Flat-Space Holography and Anisotrpic Conformal Infinity

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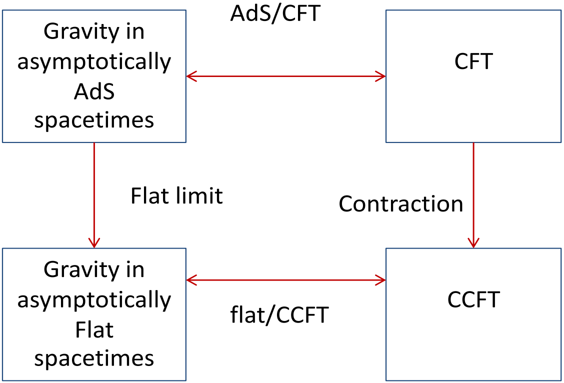
Based on:

- A. Bagchi, R. F. ,JHEP 1210 (2012) 092 [arXiv:1203.5795]
- A. Bagchi, S. Detournay, R. F. , Joan Simon ,Phys.Rev.Lett. 110 (2013) 141302[arXiv:1208.4372]
- R. F. , A. Naseh,JHEP 1403 (2014) 005 [arXiv:1312.2109]
- R. F. , A. Naseh, JHEP 06 (2014) 134 [arXiv:1404.3937]
- R. F. , A. Naseh, S. Rouhani [arXiv:1511.01774].
- R. F. , Y. Izadi, [arXiv:1603.04137]
- R. F. , P. Karimi, work in progress

Motivation

- It is of interest to explore whether holography exists beyond the known example of AdS/CFT.
- A step in this direction is study of holography for asymptotically flat spacetimes.
- An approach is taking limit from the AdS/CFT calculation.
  - Flat-space limit of AAdS spacetimes $\Rightarrow$ asymptotically flat space times
  - What is the corresponding operation in the CFT?
- Our Proposal:
  It is a field theory with contracted Conformal Symmetry.
  (Galilean Conformal symmetry in 2d)
- This correspondence: BMS/GCA or Flat/CCFT


Gravity in asymptotically AdS spacetimes
AdS/CFT
CFT
Flat limit
Contraction
Gravity in asymptotically Flat spacetimes
flat/CCFT
CCFT

Why contraction?

- Asymptotic symmetry of asymptotically AdS spacetimes in d+1 dimensions= Conformal symmetry in d dimensions
- Non-trivial ASG for asymptotically Minkowski spacetimes at null infinity in three and four dimensions:
- BMS_3:

$$[L_m, L_n] = (m-n)L_{m+n}, \quad [L_m, M_n] = (m-n)M_{m+n}, \quad [M_m, M_n] = 0. \tag{1}$$

[Ashtekar, Bicak, Schmidt 1996], [Barnich,Compere 2006]

- BMS_4:

$$[l_m, l_n] = (m-n)l_{m+n}, \quad [\bar{l}_m, \bar{l}_n] = (m-n)\bar{l}_{m+n}, \quad [l_m, \bar{l}_n] = 0,$$
$$[l_l, T_{m,n}] = (\frac{l+1}{2} - m)T_{m+l,n}, \quad [\bar{l}_l, T_{m,n}] = (\frac{l+1}{2} - n)T_{m,n+l}.$$

[Bondi, van der Burg, Metzner; Sachs 1962], [Barnich,Troessaert 2010]

Why contraction?

- Generators of two dimensional CFT on the cylinder:

$$\mathcal{L}_n = -e^{nw}\partial_w, \quad \bar{\mathcal{L}}_n = -e^{n\bar{w}}\partial_{\bar{w}} \quad (w = t + ix, \bar{w} = t - ix) \tag{2}$$

- Define $L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \quad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n})$.
- Use a spacetime contraction: $t \to \epsilon t, x \to x$.
- Final generators in the $\epsilon \to 0$ limit:

$$L_n = -e^{inx}(i\partial_x + nt\partial_t), \quad M_n = -e^{inx}\partial_t \tag{3}$$

- The resultant algebra in the $\epsilon \to 0$ limit is BMS_3 with central charges: $c_{LL} = C_1 = c - \bar{c}, \quad c_{LM} = C_2 = \epsilon(c + \bar{c})$
[A. Bagchi, R. F. (2012)]

Next Step: Flat limit of BTZ

- No asymptotically flat black hole solutions in the three dimensional Einstein gravity. [D. Ida, 2000]
- The flat limit of BTZ is well-defind!
- BTZ black holes:

$$ds^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2\ell^2}dt^2 + \frac{r^2\ell^2}{(r^2-r_+^2)(r^2-r_-^2)}dr^2 + r^2\left(d\phi - \frac{r_+ r_-}{\ell r^2}dt\right)^2$$

- $\ell \to \infty$:

$$r_- \to r_0 = \sqrt{\frac{2G}{M}}|J|, \qquad r_+ \to \ell \hat{r}_+ = \ell\sqrt{8GM} \tag{4}$$

