Thermalization of mutual information in a hyperscaling violating background

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Based on: M. Alishahiha, Mohammadi and M.R.T, JHEP (2015), arXiv:1406.7677
M.R.T, JHEP (2016), arXiv: 1512.04104

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Outlook

Plan of the talk:

1- Part I

- Motivation
- A short review:
  - § Entanglement Entropy and Mutual information
  - § Hyperscaling violating backgrounds

2- Part II

- Thermalization:
  - § Time evolution of mutual information for two disjoint strips
  - § The results

Motivations

- Study of systems which are far from equilibrium is important.

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  In this way, the main questions are:

§ What kind of quantities can probe the process of thermalization, and,

§ How fast the process is

E.g, in the context of quantum information:
How does quantum information spread in a strongly coupled system, far from equilibrium?

Motivations

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In other word:

- One can use such quantities to study the approach to equilibrium.

Motivations

Quantum information is encoded in entanglement. What is/are the measure/s to probe the time evolution of quantum entanglement?

It is claimed that Entanglement Entropy (EE), Mutual Information (MI) can indeed be used as good measures.

In other word:

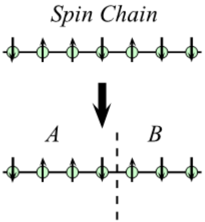
- § One can use such quantities to study the approach to equilibrium.
- § Unfortunately, in many situations in QFT, these quantities are extremely difficult to compute. However, fortunately,
- § for strongly coupled systems, one of our best analytical tools is holography.

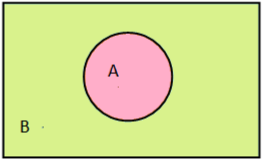
As I will briefly review: In holographic methods, entanglement entropy can be obtained from the area of an extremal surface in the dual geometry

A brief review

What is the EE?

Consider a quantum mechanical system at zero temperature, pure ground state $|\psi\rangle$ describes the total quantum system. Suppose we divide the system into two parts ($\mathcal{H}_{\text{tot.}} = \mathcal{H}_A \otimes \mathcal{H}_B$). Then one can use EE as a measure of correlations between subsystems.





[Nishiokaa, Ryu and Takayanagi, (2009)]

review...

- For pure state the density matrix is $\rho_{tot} = |\psi\rangle\langle\psi|, \rightarrow$ $\rho_A = Tr_B \rho_{\mathrm{tot.}}$ integrating out the degrees of freedom living in its geometrical complement B
- EE of the region A becomes: $S_{EE} = -Tr \rho_A \ln \rho_A$

EE is a UV-divergent quantity, for $d > 2$ the coefficient of divergent term is proportional to the area of entangling surface

$$S_{EE} \propto \frac{\mathcal{A}_A}{\epsilon^{d-2}} + \cdots$$

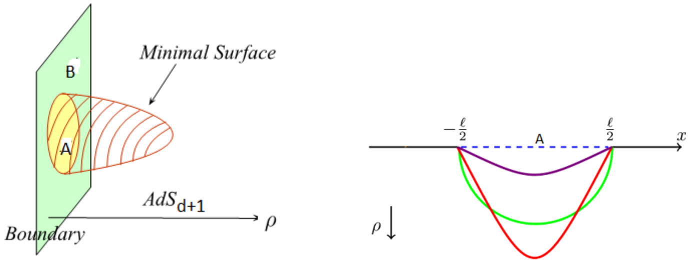
In CFT_2, one obtains

$$S_{EE} = \frac{c}{3} \log \frac{\ell}{\epsilon}$$

where ϵ is the UV cut-off.

