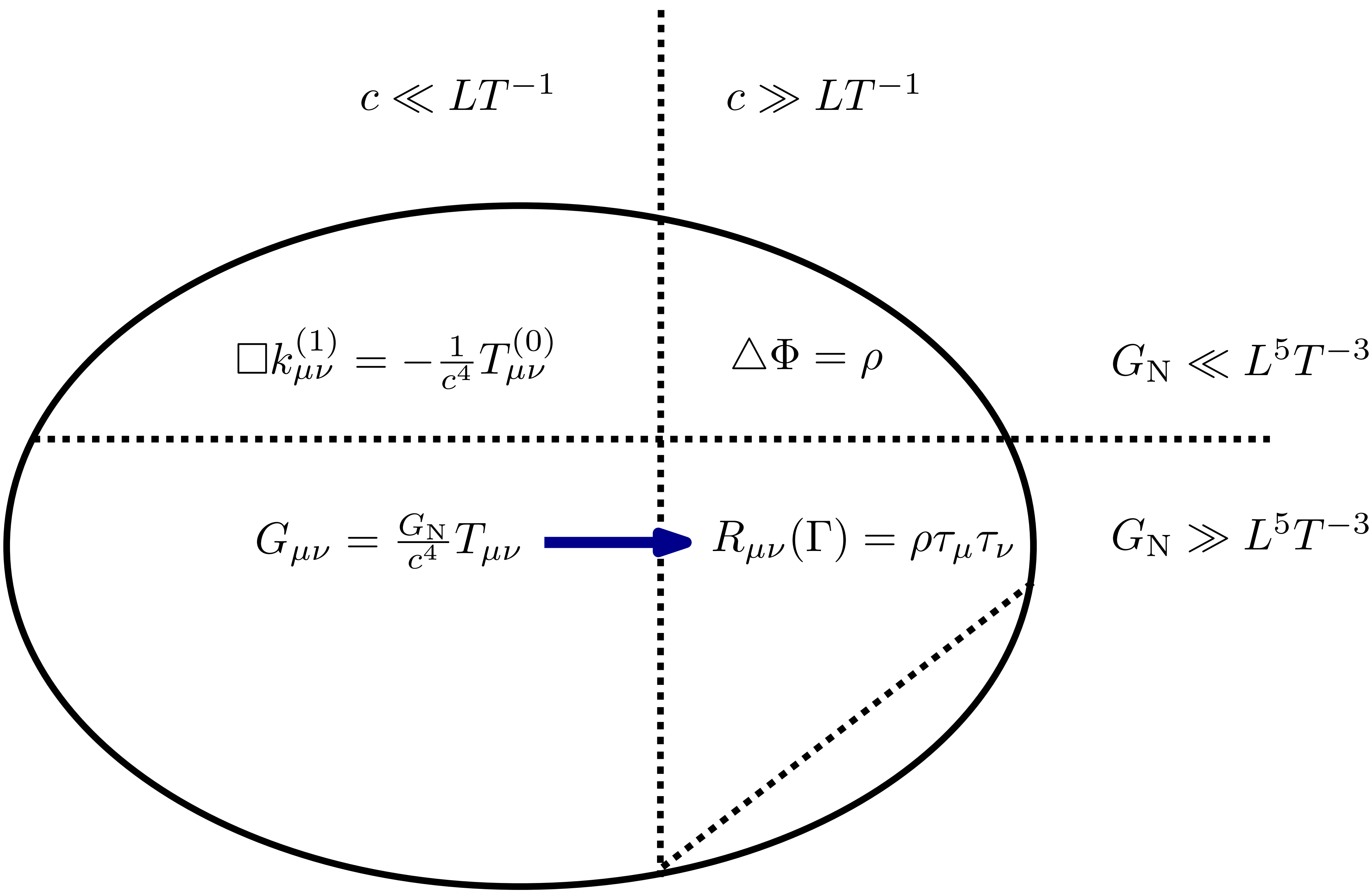
  EOM: $R_{\mu\nu}(\Gamma) = \tau_\mu \tau_\nu\, \rho$