- Flat BTZ: a cosmological solution of 3d Einstein gravity

$$ds^2_{FBTZ} = \hat{r}_+^2 dt^2 - \frac{r^2 dr^2}{\hat{r}_+^2(r^2 - r_0^2)} + r^2 d\phi^2 - 2\hat{r}_+ r_0 dt d\phi \tag{5}$$

- FBTZ is a shifted- boost orbifold of $R^{1,2}$ [Cornalba, Costa(2002)]
- $r = r_0$ is a cosmological horizon with $T = \frac{\hat{r}_+^2}{2\pi r_0}, S = \frac{\pi r_0}{2G}$

Dual field theory analysis: a Cardy-like formula

- 3d asymptotically flat spacetimes $\longrightarrow$ states of field theory with BMS symmetry.
- The states are labelled by eigenvalues of L_0 and M_0:

$$L_0|h_L, h_M\rangle = h_L|h_L, h_M\rangle, \qquad M_0|h_L, h_M\rangle = h_M|h_L, h_M\rangle,$$

where

$$h_L = \lim_{\epsilon\to 0}(h - \bar{h}) = J, \quad h_M = \lim_{\epsilon\to 0}\epsilon(h + \bar{h}) = GM + \frac{1}{8} \quad (6)$$

- Degeneracy of states with (h_L, h_M) $\longrightarrow$ Entropy of cosmological solution with $(r_0, \hat{r}_+)$

Dual field theory analysis: a Cardy-like formula

- Is there any Cardy-like formula? YES
- Modular invariance of CCFT partition function results in

$$S = \log d(h_L, h_M) = 2\pi \left(h_L \sqrt{\frac{C_M}{2h_M}} + C_L \sqrt{\frac{h_M}{2C_M}} \right) \tag{7}$$

[A. Bagchi, S. Detournay, R. F. , Joan Simon (2012)]

- In agreement with the entropy of cosmological solution!

Flat-space Stress Tensor

- Use this method for calculation of flat-space Stress Tensor.
- Starting point is AdS/CFT and Brown and York's quasi-local stress tensor:

$$T^{\mu\nu} = \frac{2}{\sqrt{-\gamma}} \frac{\delta S}{\delta \gamma_{\mu\nu}}, \tag{8}$$

- The gravitational action is

$$S = \frac{1}{16\pi G} \int_{\mathcal{M}} d^3x \sqrt{-g} \left(R - \frac{2}{\ell^2} \right) - \frac{1}{8\pi G} \int_{\partial\mathcal{M}} d^2x \sqrt{-\gamma}\, \mathcal{K} + \frac{1}{8\pi G} S_{ct}(\gamma_{\mu\nu}), \tag{9}$$

where

$$S_{ct} = -\frac{1}{\ell} \int_{\partial\mathcal{M}} d^2x \sqrt{-\gamma}. \tag{10}$$

Flat-space Stress Tensor

- We need a proper coordinate with well-defined flat-space limit.
- A choice is BMS gauge:

$$ds^2 = \left(-\frac{r^2}{\ell^2} + \mathcal{M}\right) du^2 - 2dudr + 2\mathcal{N}dud\phi + r^2 d\phi^2, \quad (11)$$

where

$$\mathcal{M}(u,\phi) = 2\left(\chi(x^+) + \bar{\chi}(x^-)\right), \qquad \mathcal{N}(u,\phi) = \ell\left(\chi(x^+) - \bar{\chi}(x^-)\right), \quad (12)$$

and $\chi, \bar{\chi}$ are arbitrary functions of $x^\pm = \frac{u}{\ell} \pm \phi$.

- The flat-space limit is well-defined:

$$ds^2 = Mdu^2 - 2dudr + 2Ndud\phi + r^2 d\phi^2. \quad (13)$$

where

$$M = \lim_{\frac{G}{\ell}\to 0} \mathcal{M} = \theta(\phi), \qquad N = \lim_{\frac{G}{\ell}\to 0} \mathcal{N} = \beta(\phi) + \frac{u}{2}\theta'(\phi), \quad (14)$$

Flat-space Stress Tensor

- Components of stress tensor for the asymptotically AdS solutions (written in the BMS gauge) are

$$
\begin{aligned}
T_{rr} &= \mathcal{O}(\frac{1}{r^2}), \qquad T_{r+} = \mathcal{O}(\frac{1}{r^2}), \qquad T_{r-} = \mathcal{O}(\frac{1}{r^2}), \\
T_{++} &= \frac{\ell}{8\pi G}\chi(x_+) + \mathcal{O}(\frac{1}{r}), \qquad T_{--} = \frac{\ell}{8\pi G}\bar{\chi}(x_-) + \mathcal{O}(\frac{1}{r}). \\
T_{+-} &= \mathcal{O}(\frac{1}{r^2}) \qquad (15)
\end{aligned}
$$

- Energy-momentum one-point function of dual CFT:

$$
\langle T_{++}\rangle = \frac{\ell\chi}{8\pi G}, \qquad \langle T_{--}\rangle = \frac{\ell\bar{\chi}}{8\pi G}, \qquad \langle T_{+-}\rangle = 0. \quad (16)
$$

