Various Time-Scales of Relaxation

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Chesler, Yaffe; Das, Nishioka, Takayanagi, Basu; Bhattacharyya, Minwalla; Abajo-Arrastia, Aparicio, Lopez; Albash, Johnson; Ebrahim, Headrick; Balasubramanian, Bernamonti, de Boer, Copland, Craps, Keski-Vakkuri, Mueller, Schafer, Shigemori, Staessens, Galli; Allias, Tonni; Keranen, Keski-Vakkuri, Thorlacius; Galante, Schvellinger; Carceres, Kundu; Wu; Garfinkle, Pando Zayas, Reichmann; Bhaseen, Gauntlett, Simons, Sonner, Wiseman; Auzzi, Elitzur, Gudnason, Rabinovici; . . . . . .

Outline:

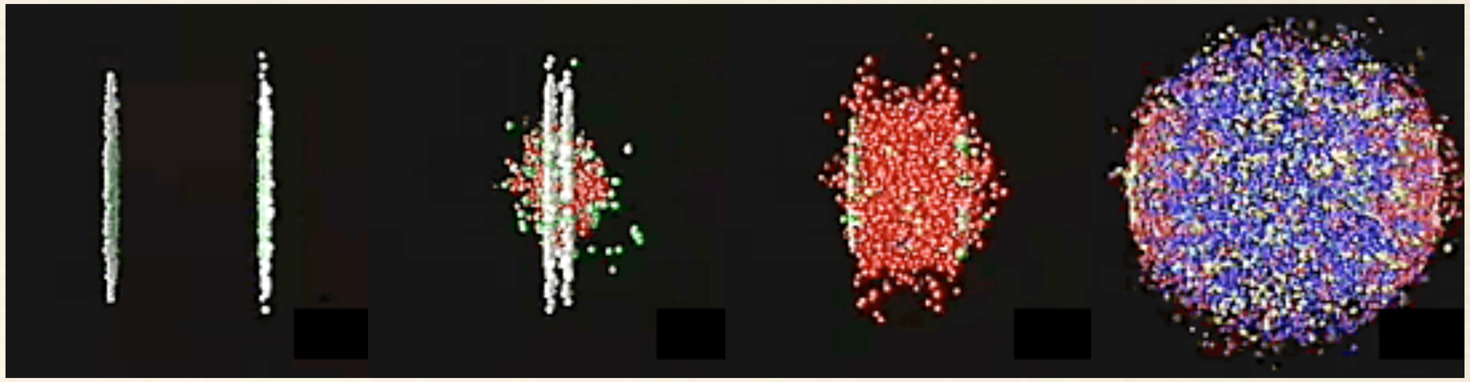
- Out-of-Equilibrium Physics
- Holographic Out-of-Equilibrium Systems
  - In the presence of a source
  - Initial out-of-equilibrium states
- Numerics
- Conclusion

Motivation:

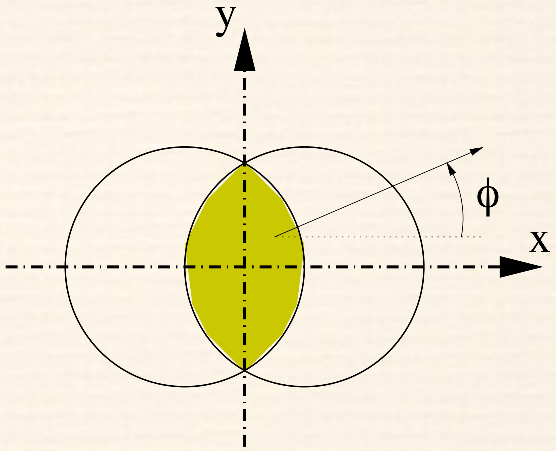
* system in equilibrium
  - non-dynamical, stationary
  - late-time state of a generic system
* time-dependent processes
  - perturbative methods
  - coarse-grained view of the system, low energy effective description, Hydrodynamics, Universal features
  - lack of adequate techniques for strongly correlated condensed matter theories, quark-gluon plasma