Manifest 1+3 dim coordinate invariance

Equal to Newtonian gravity when

$$\tau_\mu = \delta^0_\mu\,,\ h^{\mu 0} = 0\,,\ h^{ij} = \delta^{ij}\,,\ \Gamma^j_{0i} = 0$$

From GR to Newton-Cartan



From GR to Newton-Cartan

- Large c expansion

$$g_{\mu\nu}(c) = \sum_{i=-1}^{\infty} \overset{(2i)}{g}_{\mu\nu} c^{-2i} \qquad g^{\mu\nu}(c) = \sum_{i=0}^{\infty} \overset{(2i)}{g}{}^{\mu\nu} c^{-2i}$$

Ansatz: $\overset{(-2)}{g}_{\mu\nu} = -\tau_\mu \tau_\nu$ (natural via $x^0 = ct$)

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- Solving invertibility leads to

$$\overset{(0)}{g}{}^{\mu\nu} = h^{\mu\nu}$$

$$\overset{(0)}{g}_{\mu\nu} = 2\tau_{(\mu} C_{\nu)} + h_{\mu\nu} \qquad \overset{(2)}{g}{}^{\mu\nu} = -\tau^\mu \tau^\nu + 2\tau^{(\mu} h^{\nu)\lambda} C_\lambda$$

where $\tau^\mu \tau_\nu + h^{\mu\lambda} h_{\lambda\nu} = \delta^\mu_\nu$

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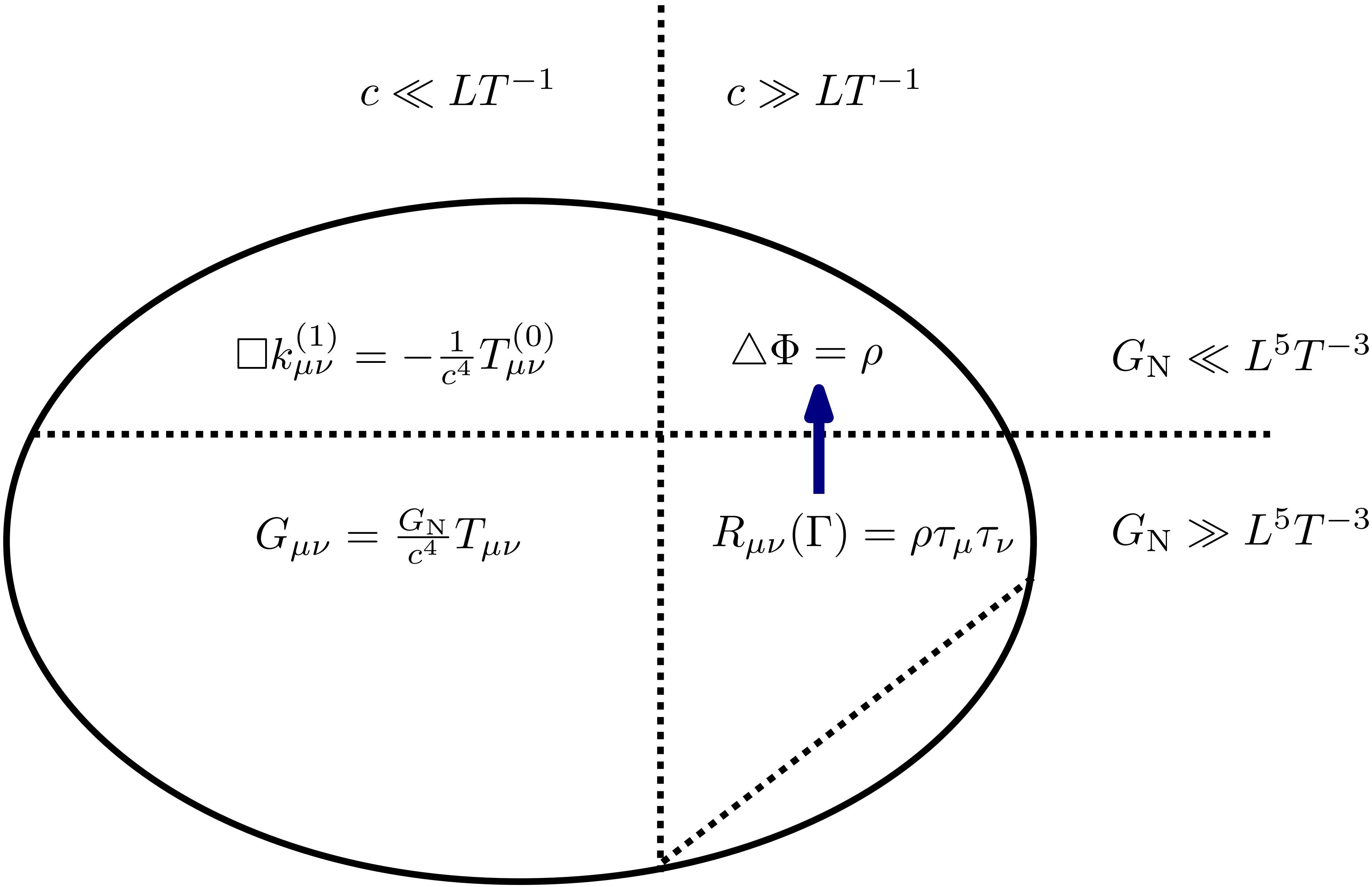
- Expand Einstein equations (for perfect fluid)

