Solution phase space and conserved charges

K. Hajian and M.M. Sheikh-Jabbari, Phys.Rev. D**93** (2016) 4, 044074, [arXiv:1512.05584].

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Recent Trends in String Theory and Related Topics
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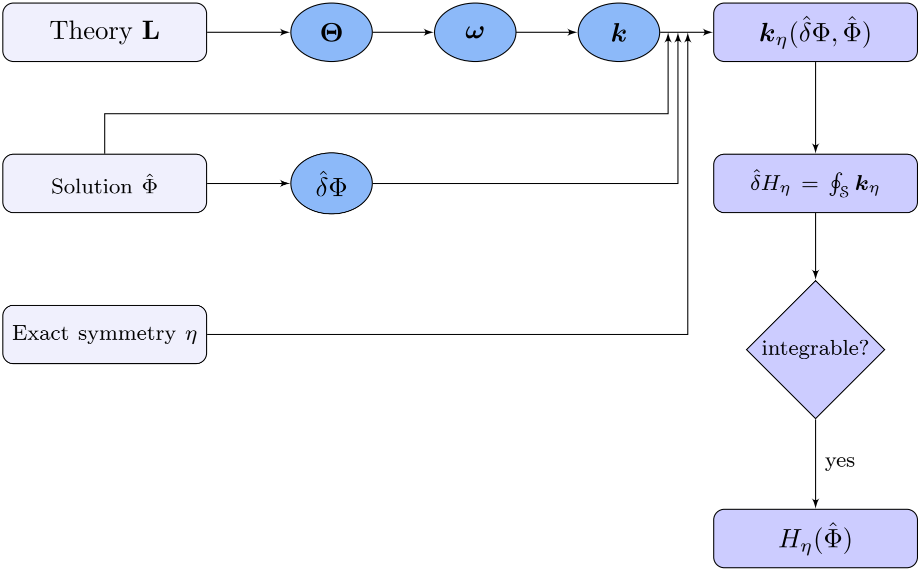
Two main lines of formulating quasi-local conserved charges:

1. Space+time $\rightarrow$ Hamiltonian formulation:
    - Komar (1958),
    - Arnowitt-Deser-Misner (ADM) (1959-62),
    - Bondi-van der Burg-Metzner (1962), Sachs (1962),
    - Regge-Teitelboim (1974),
    - Brown-York, (1993), Balasubramanian-Kraus (1999),
    - Abbott-Deser (1982),
    - Deser-Tekin (2003),
    - Kim-Kulkarni-Yi (2013).
2. Spacetime $\rightarrow$ Lagrangian formulation:
    - Crnkovic-Witten, (1987),
    - Ashtekar-Bombelli-Koul, (1987),
    - Lee-Iyer-Wald-Zoupas, (1990,93,99),
    - Barnich-Brandt (2002),

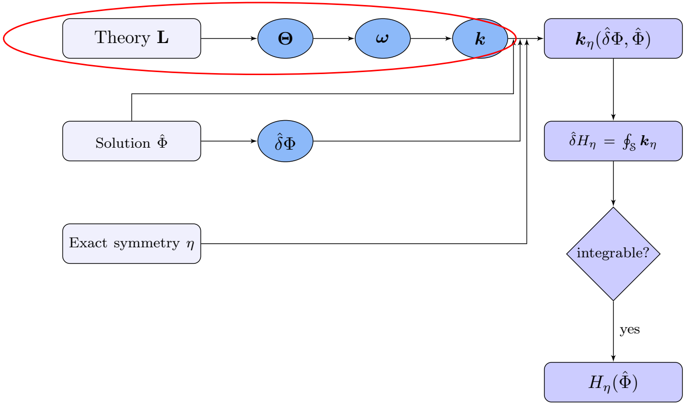
- In this talk, we will focus on the Lagrangian formulation.

Conserved charges, which will be introduced,:

- are calculable:
  - in any covariant gravitational theory,
  - in any dimensions,
  - in any asymptotics,
- are covariant,
- are unambiguous,
- are independent of the chosen codimension-2 surface of integration,
- are regular automatically,
- put entropy and electric charge in a single formulation with mass and angular momenta,
- relax the calculation of entropy over the horizon,
- make the first law equivalent to an identity between generators.


Theory $\mathbf{L}$
$\boldsymbol{\Theta}$
$\boldsymbol{\omega}$
$\boldsymbol{k}$
$\boldsymbol{k}_\eta(\hat{\delta}\Phi, \hat{\Phi})$
Solution $\hat{\Phi}$
$\hat{\delta}\Phi$
$\hat{\delta}H_\eta = \oint_{\mathcal{S}} \boldsymbol{k}_\eta$
Exact symmetry η
integrable?
yes
$H_\eta(\hat{\Phi})$

1. A review on covariant phase space formulation
2. Solution phase space
3. Conserved charges associated with exact symmetries
4. Application: Kerr-Newman-(A)dS charges and first law(s)