As mentioned, EE in quantum field theories is an extremely difficult quantity to compute, However, in the context of AdS/CFT, one can apply the RT prescription to calculate the HEE. One should find a minimal area surface in the bulk (asymptotically AdS geometry) whose boundary coincides with the boundary of the entangling region.

$$S_{EE} = \frac{\min(\text{Area})}{4G_N^{d+1}}.$$



[Nishiokaa, Ryu and Takayanagi, (2009), Mohammadi PhD thesis 2014]

review (RT proposal: An example)

According to the AdS/CFT correspondence, strongly coupled CFT_d is dual to AdS_{d+1}. In Poincare coordinates one has:

$$ds^2_{d+1} = \frac{R^2}{\rho^2}\Big(-dt^2 + d\rho^2 + dx_1^2 + dx_i^2\Big).$$

The area is given by

$$Area(\gamma_A) = \int_{\gamma_A} d^{d-1}\xi \sqrt{det\, h},$$

where h stands for the induced metric on the hypersurface. For example in AdS_3/ CFT_2, one obtains

$$S_{EE} = \frac{\min(\mathrm{Area})}{4G_N^{d+1}} = \frac{R}{2G_N}\ln\frac{\ell}{\epsilon},$$

noting that $c = \frac{3R}{2G_N}$, one recovers the CFT result.

Mutual Information

Another powerful entanglement measure is the Mutual Information (MI). For two disjoint entangling regions A_1 and A_2 it is defined by

$$I(A_1, A_2) = S_{A_1} + S_{A_2} - S_{A_1 \cup A_2},$$

where $S_{A_1 \cup A_2}$ is the entanglement entropy for the union of two entangling regions.
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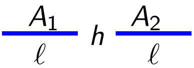
- MI is finite for two disjoint system and it is positive
- it is expected that MI carries more relevant content of CFT [Calabrese, Cardy, Toni, J.Sat.Mech (2009)]

Setup

To fix our notation let us suppose two parallel infinite strips with the equal width ℓ separated by a distance h in a $D+1$-dimensional field theory, $h \ll \ell$



$$I(A_1, A_2) = 2S(\ell) - S(2\ell + h) - S(h),$$

one needs to compute the entanglement entropy of three strips with widths h, ℓ and $2\ell + h$. To do so we first recall the computation of EE in hyperscaling violating background for a strip defined by

$$-\frac{\ell}{2} \leq x_1 \equiv x \leq \frac{\ell}{2}, \qquad 0 \leq x_a \leq L, \quad a = 2, \cdots, D$$

Hyperscaling Violating Backgrounds

The metric is given by:

$$ds^2_{D+2} = r^{-2\frac{\theta}{D}}\Big(-r^{2z}f(r)dt^2 + \frac{dr^2}{r^2 f(r)} + r^2\sum_{i=1}^{D} dx_i^2\Big),$$

There are two parameters: scaling violating parameter θ and dynamical exponent z.

There is no conformal symmetry, actually the metric transforms as $ds^2 = \zeta^{\frac{2\theta}{D}} ds^2$ under the following scaling

$$t \to \zeta^z t, \qquad \vec{x} \to \zeta\vec{x}, \qquad r \to \zeta r.$$

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Such metric seems to be important:

- One receives the Lifshitz geometry by setting $\theta = 0$
- It has some applications in condensed matter systems, e.g., in describing the critical quantum behavior (Lifshitz scaling), and log behavior of EE in a systems with Fermi surface
- The generalization of holography methods to geometries which are not asymptotically AdS

Thermalization

Let us consider a system which undergoes a sudden change which might be caused by turning on uniform density of sources for a short time interval and then turning it off (quantum quench).
Evolution towards an equilibrium state after a global quantum quench is an example of the thermalization

From the Hologrphic point of view

Thermalization after quench can be mapped to black hole formation via gravitational collapse of a thin null shell of matter into the bulk. This shell will eventually collapse to form a black hole, which is the gravity dual to a thermal state in the CFT.

$$\text{Black hole} \quad \leftrightsquigarrow \quad \text{thermal state},$$
$$\text{AdS solution} \quad \leftrightsquigarrow \quad \text{vacuum state}$$