$$\overset{(-4)}{R}_{\mu\nu} = \overset{(-2)}{R}_{\mu\nu} = 0 \longleftarrow \text{automatic}$$

$$\overset{(0)}{R}_{\mu\nu} = \overset{(0)}{\mathcal{T}}_{\mu\nu} \quad \Leftrightarrow \quad R_{\mu\nu}(\Gamma) = \rho\, \tau_\mu \tau_\nu$$

Newton - Cartan $\supsetneq$ Newtonian Gravity



Newton - Cartan $\supsetneq$ Newtonian Gravity

The dof's of NC manifest themselves after partial gauge-fixing

$$\text{Diff}_4 \to \text{Diff}_3(t)$$

$$\begin{aligned} R_{ij} &= 0 \\ -\nabla^j K_{ji} &= 2h^{jk}\nabla_{[i}\dot{h}_{j]k} \\ -\nabla^i G_i &= \frac{1}{2}h^{ij}\ddot{h}_{ij} + \frac{1}{4}\dot{h}_{ij}\dot{h}^{ij} - \frac{1}{4}K_{ij}K^{ij} + 4\pi\rho \end{aligned}$$

where $\quad G_i = -\partial_i \Phi - \dot{C}_i \qquad \Phi = -C_0$

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Standard argument:

1) In 3d Ricci flat = flat
2) C_i is harmonic, hence constant
3) NC=NG

Conclusion: nonrelativistic non-Newtonian gravity doesn't exist

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What about higher dimensions?

dimensional reduction

Or does it?

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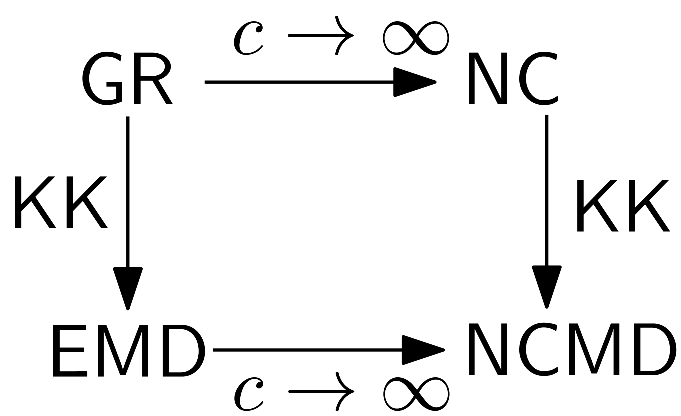
dimensional reduction

We showed [arXiv: 1512.03799]