Thermalization in QGP

* Collision of two heavy nuclei (Gold or Lead) at the relativistic speed



* Production of an Anisotropic Plasma



* Hydrodynamics applies after a very short time-scale, 1 fm

Rapid Thermalization

* Strongly Coupled Plasma

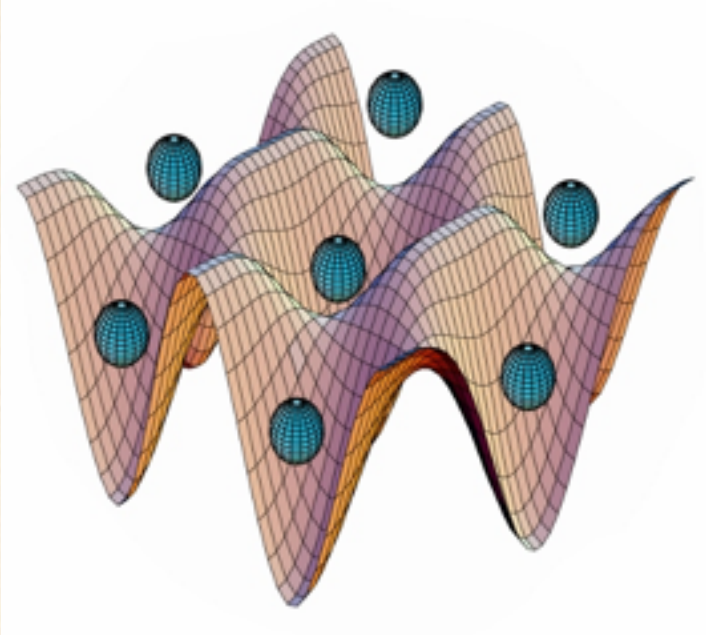
Shuryak (2003,2004); Heinz (2004); Luzum, Romatschke (2008)

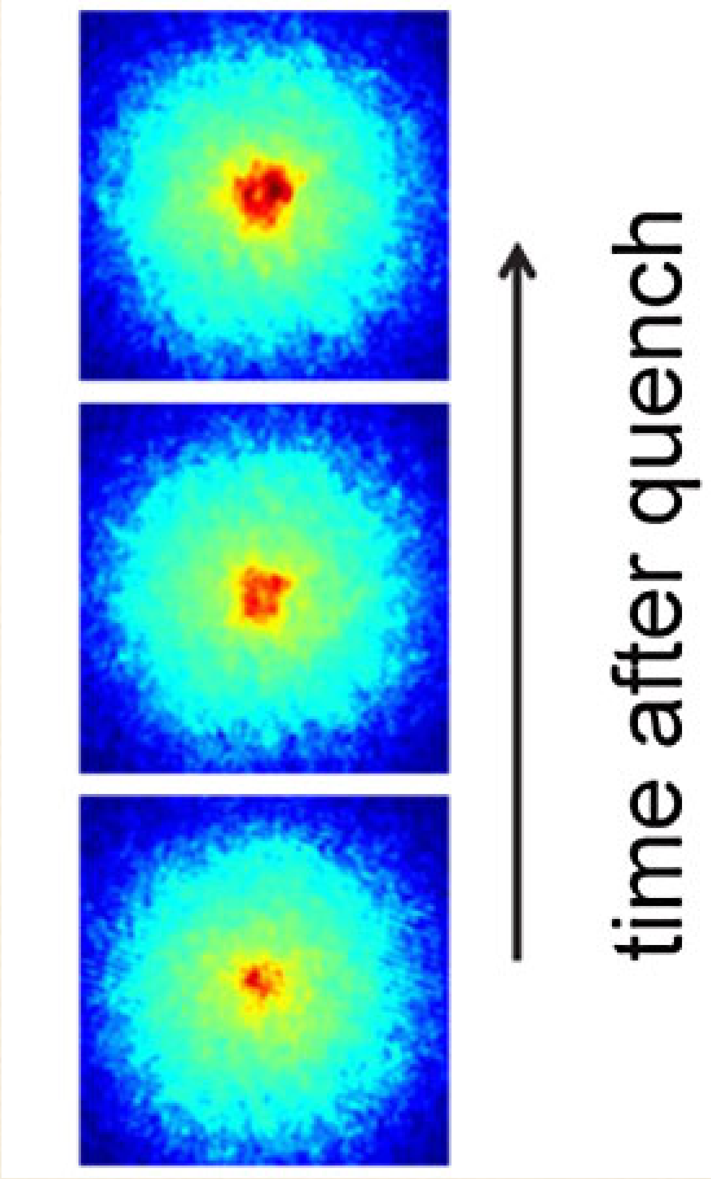
Cold Atom Systems:

Ultracold atomic gases well suited for quantum quench experiments:

- possibility of rapid quantum quench by dynamically varying microscopic Hamiltonian parameters

- near perfect isolation from environment





Real-Time Non-Equilibrium Experimental Studies

Smith, Beattie, Moulder, Campbell, Hadzibabic (2012)

- What are the organizing principles for out-of-equilibrium systems?
  - Theoretical progress has been made for variety systems; 2d CFT, free field, integrable models.
  - Still seeking more applicable techniques.
- Perhaps re-organization of the problem will lead to new insights.
- Focus on field theories with holographic dual. Two dual theories are just different descriptions of the same physical system.
- What can AdS/CFT correspondence offer?
  - strongly coupled field theories
  - real-time analysis
  - finite temperature
  - general space-time dimensions