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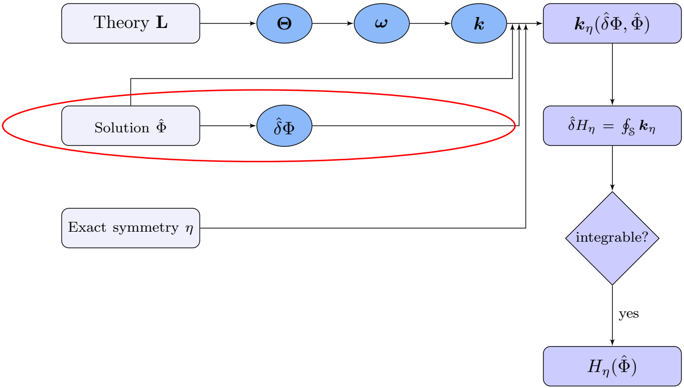

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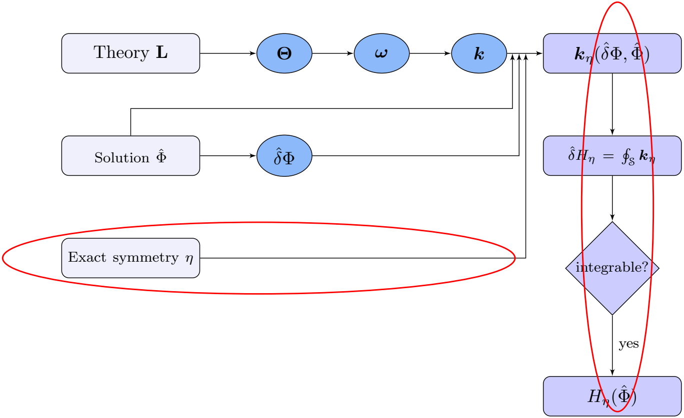

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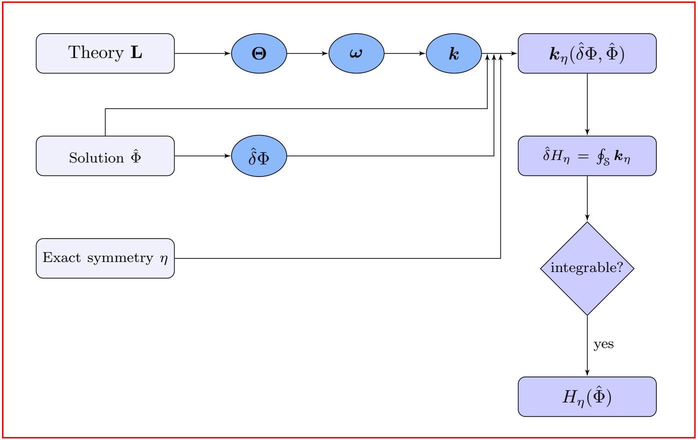
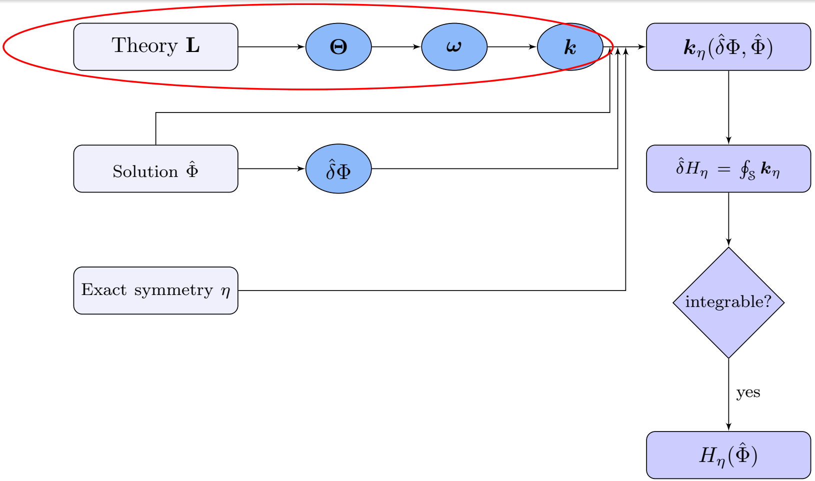


1 A review on covariant phase space method

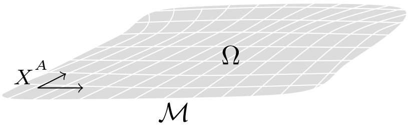


Phase space is a manifold $\mathcal{M}$ equipped with $\Omega = \Omega_{AB}\,\delta X^{A}\,\delta X^{B}$ such that:

1. $\Omega_{AB} = -\Omega_{BA}$
2. Ω be closed:
$$\delta\Omega = 0$$
3. Ω be non-degenerate:
$$\Omega_{AB}\,v^{B} = 0 \;\Leftrightarrow\; v^{B} = 0$$

- Ω is invertible:
$$\Omega^{AB} = (\Omega^{-1})_{AB}$$

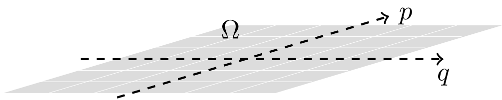
Phase Space



Manifold $\mathcal{M}$ with symplectic 2-form Ω

Particle in one dimension

- $X^1 = q \quad , \quad X^2 = p$
- $\Omega_{AB} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
- $\{q, p\} = 1,$
  $\{q, q\} = \{p, p\} = 0$