Time-dependent backgrounds

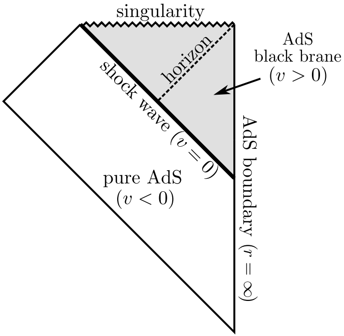
It is known that after the quench, the geometry of the bulk is given by the time-dependent AdS-Vaidya geometry:

$$ds^2 = -[r^2 - r_H^2\,\theta(v)]dv^2 + 2dr\,dv + r^2 dx^2,$$

which is the AdS-Vaidya metric, where v is the null coordinate. Because of the $\theta(v)$, one receives:

- AdS geometry for $v < 0$,
- AdS-BH geometry for $v > 0$

AdS-Vaidya metric



The causal structure of the thin-shell AdS_3-Vaidya spacetime in the Poincare patch. [Fig. from Bernamonti, et. al 2012]

One point: According to the RT-proposal, in order to compute the EE one should find the minimal surface in the bulk, however, in time-dependent case, one should use the covariant proposal of finding the extremum hypersurface in the bulk.

Thermalization

The scales are t, ℓ and $\rho_H (\equiv r_H^{-1})$.
When ℓ and $t \gtrsim \rho_H$, but $t < \frac{\ell}{2}$, the extremal surface starts intersecting the null shell behind the horizon and $\mathcal{A} = \mathcal{A}_{AdS} + \mathcal{A}_{BH}$.
In computing the extremal surface one should consider three regions: AdS-BH ($v > 0$), null shell ($v = 0$) and AdS ($v < 0$)

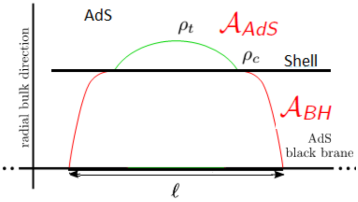


Figure: by increasing ℓ, hypersurface cross the shell and continue into the pure AdS geometry

So the crucial fact is to compute the following area in the bulk: $\mathcal{A} = \mathcal{A}_{AdS} + \mathcal{A}_{BH}$ boundary conditions are

$$\begin{aligned} \rho(\frac{\ell}{2}) &= 0, \qquad v(\frac{\ell}{2}) = t, \qquad \rho'(0) = 0, \\ v'(0) &= 0, \qquad \rho(0) = \rho_t, \qquad v(0) = v_t, \end{aligned}$$

Finally, the extremal area reads: $\xi \equiv \frac{\rho}{\rho_t}$

$$\mathcal{A} = \frac{L^{D-1}}{\rho_t^{D-1}} \Big(\int_0^{\frac{\rho_c}{\rho_t}} \frac{d\xi}{\xi^{2D} \sqrt{V_{eff}(\rho_t \xi)}} + \int_{\frac{\rho_c}{\rho_t}}^1 \frac{d\xi}{\xi^D \sqrt{1 - \xi^{2D}}} \Big)$$

$$V_{eff}(\rho_t \xi) \equiv E^2 + \left(\frac{1}{\xi^{2D}} - 1 \right) \left(1 - (\frac{\rho_t}{\rho_H})^{D+1} \xi^{D+1} \right),$$

$$E = -\frac{1}{2} (\frac{\rho_c}{\rho_t})^{D+1} \sqrt{(\frac{\rho_t}{\rho_c})^{2D} - 1}$$

Now one can study the time evolution of the EE by computing the hypersurface $\mathcal{A}$.