Holographic Out-of-Equilibrium Systems:

- There are two methods to produce out-of-equilibriums systems:
  - injecting energy by turning on a source $H_\lambda = H_0 + \lambda\, \delta H$
  - starting from out-of-equilibrium initial states
- In the gauge/gravity framework these correspond to:
  - deforming the boundary field theory by a time-dependent coupling $\mathcal{L}_{SYM} + \lambda_\Delta(t)\ \mathcal{O}_\Delta$
  - gravity configuration on the initial time-slice

Deforming the SYM Action:

S. Bhattacharyya, S. Minwalla; 2009

P. Chesler, L. Yaffe; 2009

A. Buchel, L. Lehner, R. Myers; 2012

$$S \to S + \int d^4x\, \phi(x)\mathcal{O}(x)$$

Time-Dependent Coupling on the Boundary

Non-Normalizable Mode of the Bulk Field Dual to the Corresponding Operator

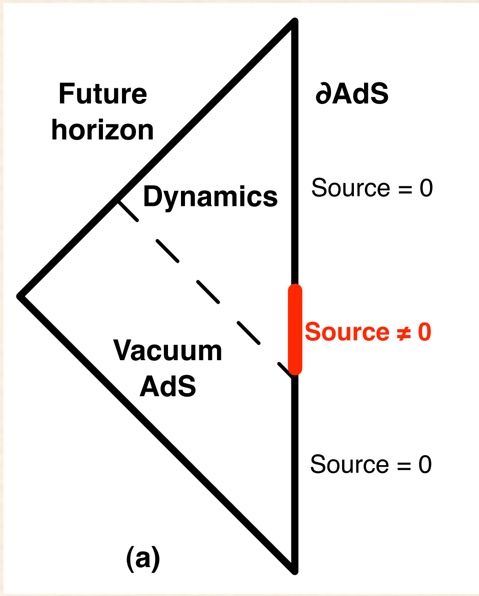
$$\phi(x) = \Phi|_{\partial AdS}(x) \equiv \lim_{z\to 0} z^{\alpha_\Phi}\Phi(x,z)$$

$$\int d^4x\, a_\mu(x) J^\mu(x)$$

$$a_\mu(x) = \lim_{z\to 0} A_\mu(z,x)$$

$$\int d^4x\, h_{\mu\nu}(x) T^{\mu\nu}(x)$$

$$g^{(b)}_{\mu\nu}(x) = \lim_{z\to 0} \frac{z^2}{R^2} g_{\mu\nu}(z,x) = \eta_{\mu\nu} + h_{\mu\nu}$$



The strongly coupled field theory evolves to an equilibrium state.

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The evolution in the bulk is described by Einstein equations plus other present field equations of motion.

Equilibrium state is field theory is dual to static black hole at temperature T.

- We are interested in an unforced relaxation to the thermal state.

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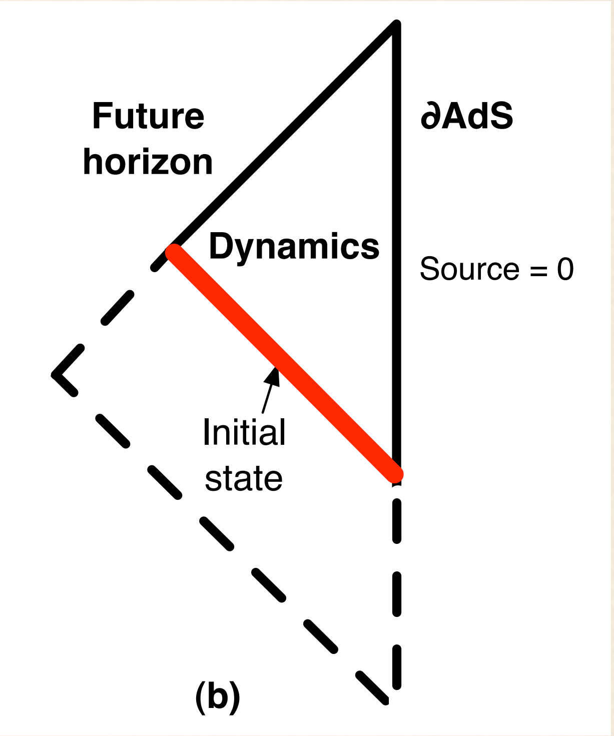
  no time-dependent source pumping energy into the system