How to read Ω from Lagrangian

$$\mathcal{L} = \mathcal{L}(q, \dot{q}) \quad \Rightarrow \quad \delta\mathcal{L} = \Big(\frac{\partial\mathcal{L}}{\partial q} - \partial_t\big(\frac{\partial\mathcal{L}}{\partial\dot{q}}\big)\Big)\delta q + \frac{\mathrm{d}}{\mathrm{d}t}(p\,\delta q)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(p\,\delta q) \quad \rightarrow \quad p\,\delta q \quad \rightarrow \quad \Omega = \delta(p\,\delta q) = \delta p \wedge \delta q\,.$$

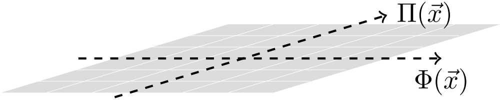
Conserved charge

- charge variation for a vector field $v \quad \Rightarrow \quad \delta H_v \equiv v \cdot \Omega$
- Example: $v = \partial_q \quad \Rightarrow \quad \delta H_v = \partial_q \cdot \Omega = \delta p \quad \Rightarrow \quad H_v = p$

Canonical fields

- $\Phi(\vec{x}) \quad , \quad \Pi(\vec{x}) \equiv \dfrac{\partial \mathcal{L}}{\partial \dot{\Phi}}$
- Poisson brackets

$$\{\Phi(\vec{x}_1), \Pi(\vec{x}_2)\} = \delta(\vec{x}_1 - \vec{x}_2)$$
$$\{\Phi(\vec{x}_1), \Phi(\vec{x}_2)\} = 0$$
$$\{\Pi(\vec{x}_1), \Pi(\vec{x}_2)\} = 0\,.$$



General covariance?

- In canonical method, it is necessary to introduce time foliations, which usually breaks the general covariance.

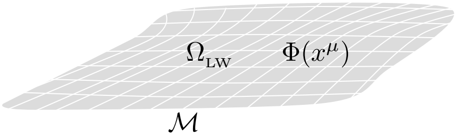
Covariant manifold $\mathcal{M}$

Given a covariant action $\mathcal{S} = \int \mathbf{L}[\Phi]$ in d-dim spacetime:

- manifold $\mathcal{M}$ is composed of dynamical field $\Phi(x^\mu)$

- $\Phi(x^\mu)$: metric $g_{\alpha\beta}(x^\mu)$, gauge field $A_\nu(x^\mu)$, scalar fields $\phi(x^\mu)$...

covariant phase space



Symplectic 2-form Ω

(1) picking the $\boldsymbol{\Theta}_{\text{LW}}$ from $\delta\mathbf{L}[\Phi] = \mathbf{E}(\Phi)\delta\Phi + \mathrm{d}\boldsymbol{\Theta}_{\text{LW}}(\delta\Phi, \Phi)$

(2) exterior derivation on the phase space:

$$\boldsymbol{\omega}_{\text{LW}}(\delta_1\Phi, \delta_2\Phi, \Phi) \equiv \delta_1\boldsymbol{\Theta}_{\text{LW}}(\delta_2\Phi, \Phi) - \delta_2\boldsymbol{\Theta}_{\text{LW}}(\delta_1\Phi, \Phi)$$

(3) the Lee-Wald symplectic form Ω would be: $\Omega_{\text{LW}} \equiv \int_\Sigma \boldsymbol{\omega}_{\text{LW}}$,

Ambiguity

Ambiguity in the definition of $\mathbf{\Theta}$ through $\delta \mathbf{L}[\Phi] = \mathbf{E}(\Phi)\delta\Phi + \mathrm{d}\mathbf{\Theta}(\delta\Phi, \Phi)$:

$$\mathbf{\Theta}(\delta\Phi, \Phi) \to \mathbf{\Theta}(\delta\Phi, \Phi) + \mathrm{d}\mathbf{Y}(\delta\Phi, \Phi)$$

- It results in an ambiguity in the symplectic structure:

$$\boldsymbol{\omega}(\delta_1\Phi, \delta_2\Phi, \Phi) \to \boldsymbol{\omega}(\delta_1\Phi, \delta_2\Phi, \Phi) + \mathrm{d}\big(\delta_2\mathbf{Y}(\delta_1\Phi, \Phi) - \delta_1\mathbf{Y}(\delta_2\Phi, \Phi)\big)$$

Conditions for independence of Ω from Σ

$$\mathrm{d}\boldsymbol{\omega}(\delta_1\Phi, \delta_2\Phi, \Phi) = 0 \qquad \text{and} \qquad \boldsymbol{\omega}(\delta_1\Phi, \delta_2\Phi, \Phi)\Big|_{\partial\Sigma} = 0\,,$$

- the former is satisfied if Φ and $\delta\Phi$ satisfy e.o.m and linearized e.o.m respectively $\Rightarrow$ we will assume on-shell-ness here after.
- for the latter, one usually needs to impose some fall-off conditions on $\delta\Phi$.