By considering all the situations that I have already mentioned, one obtains a general rule for time evolution of EE: [Liu and Suh, PRL 2014, PRD 2014, Alishahiha, Mohammadi and M.R.T JHEP 2016]

- For $\ell \ll \rho_H$ (Note that $\rho_H = r_H^{-1}$)

$$\begin{aligned} \text{Early times}\,(t \ll \rho_H) \qquad & \Delta S_{EE} \sim \mathcal{E}t^2, \\ t \gtrsim \frac{\ell}{2} \qquad & \Delta S_{EE} \sim \mathcal{E}\ell^2, \end{aligned}$$

  where $\mathcal{E}$ is the energy density

- Whereas for $\ell \gg \rho_H$

$$\begin{aligned} \text{Early times}\,(t \ll \rho_H) \qquad & \Delta S_{EE} \sim \mathcal{E}t^2, \\ \text{After local Eq. itermediate time}\,(\rho_H < t < \frac{\ell}{2}) \qquad & \Delta S_{EE} \sim S_{th}t, \\ \text{Saturation}\,(t > \frac{\ell}{2}) \qquad & \Delta S_{EE} \sim S_{th}\ell. \end{aligned}$$

Note that these behaviors of HEE are independent of dim., collapsing matter...

Thermalization in HSV backgrounds

What about time-dependent HSV background:

$$ds^2_{D+2} = \rho^{2\frac{\theta-D}{D}}\Big(-\rho^{2-2z}f(\rho,v)dv^2 - 2\rho^{1-z}d\rho dv + \sum_{i=1}^{D} dx_i^2\Big),$$

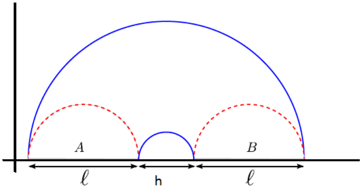
$$f(\rho,v) = 1 - m(v)\rho^{d-1+z}$$

- Time evolution of the EE in this background has been analyzed [Alishahiha, Faraji, Mohammadi PRD 2014]

— — — — — — — — — — —

Question: In this time-dependent background, what is the time evolution of MI?

Let us consider two strips with widths ℓ separated by distance h with condition $h \ll \ell$. Therefore, the scales are: ℓ, h, $2\ell + h$ and ρ_H,



Depending on the size of ρ_H, one may distinguish four main possibilities for the order of scales

I) $\rho_H \ll \frac{h}{2} \ll \frac{\ell}{2} < \ell + \frac{h}{2}$,

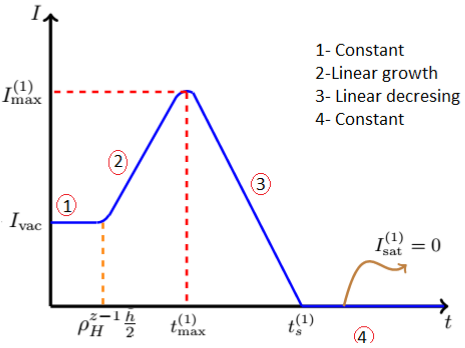
II) $\frac{h}{2} \ll \rho_H \ll \frac{\ell}{2} < \ell + \frac{h}{2}$,

III) $\frac{h}{2} \ll \frac{\ell}{2} < \rho_H < \ell + \frac{h}{2}$,

IV) $\frac{h}{2} \ll \frac{\ell}{2} < \ell + \frac{h}{2} \ll \rho_H$.

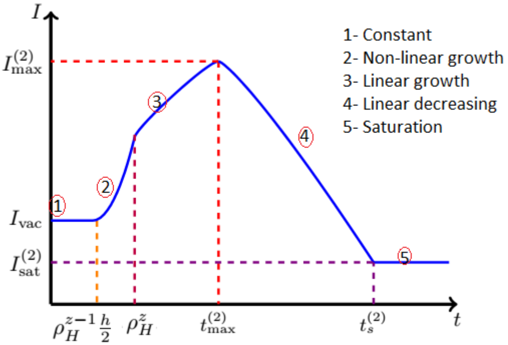
First regime: $\rho_H \ll \frac{h}{2} \ll \frac{\ell}{2} < \ell + \frac{h}{2}$

- $t \ll \rho_H^z$,
- $\rho_H^z \ll t \ll \rho_H^{z-1}\frac{h}{2}$,
- $\rho_H^{z-1}\frac{h}{2} \ll t \ll \rho_H^{z-1}\frac{\ell}{2}$,
- $\rho_H^{z-1}\frac{\ell}{2} < t < \rho_H^{z-1}(\ell + \frac{h}{2})$,
- Saturation time.