- One needs to specify the initial out-of-equilibrium state.
- Initial state in the field theory is dual to bulk field configurations on an initial time-slice.
- The form of the bulk metric is determined by the ansatz for the boundary stress tensor:
  - homogeneity assumption; not preceded by a hydrodynamic phase
  - rotational symmetry in two of the space-like directions
  - flatness of the boundary metric
  - The most general conserved and traceless stress tensor is:

$$\langle T_{\mu\nu}\rangle = \frac{N_c^2}{2\pi^2}\mathrm{diag}\Big[\mathcal{E},\, \mathcal{P}_{\mathrm{L}}(t),\, \mathcal{P}_{\mathrm{T}}(t),\, \mathcal{P}_{\mathrm{T}}(t)\Big]$$

$$\mathcal{P}_{\mathrm{L}}(t) = \tfrac{1}{3}\mathcal{E} - \tfrac{2}{3}\Delta\mathcal{P}(t)$$

$$\mathcal{P}_{\mathrm{T}}(t) = \tfrac{1}{3}\mathcal{E} + \tfrac{1}{3}\Delta\mathcal{P}(t)$$

homogeneity + conservation of the stress tensor → $\partial_t \varepsilon = 0$ → energy density as part of initial conditions

The only possible static state with finite energy density is the isotropic and homogeneous plasma.

→ The final state is already known from the start.

Heller, Mateos, van der Schee, Trancanelli; 2012

Heller, Mateos, van der Schee, Triana; 2013

- We are interested in studying different relaxation time-scales.
- both metric and scalar field in the bulk:

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(\mathcal{R} + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 \right)$$

- The symmetries of the stress tensor suggests:

$$ds_5^2 = 2drdt - A(t,r)dt^2 + \Sigma(t,r)^2 \left(e^{-2B(t,r)} dx_L^2 + e^{B(t,r)} d\mathbf{x}_T^2 \right)$$

$$\phi = \phi(t,r)$$

- Since there is no source terms, different initial conditions with the same value of energy density must relax to homogeneous and isotropic black hole:

$$ds^2 = 2dtdr - r^2 \left(1 - \frac{(\pi T)^4}{r^4} \right) dt^2 + r^2 d\vec{x}^2$$

$$4\mathcal{E} = 3\pi^4 T^4$$

- The field ansatz should satisfy Einstein plus scalar equations of motion.

- Solving the equations of motion in the near boundary expansion gives the link between the form of the expectation value of the boundary operators and their dual bulk fields:

$$A(t,r) = r^2 + \frac{a_4}{r^2} - \frac{2b_4(t)^2}{7r^6} + \ldots,$$
$$B(t,r) = \frac{b_4(t)}{r^4} + \frac{\partial_t b_4(t)}{r^5} + \ldots,$$
$$\Sigma(t,r) = r - \frac{b_4(t)^2}{7r^7} + \ldots,$$
$$\phi(t,r) = \frac{\phi_2(t)}{r^3} + \ldots,$$

$$\Delta\mathcal{P}(t) = 3b_4(t), \qquad \mathcal{E} = -\frac{3a_4}{4},$$
$$-2\phi_2(t) = 16\pi G_5 \langle \mathcal{O} \rangle$$

- The residual diffeomorphism freedom $r \to r + f(t)$is fixed by the absence of the term proportional to r in the near boundary expansion of A(t,r).
- Note that we have assumed $m^2 = -3$.
- The normalizable time-dependent modes $a_4,\ b_4(t),\ \phi_2(t)$ are determined by evolving the metric components and the scalar field forward in time from appropriate initial configurations.

Equations of Motion:

- Using the definition of derivative along outgoing radial null geodesics:
$$\dot{h} \equiv \partial_t h + \frac{1}{2} A \partial_r h$$

- the equations of motion get the nested form:

$$0 = 2\partial_r(\dot{\phi}) + \frac{3\partial_r \Sigma}{\Sigma}\dot{\phi} + \frac{3\partial_r \phi}{\Sigma}\dot{\Sigma} - m^2\phi$$

$$0 = 2\partial_r(\dot{B}) + \frac{3\partial_r \Sigma}{\Sigma}\dot{B} + \frac{3\dot{\Sigma}}{\Sigma}\partial_r B$$

$$0 = \Sigma\partial_r(\dot{\Sigma}) + 2\dot{\Sigma}\partial_r\Sigma - 2\Sigma^2 + \frac{1}{12}m^2\phi^2\Sigma^2$$

$$0 = \partial_r^2 A - \frac{12}{\Sigma^2}\dot{\Sigma}\partial_r\Sigma + 3\dot{B}\partial_r B + 4 + \dot{\phi}\partial_r\phi - \frac{1}{6}m^2\phi^2$$

evolution equations for the initial-value problem; specify the form of the metric and scalar field on the neighboring time slice.

constraint equations; the evolution equations guarantee that they are obeyed provided that:

$$0 = \partial_r^2\Sigma + \frac{1}{6}\Sigma\left(3(\partial_r B)^2 + (\partial_r\phi)^2\right)$$

holds on the initial time slice.

$$0 = \ddot{\Sigma} - \frac{1}{2}\partial_r A\dot{\Sigma} + \frac{1}{6}\Sigma(3\dot{B}^2 + \dot{\phi}^2)$$

holds on the boundary.