- Let us put diff and gauge transformations in a single set $\epsilon \equiv \{\xi, \lambda\}$:

$$\delta_\epsilon \Phi = \mathcal{L}_\xi \Phi + \delta_\lambda A$$

Charge variation associated with ϵ

- Any generator $\epsilon \equiv \{\xi, \lambda\}$ (with arbitrary $\delta\epsilon$) induces a vector field $v = \delta_\epsilon \Phi$ on the $T\mathcal{M}$.
- The charge variation for the ϵ is defined as

$$\delta H_\epsilon \equiv v \cdot \Omega = \int_\Sigma \big(\delta^{[\Phi]} \boldsymbol{\Theta}(\delta_\epsilon \Phi, \Phi) - \delta_\epsilon \boldsymbol{\Theta}(\delta\Phi, \Phi)\big)$$

Fundamental theorem of the covariant phase space formalism

$$\delta^{[\Phi]} \boldsymbol{\Theta}(\delta_\epsilon \Phi, \Phi) - \delta_\epsilon \boldsymbol{\Theta}(\delta\Phi, \Phi) = \mathrm{d}\boldsymbol{k}_\epsilon(\delta\Phi, \Phi)$$

Stokes theorem $\Rightarrow$ $\boxed{\delta H_\epsilon = \oint_{\partial\Sigma} \boldsymbol{k}_\epsilon(\delta\Phi, \Phi)}$

Charge conservation and integrability

Conservation conditions

For all $\Phi \in \mathcal{M}$ and $\delta\Phi \in T\mathcal{M}$:

$$\mathrm{d}\boldsymbol{\omega}(\delta\Phi, \delta_\epsilon\Phi, \Phi) = 0\,, \qquad \boldsymbol{\omega}(\delta\Phi, \delta_\epsilon\Phi, \Phi)\Big|_{\partial\Sigma} = 0\,.$$

- Not any generator ϵ has conserved charge.

Integrability condition

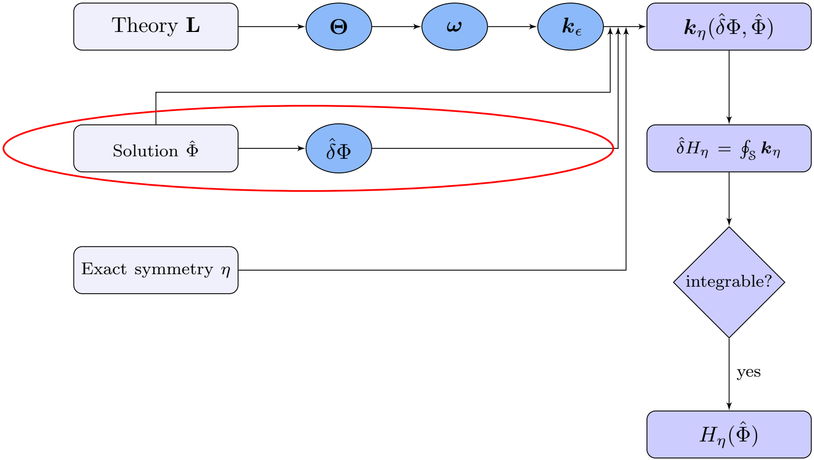
For all $\delta_1\Phi, \delta_2\Phi \in T\mathcal{M}$:

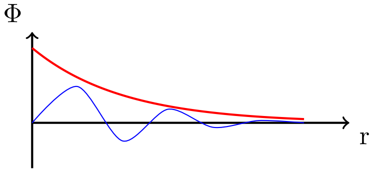
$$(\delta_1\delta_2 - \delta_2\delta_1)H_\epsilon = 0\,, \;\Rightarrow\; \oint_{\partial\Sigma}\Big(\xi\cdot\boldsymbol{\omega}(\delta_1\Phi, \delta_2\Phi, \Phi) + \boldsymbol{k}_{\delta_1\epsilon}(\delta_2\Phi, \Phi) - \boldsymbol{k}_{\delta_2\epsilon}(\delta_1\Phi, \Phi)\Big) = 0.$$

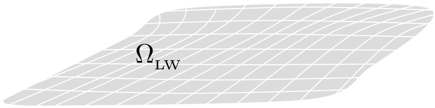
- Not any generator ϵ has integrable charge.

- Lee and Wald (1990), Wald and Zoupas (2000), G. Compère *et al.* (2016).

2 Solution phase space



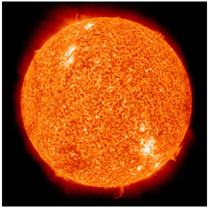




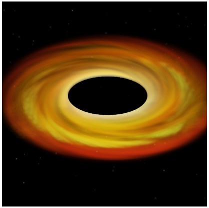
$\mathcal{M} = ?$

- In the standard covariant phase space formulation, $\mathcal{M}$ is introduced by some fall-off conditions.
- The fall-off conditions, although restrict the manifold, but do not identify it completely.