Here is NCMD

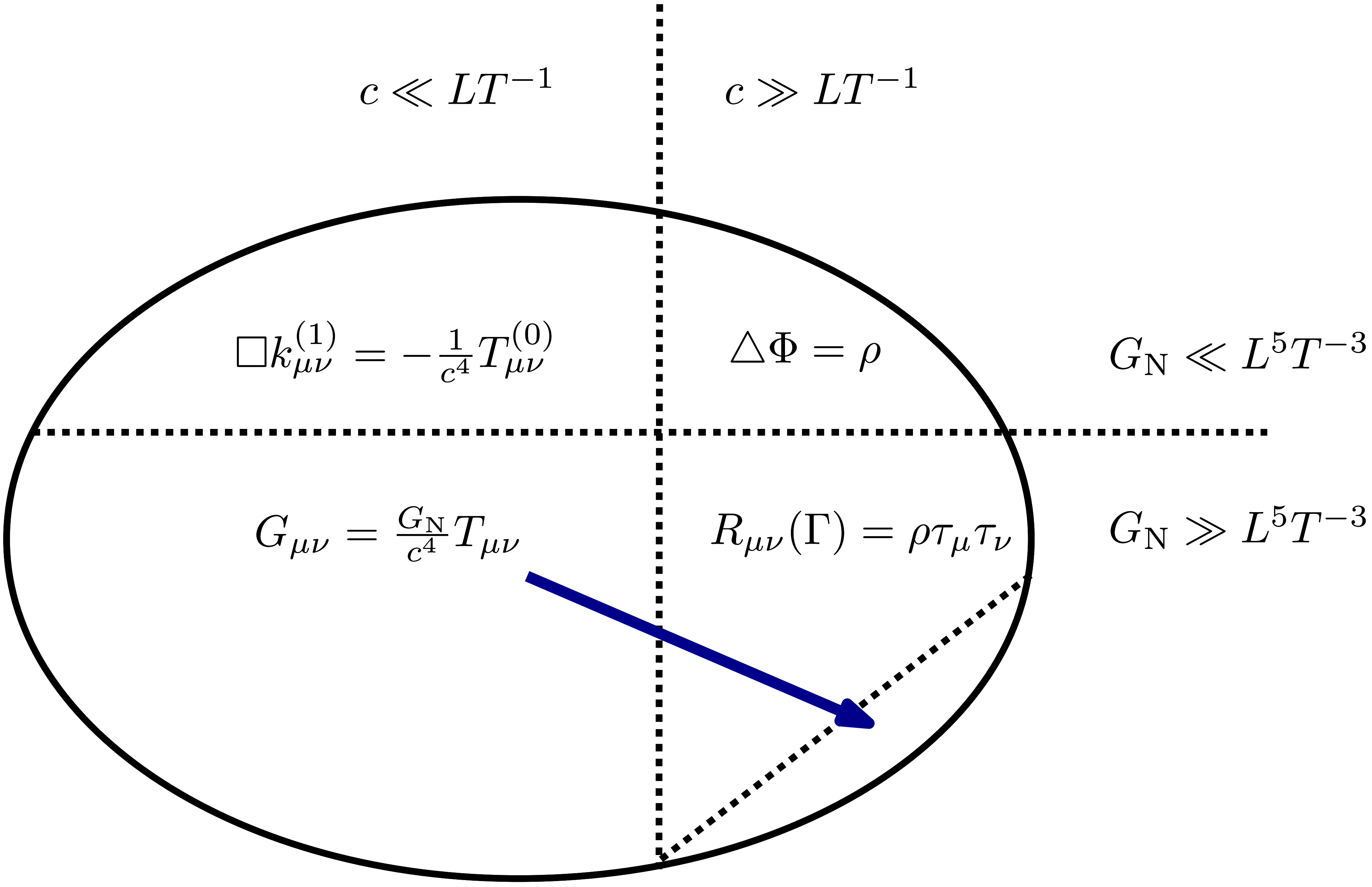
$$\nabla_i \partial^i \Omega = \frac{1}{4}\Omega^3 F_{ij} F^{ij} \qquad \nabla_i \left(\Omega^3 E^i\right) = \frac{1}{2}\Omega^3 K_{ij} F^{ij} \quad \nabla_i \left(\Omega^3 F^{ij}\right) = 0$$