Initial Conditions:

1. satisfy the constraint equation:

$$0 = \partial_r^2 \Sigma + \frac{1}{6}\Sigma\left(3(\partial_r B)^2 + (\partial_r \phi)^2\right)$$

2. right form of the near boundary expansion:

$$B(t,r) = \frac{b_4(t)}{r^4} + \frac{\partial_t b_4(t)}{r^5} + \dots$$

3. bulk regularity

* The initial conditions are given for the metric anisotropic function, B(t,r), and the scalar field, $\phi(t,r)$, on the initial time slice.

* The functions we have considered are:

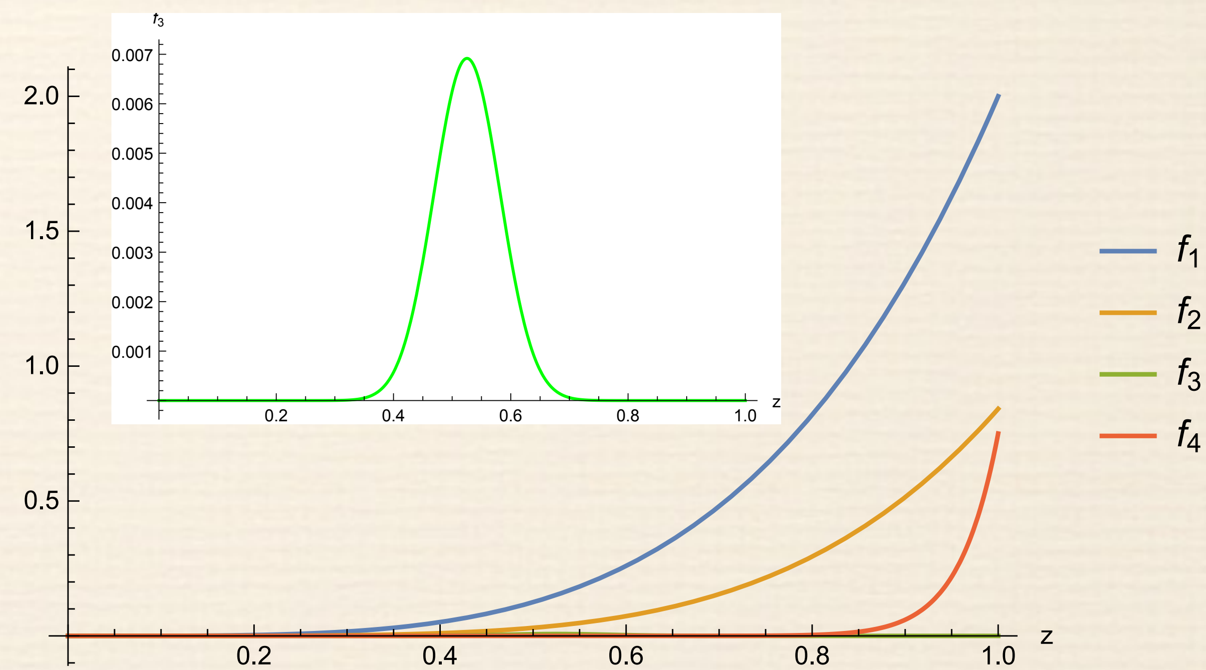
$$f_1(t=0,z) = \frac{8}{3}\mathcal{E}z^4,$$
$$f_2(t=0,z) = \frac{4}{3}\mathcal{E}z^4 \sin z,$$
$$f_3(t=0,z,\beta) = \frac{2}{15}\,\mathcal{E}z^4 \exp\left[\frac{-150}{z_h^2}(z-\beta z_h)^2\right],$$
$$f_4(t=0,z) = \mathcal{E}z^{24}$$

* $\Sigma(t,r)$ is a convex function and must vanish for $r \geq 0$.

$$\Sigma(t,r) = r - \frac{b_4(t)^2}{7r^7} + \dots$$

* We should make sure where $\Sigma(t,r)$ vanishes is hidden behind the event horizon on the initial time slice.