Nebula

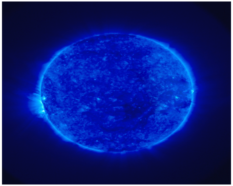


Star



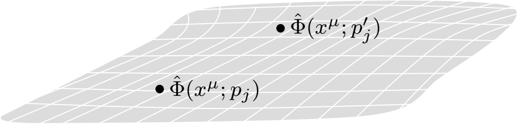
Black hole

Solution phase space



Focussing on a set of (black Hole) solutions identified by some parameters $\hat{\Phi}(x^\mu; p_j)$

solution phase space



$\hat{\mathcal{M}}\,,\hat{\Omega}_{\text{LW}}$

- Manifold $\mathcal{M}$ $\longrightarrow$ "solution phase space manifold" $\hat{\mathcal{M}}$ built of $\hat{\Phi}(x^\mu; p_i)$
- Ω $\longrightarrow$ Ω_{LW} confined to $\hat{\mathcal{M}}$
- $T\mathcal{M}$ $\longrightarrow$ $T\hat{\mathcal{M}}$ is spanned by "parametric variations": $\hat{\delta}\Phi \equiv \frac{\partial\hat{\Phi}}{\partial p_j}\delta p_j$

Kerr solution phase space

- **Theory:** Einstein-Hilbert $\mathcal{L} = \frac{1}{16\pi G} R$
- **Dynamical field $\hat{\Phi}$:** the metric $g_{\mu\nu}$
- **The $\boldsymbol{k_\xi}$:** It is dual to

$$k_\xi^{\mu\nu} = \frac{1}{16\pi G}\Big[\xi^\nu \nabla^\mu h - \xi^\nu \nabla_\tau h^{\mu\tau} + \xi_\tau \nabla^\nu h^{\mu\tau} + \frac{1}{2} h \nabla^\nu \xi^\mu - h^{\tau\nu}\nabla_\tau \xi^\mu\Big] - [\mu \leftrightarrow \nu]$$

in which $h^{\mu\nu} \equiv g^{\mu\sigma} g^{\nu\tau}\delta g_{\sigma\tau}$ and $h \equiv h^\mu{}_\mu$.

- **Manifold $\hat{\mathcal{M}}$:**

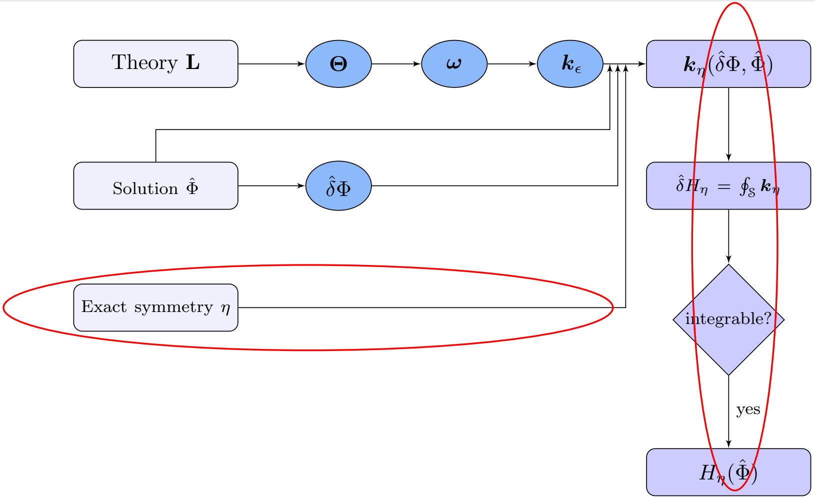
$$\mathrm{d}s^2 = -(1-f)\mathrm{d}t^2 + \frac{\rho^2}{\Delta_r}\mathrm{d}r^2 + \rho^2 \mathrm{d}\theta^2 - 2fa\sin^2\theta\,\mathrm{d}t\mathrm{d}\varphi + \big(r^2 + a^2 + fa^2\sin^2\theta\big)\sin^2\theta\,\mathrm{d}\varphi^2\,,$$

$$\rho^2 \equiv r^2 + a^2\cos^2\theta\,, \qquad \Delta_r \equiv r^2 + a^2 - 2Gmr\,, \qquad f \equiv \frac{2Gmr}{\rho^2}\,,$$

- **Parameters:** $p_j = \{m, a\}$
- **Parametric variations $\hat{\delta}\Phi$:**

$$\hat{\delta} g_{\mu\nu} = \frac{\partial \hat{g}_{\mu\nu}}{\partial m}\delta m + \frac{\partial \hat{g}_{\mu\nu}}{\partial a}\delta a$$

3 Conserved charges associated with exact symmetries



Symplectic symmetry generator

Definition: A generator $\epsilon = \{\xi, \lambda\}$ is called symplectic symmetry generator if

$$\boldsymbol{\omega}(\delta\Phi, \delta_\epsilon\Phi, \Phi) = 0$$

on-shell for all Φ and $\delta\Phi$ in $\mathcal{M}$ and $T\mathcal{M}$.

Two nice features

- **Conservation:** symplectic symmetry generator $\longrightarrow$ conservation is guaranteed:

$$\mathrm{d}\boldsymbol{\omega}(\delta\Phi, \delta_\epsilon\Phi, \Phi) = 0\,, \qquad \boldsymbol{\omega}(\delta\Phi, \delta_\epsilon\Phi, \Phi)\Big|_{\partial\Sigma} = 0 \;\checkmark$$

- **Independence of $\boldsymbol{\partial\Sigma}$:** symplectic symmetry generator $\longrightarrow$ the δH_ϵ is independent of chosen codimension-2 surface of integration:

$$\oint_{\mathcal{S}_2} \boldsymbol{k}_\epsilon(\delta\Phi, \Phi) - \oint_{\mathcal{S}_1} \boldsymbol{k}_\epsilon(\delta\Phi, \Phi) = \int_\Sigma \boldsymbol{\omega}(\delta\Phi, \delta_\xi\Phi, \Phi) = 0$$

Non-exact and exact symplectic symmetry generators

Symplectic symmetry generators are composed of two sets:

1. **non-exact symmetries:** $\chi = \{\xi, \lambda\}$ is called non-exact symmetry if

$$\exists \Phi \in \mathcal{M} : \qquad \delta_\chi \Phi \neq 0$$

2. **exact symmetries:** $\eta = \{\zeta, \lambda\}$ is called exact symmetry if

$$\forall \Phi \in \mathcal{M} : \qquad \delta_\eta \Phi = 0$$

- Exact symplectic symmetries are in our main focus in the "solution phase space method".