$$\Omega R_{ij} = \nabla_i \partial_j \Omega + \frac{1}{2}\Omega^3 F_i{}^k F_{jk}$$

$$\nabla_j \left(\Omega K^j{}_i\right) = -\Omega\, \partial_i (h^{jk} \dot{h}_{jk}) + \nabla^j (\Omega \dot{h}_{ij}) - 2\partial_i \dot{\Omega} + \Omega^3 F_{ij} E^j$$

$$-\nabla_i \left(\Omega G^i\right) = \frac{1}{4}\Omega \left(\dot{h}^{ij} \dot{h}_{ij} - K_{ij} K^{ij} + 2 h^{ij} \ddot{h}_{ij}\right) + \frac{1}{2}\Omega^3 E_i E^i + \ddot{\Omega}$$

From GR to Twistless Torsional NC



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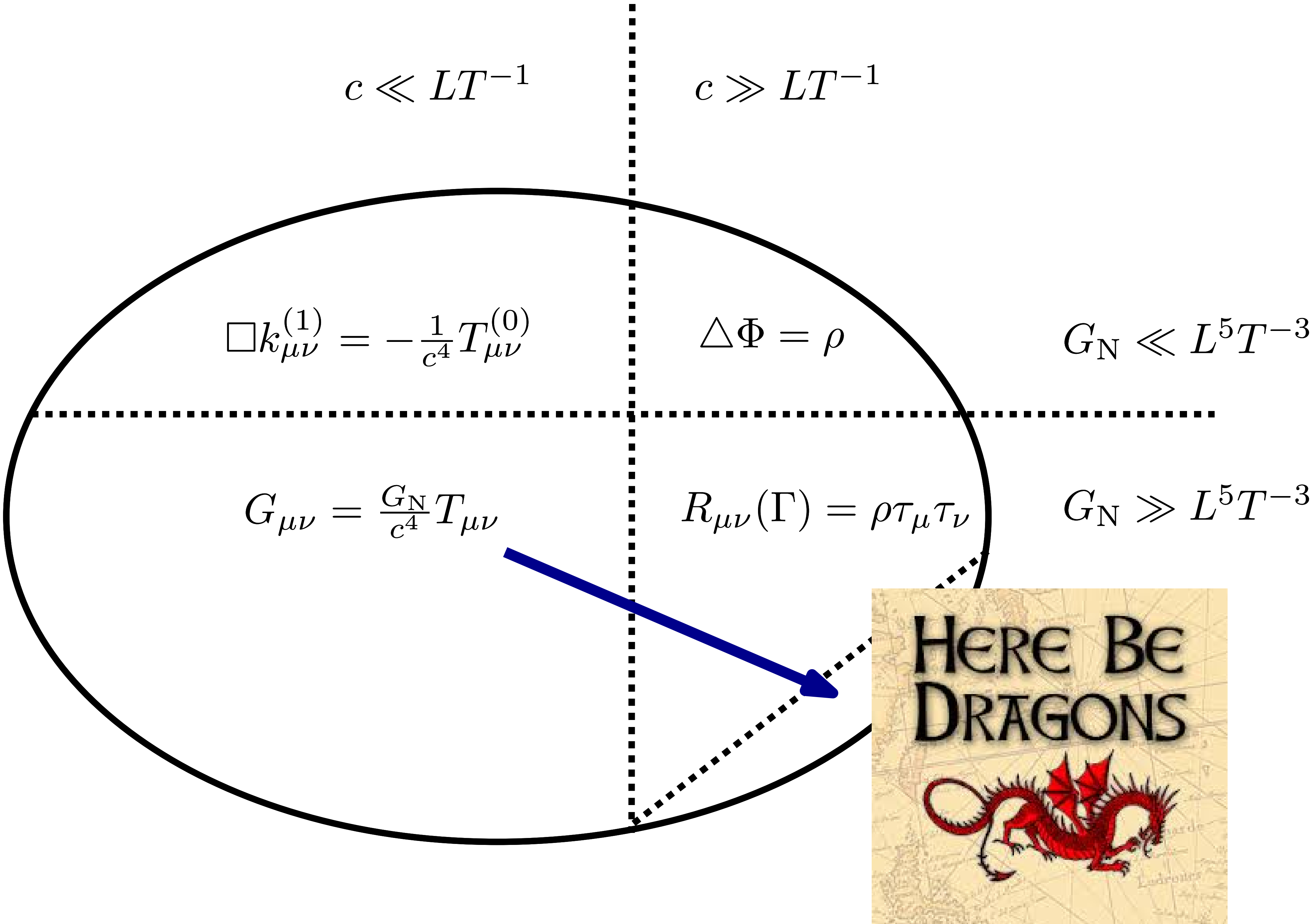