specifying the energy density, ε



Solving Bulk Equations of Motion (Numerics):

$$0 = \partial_r^2\Sigma + \frac{1}{6}\Sigma\left(3(\partial_r B)^2 + (\partial_r\phi)^2\right)$$

initial out-of-equilibrium state

↓

given geometry and scalar field on the initial time-slice

$B(t=0,r)$ & $\phi(t=0,r)$ → $\Sigma(t=0,r)$

$$0 = \Sigma\partial_r(\dot{\Sigma}) + 2\dot{\Sigma}\partial_r\Sigma - 2\Sigma^2 + \frac{1}{12}m^2\phi^2\Sigma^2$$ → $\dot{\Sigma}(t=0,r)$

$$0 = 2\partial_r(\dot{\phi}) + \frac{3\partial_r\Sigma}{\Sigma}\dot{\phi} + \frac{3\partial_r\phi}{\Sigma}\dot{\Sigma} - m^2\phi$$ → $\dot{\phi}(t=0,r)$

Pseudo-Spectral Method

$$0 = 2\partial_r(\dot{B}) + \frac{3\partial_r\Sigma}{\Sigma}\dot{B} + \frac{3\dot{\Sigma}}{\Sigma}\partial_r B$$ → $\dot{B}(t=0,r)$

$$0 = \partial_r^2 A - \frac{12}{\Sigma^2}\dot{\Sigma}\partial_r\Sigma + 3\dot{B}\partial_r B + 4 + \dot{\phi}\partial_r\phi - \frac{1}{6}m^2\phi^2$$ → $A(t=0,r)$

$$\dot{h} \equiv \partial_t h + \frac{1}{2}A\partial_r h$$ → $\partial_t\phi$ & $\partial_t B$

Proceed to the next time step using a finite difference scheme.

$B(t=\delta t,r)$ & $\phi(t=\delta t,r)$

What are we going to get?

* Thermalization Criterion:

The event horizon radius is: $r - r_{EH}(t) = 0$

with the null normal vector: $r'_{EH}(t) - \frac{1}{2}A(t, r_{EH}(t)) = 0$

$$\epsilon(t) = \left|\frac{r_{EH}(t) - \pi T}{\pi T}\right|$$

$$\epsilon(t_{th}) < 5 \times 10^{-4}$$

$$s_{EH}(t) \propto \Sigma\left(t, r_{EH}(t)\right)^3$$

* Isotropization Criterion:

$$\delta(t) = \frac{\Delta\mathcal{P}(t)}{\mathcal{E}}$$

$$\delta(t_{iso}) < 5 \times 10^{-4}$$

* Equilibration Criterion:

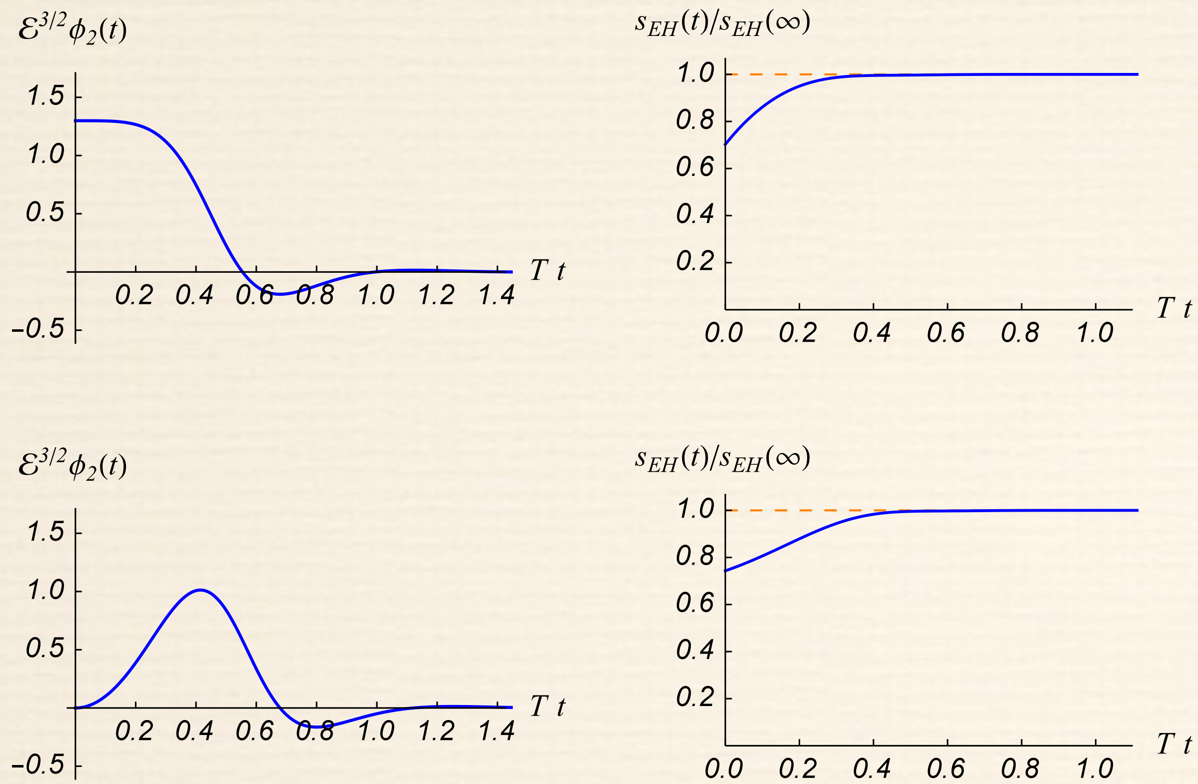
$$\sigma(t) = \left|\mathcal{E}^{3/2}\phi_2(t)\right|$$