No ambiguity

- Exact symmetries $\longrightarrow$ charges are unambiguous:

linear in $\delta_\eta \Phi = 0$

$$\boldsymbol{\omega}(\delta\Phi, \delta_\eta\Phi, \Phi) \to \boldsymbol{\omega}(\delta\Phi, \delta_\eta\Phi, \Phi) + \mathrm{d}\big(\delta_\eta \mathbf{Y}(\delta\Phi, \Phi) - \delta\mathbf{Y}(\delta_\eta\Phi, \Phi)\big)$$

Putting pieces together

covariant phase space formulation		solution phase space		exact symmetries
$\boldsymbol{k}_\epsilon(\delta\Phi,\Phi)$	+	$\hat{\Phi}(x^\mu,p_j)\,,\;\hat{\delta}\Phi$	+	$\eta=\{\zeta,\lambda\}$

Conserved charge associated with η

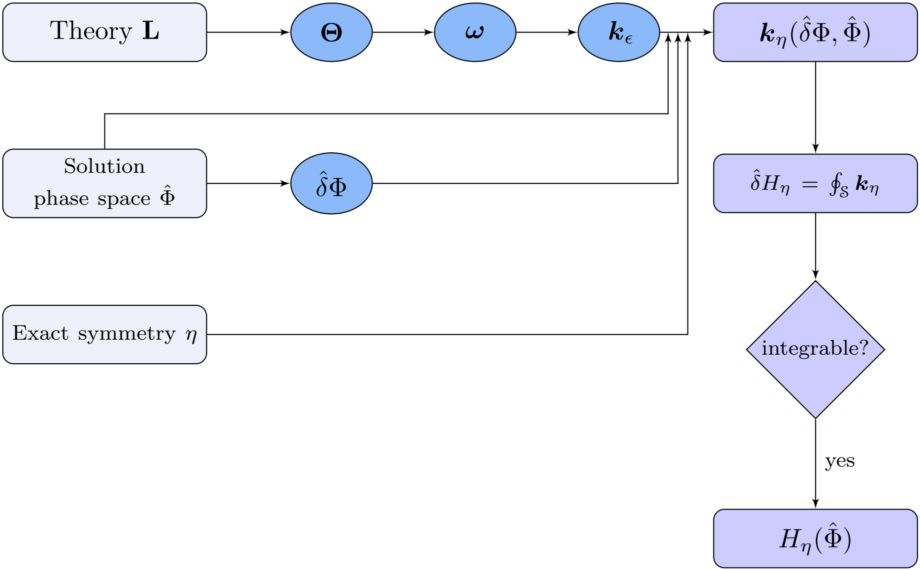
Conserved charge associated with the exact symmetry $\eta=\{\zeta,\lambda\}$:

$$\hat{\delta}H_\eta=\oint_{\mathcal{S}}\boldsymbol{k}_\eta(\hat{\delta}\Phi,\hat{\Phi})\,.$$

Integrability condition:

$$\oint_{\mathcal{S}}\Big(\zeta\cdot\hat{\boldsymbol{\omega}}(\hat{\delta}_1\Phi,\hat{\delta}_2\Phi,\hat{\Phi})+\boldsymbol{k}_{\hat{\delta}_1\eta}(\hat{\delta}_2\Phi,\Phi)-\boldsymbol{k}_{\hat{\delta}_2\eta}(\hat{\delta}_1\Phi,\hat{\Phi})\Big)=0,\quad\forall\hat{\delta}_{1,2}\Phi\text{ and }\forall\hat{\Phi}.$$

If integrable, then $H_\eta[\hat{\Phi}]=\int_{\bar{p}}^{p}\hat{\delta}H_\eta+H_\eta[\bar{\Phi}]$



Theory $\mathbf{L}$
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Solution phase space $\hat{\Phi}$
$\hat{\delta}\Phi$
$\hat{\delta}H_\eta = \oint_S \boldsymbol{k}_\eta$
Exact symmetry η
integrable?
yes
$H_\eta(\hat{\Phi})$

Thermodynamical conserved charges

- **Mass:**

$$\eta_M = \{\partial_t, 0\} \quad \longrightarrow \quad \hat{\delta} M \equiv \hat{\delta} H_\eta$$

- **Angular momentum:**

$$\eta_J = \{\partial_\varphi, 0\} \quad \longrightarrow \quad \hat{\delta} J \equiv -\hat{\delta} H_\eta$$

- **Electric charge:**

$$\eta_Q = \{0, 1\} \quad \longrightarrow \quad \hat{\delta} Q \equiv \hat{\delta} H_\eta$$