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- Solving invertibility ✓
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↑

Torsional Newton - Cartan geometry

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$$\overset{(0)}{R}_{\mu\nu} = \overset{(nc)}{R}_{\mu\nu} + D^\rho\left(\hat{h}_{\rho\nu}\hat{a}_\mu\right) - \tfrac{1}{2} a^\rho \overset{(nc)}{\nabla}_\rho \hat{h}_{\mu\nu} - a_\rho \tau_{(\mu} \overset{(nc)}{\nabla}_{\nu)} \hat{\tau}^\rho + \tfrac{1}{2}\tau_\mu \tau_\nu a^2 \hat{\tau}^2$$

Example

Schwarzschild

$$\begin{aligned} ds^2 &= -c^2\left(1-\frac{2m}{c^2\,r}\right)dt^2+\left(1-\frac{2m}{c^2\,r}\right)^{-1}dr^2+r^2d\Omega^2 \\ &= -c^2dt^2+\frac{2m}{r}dt^2+dr^2+r^2d\Omega^2+\mathcal{O}(c^{-2}) \end{aligned}$$

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- This solves NC eom
- $\Phi=C_0=\frac{m}{r}$ Newtonian gravity of point mass
- $d\tau=0 \quad\Rightarrow\quad a_\mu=0$ no torsion

Example

Schwarzschild (extremely massive)

$$ds^2 = -c^2\left(1-\frac{2M}{r}\right)dt^2+\left(1-\frac{2M}{r}\right)^{-1}dr^2+r^2d\Omega^2$$

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$$\tau_\mu = \left(1-\tfrac{2M}{r}\right)^{1/2}\delta^t_\mu, \qquad h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \left(1-\frac{2M}{r}\right)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix} \qquad C_\mu = 0$$

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- This solves TTNC eom
- $\Phi = C_0 = 0$ vanishing Newtonian potential !
- $d\tau \neq 0 \quad \Rightarrow \quad a_\mu = -\frac{M}{r^2}\left(1-\frac{2M}{r}\right)^{-1}\delta^r_\mu$ non-vanishing torsion!
- curved spatial geometry!

Nonrelativistic non-Newtonian gravity

Summary & outlook

Two punch lines:

- nonrelativistic gravity is more than a Newtonian potential
- Appearance of TTNC out of GR

In progress/to do:

- precise relation with bottom up TTNC
- add energy momentum
- gauge fixed version
- precise physical interpretation
- real world applications?

Gaussian normal coordinates

There always exist coordinates such that (on a patch)

$$ds^2 \quad = \quad -c^2 d\sigma^2 + g_{ij} dx^i dx^j$$

$\Rightarrow$ doesn't that imply we can always remove the torsion?

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The transformation to GN coordinates is not compatible with (standard) large c expansion

Schwarzschild (extremely massive)

$$\begin{aligned} ds^2 &= -c^2 d\sigma^2 + \frac{2M}{r(\sigma,\rho)} d\rho^2 + r(\sigma,\rho)^2 d\Omega^2 \\ &= -c^2 d\sigma^2 + c^{4/3} \left(\frac{9}{2} M\right)^{2/3} \sigma^{4/3} d\Omega^2 + \mathcal{O}(c^{1/3}) \end{aligned}$$

$$r(\sigma,\rho) = \left(-\tfrac{3}{2}\sqrt{2M}(c\sigma + \rho)\right)^{2/3}$$