$$\sigma(t_{eq}) < 5 \times 10^{-4}$$

Numerical Results:

$$a_4 = -1 \rightarrow \varepsilon = \frac{3}{4}$$

- Isotropic case



$$f_1(t=0,z) = \frac{8}{3}\mathcal{E} z^4$$

$$f_2(t=0,z) = \frac{4}{3}\mathcal{E} z^4 \sin z$$

Time-scales of relaxation for $\phi_i = f_3(0,z,\beta_\phi) = \frac{2}{15}\,\mathcal{E} z^4 \exp\left[\frac{-150}{z_h^2}(z-\beta z_h)^2\right]$

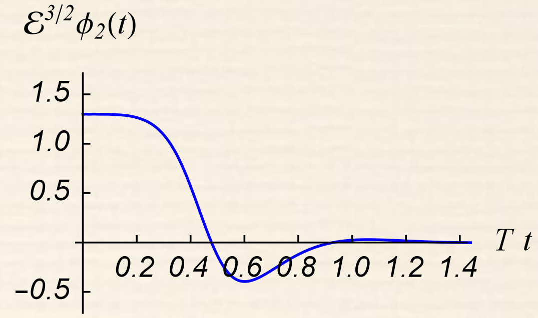
β_ϕ	t_{eq}	t_{th}
1/6	0.633236	0
1/2	1.05992	0.079232
5/6	1.57635	0.592548

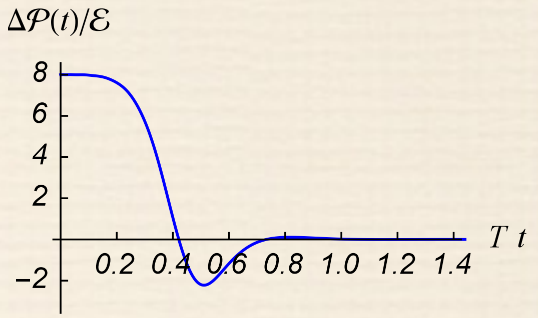
$$t_{eq} > t_{th}$$

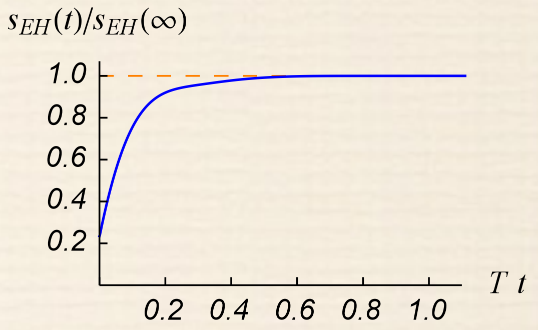
Anisotropic Case

Time-scales of relaxation for $\phi_i = f_3(0, z, \beta_\phi)$ and $B_i = f_3(0, z, 0.5)$

β_ϕ	t_{eq}	t_{th}	t_{iso}
1/6	0.43656	0.255016	1.42388
1/3	0.92813	0.255528	1.42386
1/2	1.1525	0.286825	1.4238
5/6	1.64636	0.632499	1.42316







$$f_1(t=0,z) = \frac{8}{3}\mathcal{E}z^4$$

Time-scales of relaxation for $\phi_i = f_3(0, z, 0.5)$ and $B_i = f_3(0, z, \beta_B)$

β_B	t_{eq}	t_{th}	t_{iso}
1/6	1.15317	0.0602394	1.01136
1/3	1.15303	0.147214	1.2986
1/2	1.1525	0.286825	1.4238
5/6	1.14811	0.606601	1.52925

$$f_3(t=0,z,\beta) = \frac{2}{15}\,\mathcal{E}z^4 \exp\left[\frac{-150}{z_h^2}(z-\beta z_h)^2\right]$$

Time-scales of relaxation for $\phi_i = f_3(0, z, \beta_\phi)$ and $B_i = f_3(0, z, 1/6)$