- **Entropies:**

$$\eta_{S_{\rm H}} = \frac{2\pi}{\kappa_{\rm H}}\{\zeta_{\rm H}, -\Phi_{\rm H}\} \quad \longrightarrow \quad \hat{\delta} S_{\rm H} \equiv \hat{\delta} H_\eta$$

First law(s)

$$\eta_{S_{\rm H}} = \mu_M \eta_M + \mu_J \eta_J + \mu_Q \eta_Q \xrightarrow{\text{linearity of } \delta H_\epsilon \text{ in } \epsilon} \delta S_{\rm H} = \mu_M \delta M - \mu_J \delta J + \mu_Q \delta Q$$

4

Application: Kerr-Newman-(A)dS charges and first law(s)

- **Theory:** $\mathcal{L} = \frac{1}{16\pi G}(R - F^2 - 2\Lambda)$
- $\boldsymbol{k_\epsilon(\delta\Phi, \Phi)}$ **:** For the theory under consideration, and for diffeomorphism+gauge transformation $\epsilon = \{\xi, \lambda\}$

$$\boldsymbol{k}_\epsilon(\delta\Phi, \Phi) = \frac{\sqrt{-g}}{2!\,2!}\,\epsilon_{\mu\nu\sigma\rho}\,(k_\epsilon^{\mathrm{EH}\,\mu\nu} + k_\epsilon^{\mathrm{M}\,\mu\nu})\;\mathrm{d}x^\sigma \wedge \mathrm{d}x^\rho$$

in which

$$k_\xi^{\mathrm{EH}\,\mu\nu} = \frac{1}{16\pi G}\Big(\Big[\xi^\nu\nabla^\mu h - \xi^\nu\nabla_\tau h^{\mu\tau} + \xi_\tau\nabla^\nu h^{\mu\tau} + \frac{1}{2}h\nabla^\nu\xi^\mu - h^{\tau\nu}\nabla_\tau\xi^\mu\Big] - [\mu\leftrightarrow\nu]\Big)\,,$$

$$k_\epsilon^{\mathrm{M}\,\mu\nu} = \frac{1}{8\pi G}\Big(\Big[(\frac{-h}{2}F^{\mu\nu} + 2F^{\mu\rho}h_\rho{}^\nu - \delta F^{\mu\nu})(\xi^\sigma A_\sigma + \lambda) - F^{\mu\nu}\xi^\rho\delta A_\rho - 2F^{\rho\mu}\xi^\nu\delta A_\rho\Big] - [\mu\leftrightarrow\nu]\Big)$$

where $h^{\mu\nu} \equiv g^{\mu\sigma}g^{\nu\tau}\delta g_{\sigma\tau}$ and $h \equiv h^\mu{}_\mu$.

- **Solution phase space $\hat{\mathcal{M}}$:**

$$\mathrm{d}s^2 = -\Delta_\theta\Big(\frac{1-\frac{\Lambda r^2}{3}}{\Xi} - \Delta_\theta f\Big)\mathrm{d}t^2 + \frac{\rho^2}{\Delta_r}\mathrm{d}r^2 + \frac{\rho^2}{\Delta_\theta}\mathrm{d}\theta^2 - 2\Delta_\theta f a \sin^2\theta\,\mathrm{d}t\mathrm{d}\varphi + \left(\frac{r^2+a^2}{\Xi} + f a^2\sin^2\theta\right)\sin^2\theta\,\mathrm{d}\varphi^2\,,$$

$$\rho^2 \equiv r^2 + a^2\cos^2\theta\,, \qquad \Delta_r \equiv (r^2+a^2)(1-\frac{\Lambda r^2}{3}) - 2Gmr + q^2\,,$$

$$\Delta_\theta \equiv 1 + \frac{\Lambda a^2}{3}\cos^2\theta\,, \qquad \Xi \equiv 1 + \frac{\Lambda a^2}{3}\,, \qquad f \equiv \frac{2Gmr}{\rho^2\Xi^2}\,,$$

$$\hat{A}_\mu \mathrm{d}x^\mu = \frac{qr}{\rho^2\Xi}(\Delta_\theta \mathrm{d}t - a\sin^2\theta\,\mathrm{d}\varphi)\,.$$

- **Parameters:** $p_j = \{m, a, q\}$
- **Parametric variations:**

$$\hat{\delta} g_{\mu\nu} = \frac{\partial \hat{g}_{\mu\nu}}{\partial m}\delta m + \frac{\partial \hat{g}_{\mu\nu}}{\partial a}\delta a + \frac{\partial \hat{g}_{\mu\nu}}{\partial q}\delta q\,, \qquad \hat{\delta} A_\mu = \frac{\partial \hat{A}_\mu}{\partial m}\delta m + \frac{\partial \hat{A}_\mu}{\partial a}\delta a + \frac{\partial \hat{A}_\mu}{\partial q}\delta q$$

Application: Kerr-Newman-(A)dS black holes

- *Mass:* $\eta_M = \{\partial_t, 0\}$

$$\hat{\delta} M = \frac{\partial\left(\frac{m}{\Xi^2}\right)}{\partial m}\delta m + \frac{\partial\left(\frac{m}{\Xi^2}\right)}{\partial a}\delta a + \frac{\partial\left(\frac{m}{\Xi^2}\right)}{\partial q}\delta q = \delta\left(\frac{m}{\Xi^2}\right) \qquad \Rightarrow \qquad M = \frac{m}{\Xi^2}\,,$$