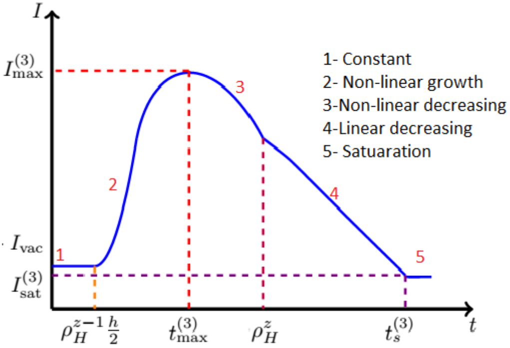
Second regime: $\frac{h}{2} \ll \rho_H \ll \frac{\ell}{2} < \ell + \frac{h}{2}$

- $t \ll \rho_H^{z-1} \frac{h}{2}$,
- $\frac{h}{2} \rho_H^{z-1} \ll t \ll \rho_H^z$,
- $\rho_H^z \ll t \ll \rho_H^{z-1} \frac{\ell}{2}$,
- $\rho_H^{z-1} \frac{\ell}{2} < t < \rho_H^{z-1} (\ell + \frac{h}{2})$,
- Saturation time



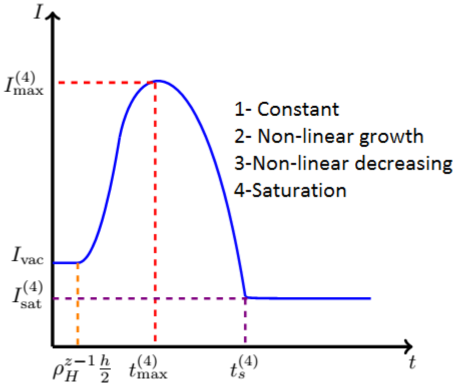
Third regime: $\frac{h}{2} \ll \frac{\ell}{2} < \rho_H < \ell + \frac{h}{2}$

In this regime the related co-dimension two hypersurfaces of $S(h)$ and $S(\ell)$ cannot probe the region near and behind the horizon and entangling regions saturate to their equilibrium values before the system reaches a local equilibrium. Thus only $S(2\ell + h)$ grows linearly with time before it reaches its equilibrium value.



Fourth regime: $\frac{h}{2} \ll \frac{\ell}{2} < \ell + \frac{h}{2} \ll \rho_H$

In this case due to the fact that all entangling regions h, ℓ and $2\ell + h$ are smaller than the radius of horizon, the corresponding entanglement entropies does not exhibit linear growth with time during the process of thermalization.



Thank You

For first regime: $d = D - \theta + 1$

$$t^{(1)}_{\rm max} \sim \frac{\ell}{2} - c_2\rho_H + c_{d-1}\frac{\rho_H^{d-1}}{\ell^{d-2}},$$

$$I^{(1)}_{\rm max} \approx I_{\rm vac} + \frac{L^{D-1}}{4G_N}\left(\frac{c_{d-1}}{\ell^{d-2}} - \frac{c_{d-1}}{h^{d-2}}\right) + \frac{L^{D-1}}{4G_N\rho_H^{d-1}}\left(\frac{\ell}{2} - \frac{h}{2}\right),$$

$$t^{(1)}_s \approx \ell - \frac{h}{2} - c_2\rho_H + \frac{c_{d-1}\rho_H^{d-1}}{(2\ell + h)^{d-2}}.$$