β_B	t_{eq}	t_{th}	t_{iso}
1/6	0.436572	0	1.01144
1/2	1.15317	0.06024	1.01136
2/3	1.27577	0.402617	1.01146
5/6	1.57775	0.591871	1.01105

$$t_{eq} < t_{th} < t_{iso},$$
$$t_{th} < t_{eq} < t_{iso},$$
$$t_{th} < t_{iso} < t_{eq}.$$

Time-scales of relaxation for $\phi_i = f_3(0, z, \beta_\phi)$ and $B_i = f_4(0, z)$

β_ϕ	t_{eq}	t_{th}	t_{iso}
1/6	0.435816	0.791326	1.70082
1/2	1.13573	0.791695	1.70092
5/6	1.56707	0.799356	1.70206

$$f_3(t=0, z, \beta) = \frac{2}{15}\,\mathcal{E} z^4 \exp\left[\frac{-150}{z_h^2}(z-\beta z_h)^2\right]$$

$$f_4(t=0, z) = \mathcal{E} z^{24}$$

$$f_\phi(t=0, z, \beta_\phi) = c_\phi\,\mathcal{E} z^4 \exp\left[\frac{-150}{z_h^2}(z-\beta_\phi z_h)^2\right]$$

$$f_B(t=0, z, \beta_B) = c_B\,\mathcal{E} z^4 \exp\left[\frac{-150}{z_h^2}(z-\beta_B z_h)^2\right]$$

$$\beta_\phi = \frac{5.6}{6}$$

$$\beta_B = \frac{0.001}{6}$$

$$c_B = \frac{1}{15}$$

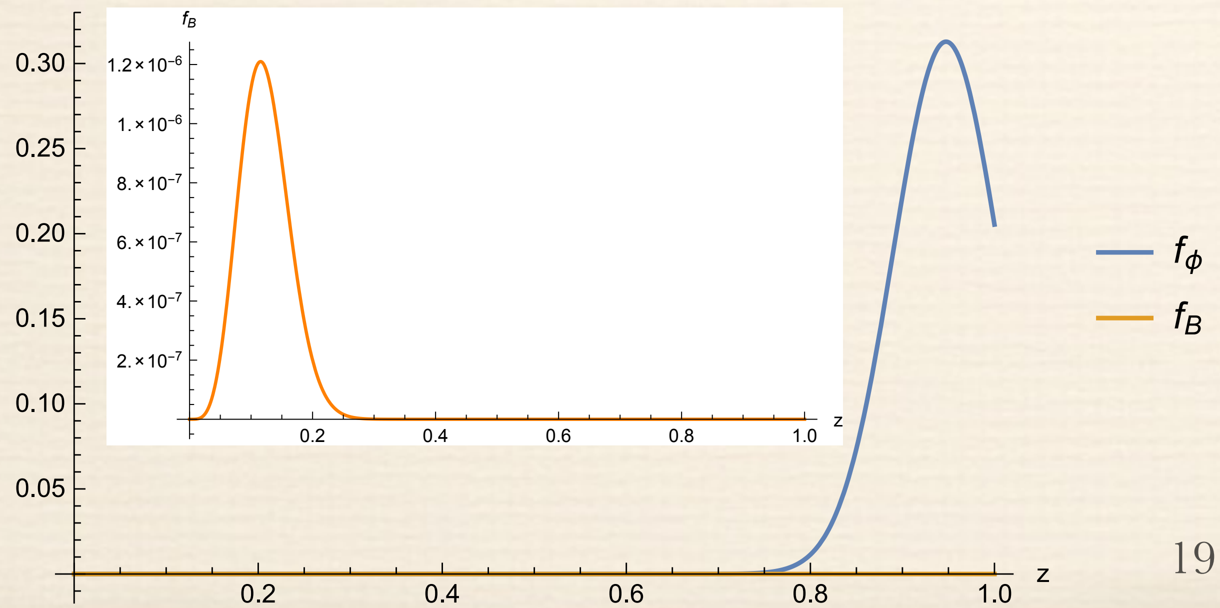
c_ϕ	t_{eq}	t_{th}	t_{iso}
10/15	1.90965	0.866807	0.744497
8/15	1.89562	0.844064	0.746243
4/15	1.84284	0.781432	0.748315

$t_{iso} < t_{th} < t_{eq}$,

$t_{eq} < t_{th} < t_{iso}$,

$t_{th} < t_{eq} < t_{iso}$,

$t_{th} < t_{iso} < t_{eq}$.



Comments:

- Gauge/Gravity duality provides a useful framework to study out-of-equilibrium strongly coupled systems.
- QGP formation and relaxation
- Local time-dependent processes
- Non-conformal theories
- Revivals and relaxation in finite size systems

Thank you