- *Angular momentum:* $\eta_J = \{\partial_\varphi, 0\}$

$$\hat{\delta} J = \frac{\partial\left(\frac{ma}{\Xi^2}\right)}{\partial m}\delta m + \frac{\partial\left(\frac{ma}{\Xi^2}\right)}{\partial a}\delta a + \frac{\partial\left(\frac{ma}{\Xi^2}\right)}{\partial q}\delta q = \delta\left(\frac{ma}{\Xi^2}\right) \qquad \Rightarrow \qquad J = \frac{ma}{\Xi^2}\,.$$

- *Electric charge:* $\eta_Q = \{0, 1\}$

$$\hat{\delta} Q = \frac{\partial\left(\frac{q}{\Xi}\right)}{\partial m}\delta m + \frac{\partial\left(\frac{q}{\Xi}\right)}{\partial a}\delta a + \frac{\partial\left(\frac{q}{\Xi}\right)}{\partial q}\delta q = \delta\left(\frac{q}{\Xi}\right) \qquad \Rightarrow \qquad Q = \frac{q}{\Xi}\,.$$

- Reference points: $M = 0$, $J = 0$ and $Q = 0$ in pure (A)dS spacetime.

- Surface gravity, angular velocity and electric potential for any horizon:

$$\kappa_{\text{H}} = \frac{r_{\text{H}}(1 - \frac{\Lambda a^2}{3} - \Lambda r_{\text{H}}^2 - \frac{a^2+q^2}{r_{\text{H}}^2})}{2(r_{\text{H}}^2 + a^2)}, \qquad \Omega_{\text{H}} = \frac{a(1 - \frac{r_{\text{H}}^2}{l^2})}{r_{\text{H}}^2 + a^2}, \qquad \Phi_{\text{H}} = \frac{q r_{\text{H}}}{r_{\text{H}}^2 + a^2}.$$

- *Entropies:* $\eta_{\text{H}} = \frac{2\pi}{\kappa_{\text{H}}}\{\zeta_{\text{H}}, -\Phi_{\text{H}}\}$ in which $\zeta_{\text{H}} = \partial_t + \Omega_{\text{H}}\partial_\varphi$

$$\hat{\delta} S_{\text{H}} = \frac{\partial\left(\frac{\pi(r_{\text{H}}^2+a^2)}{G\,\Xi}\right)}{\partial m}\delta m + \frac{\partial\left(\frac{\pi(r_{\text{H}}^2+a^2)}{G\,\Xi}\right)}{\partial a}\delta a + \frac{\partial\left(\frac{\pi(r_{\text{H}}^2+a^2)}{G\,\Xi}\right)}{\partial q}\delta q = \delta\left(\frac{\pi(r_{\text{H}}^2 + a^2)}{G\,\Xi}\right),$$

$$\Rightarrow \qquad S_{\text{H}} = \frac{\pi(r_{\text{H}}^2 + a^2)}{G\,\Xi}.$$

- Reference points:
  - Event horizons: $S_{\text{H}} = 0$ on pure (A)dS.
  - Cosmological horizons: $S_{\text{H}} = \frac{\pi l^2}{G}$ on pure dS.

First law(s)

$$\eta_{\rm H} = \frac{2\pi}{\kappa_{\rm H}}\{\partial_t, 0\} + \frac{2\pi\Omega_{\rm H}}{\kappa_{\rm H}}\{\partial_\varphi, 0\} - \frac{2\pi\Phi_{\rm H}}{\kappa_{\rm H}}\{0, 1\}\,,$$

- *First law(s):* linearity of δH_η in η, for each one of the horizons, results to

$$\delta S_{\rm H} = \frac{2\pi}{\kappa_{\rm H}}\delta M - \frac{2\pi}{\kappa_{\rm H}}\Omega_{\rm H}\delta J - \frac{2\pi}{\kappa_{\rm H}}\Phi_{\rm H}\delta Q$$

which by Hawking temperature(s) $T_{\rm H} = \frac{\kappa_{\rm H}}{2\pi}$ yields the first law(s)

$$\delta M = T_{\rm H}\delta S_{\rm H} + \Omega_{\rm H}\delta J + \Phi_{\rm H}\delta Q\,.$$

- Notice that thermodynamics of Kerr-AdS, Kerr and Kerr-dS has been unified.
  K. Hajian, *"Conserved Charges and First Law of Thermodynamics for Kerr-de Sitter Black Holes,"* arXiv:1602.05575 [gr-qc].

Conclusion

covariant phase space formulation		solution phase space		exact symmetries
$\boldsymbol{k}_\epsilon(\delta\Phi,\Phi)$	$+$	$\hat{\Phi}(x^\mu, p_j)\,,\ \hat{\delta}\Phi$	$+$	$\eta=\{\zeta,\lambda\}$

Conserved charges in solution phase space method:

- are calculable:
  - in any covariant gravitational theory,
  - in any dimensions,
  - in any asymptotics,
- are covariant,
- are unambiguous,
- are independent of the chosen codimension-2 surface of integration,
- are regular automatically,
- put entropy and electric charge in a single formulation with mass and angular momenta,
- make the first law equivalent to an identity between generators.

Thanks for your attention