- In the coordinate $\{u, \phi\}$ we have

$$
\begin{aligned}
\langle T_{uu}\rangle &= \frac{\chi + \bar{\chi}}{8\pi G\ell} = \frac{\mathcal{M}}{16\pi G\ell}, \qquad \langle T_{u\phi}\rangle = \frac{\chi - \bar{\chi}}{8\pi G} = \frac{\mathcal{N}}{8\pi G\ell}, \quad (17) \\
\langle T_{\phi\phi}\rangle &= \frac{\ell(\chi + \bar{\chi})}{8\pi G} = \frac{\ell\mathcal{M}}{16\pi G}. \quad (18)
\end{aligned}
$$

Flat-space Stress Tensor

- $\ell \to \infty$ is not well-defined!
- However, the combinations

$$T_1 = \lim_{G/\ell \to 0} \frac{G}{\ell}(T_{++} + T_{--}), \qquad T_2 = \lim_{G/\ell \to 0}(T_{++} - T_{--}), \tag{19}$$

are finite in the flat limit.

- We define the flat-space stress tensor, $\tilde{T}_{ij}$, by

$$T_1 = (\tilde{T}_{++} + \tilde{T}_{--}), \qquad T_2 = (\tilde{T}_{++} - \tilde{T}_{--}), \qquad \tilde{T}_{+-} = 0, \tag{20}$$

where $x^{\pm} = \frac{u}{G} \pm \phi$ [R.F. , Ali Naseh,2013]

Flat-space Stress Tensor

- Contraction of $\nabla^i T_{ij} = 0$ results in

$$\partial_t T_1 = 0, \qquad \partial_x T_1 - \partial_t T_2 = 0, \tag{21}$$

where t is the contracted time.

- The above equations are consistent with the Einstein equations in the bulk side .
- A useful result:

$$\tilde{T}_{uu} = \lim_{\frac{G}{\ell}\to 0} \frac{\ell}{G} T_{uu}, \quad \tilde{T}_{u\phi} = \lim_{\frac{G}{\ell}\to 0} \frac{\ell}{G} T_{u\phi}, \quad \tilde{T}_{\phi\phi} = \lim_{\frac{G}{\ell}\to 0} \frac{G}{\ell} T_{\phi\phi}. \tag{22}$$

- The flat space stress tensor $\tilde{T}_{ij}$ is given by appropriate scaling of AdS counterpart.

Flat-space Stress Tensor

- The geometry of spacetime which contracted theory lives on it, is the same as parent theory.
- Its time coordinate is given by contraction of original one.
- Conserved charges of symmetry generators are given by

$$Q_\xi = \int_\Sigma d\phi \sqrt{\sigma} v^\mu \xi^\nu \tilde{T}_{\mu\nu}, \tag{23}$$

- The final result is consistent with the known results.

Anisotropic conformal infinity

- Asymptotically flat spacetimes which was used for calculation of holographic stress tensor:

$$ds^2 = Mdu^2 - 2dudr + 2Ndud\phi + r^2 d\phi^2. \tag{24}$$

where

$$M = \lim_{\frac{G}{\ell}\to 0} \mathcal{M} = \theta(\phi), \qquad N = \lim_{\frac{G}{\ell}\to 0} \mathcal{N} = \beta(\phi) + \frac{u}{2}\theta'(\phi). \tag{25}$$

- CCFT lives on

$$d\tilde{s}^2 = -du^2 + G^2 d\phi^2 \tag{26}$$

- It is clear that the geometry which CCFT lives on it, is not given by isotropic scaling of generic asymtotically flat spacetimes (24)!

Anisotropic conformal infinity

- However, an anisotropic scaling like below is possible:

$$g_{\phi\phi} \to \Omega^{2z} g_{\phi\phi}, \quad g_{u\phi} \to \Omega^2 g_{u\phi}, \quad g_{ur} \to \Omega^2 g_{ur}, \quad g_{uu} \to g_{uu}. \tag{27}$$

where

$$z = 2, \qquad \Omega = \frac{M^{\frac{1}{4}}\sqrt{G}}{\sqrt{r}} \tag{28}$$

- The final metric is conformal to the metric of spacetime which dual CCFT lives on it.

Anisotropic conformal infinity

The anisotropic scaling defined in this way is well-defined:

- Form of the asymptotically flat spacetimes defined as (24) are preserved under the transformation generated by

$$\chi^u = F, \quad \chi^\phi = Y - \frac{1}{r}\partial_\phi F, \quad \chi^r = -r\partial_\phi Y + \partial_\phi^2 F - \frac{1}{r}N\partial_\phi F \tag{29}$$

  where $F = T + u\partial_\phi Y$ and T and Y are arbitrary functions of ϕ coordinate.
- It is not difficult to show that: if ω generates the infinitesimal anisotropic conformal scaling for metric $g_{\mu\nu}$ then $\omega + \delta_\chi \omega$ scales metric $g_{\mu\nu} + \delta_\chi g_{\mu\nu}$ to a geometry which differs from the scaling of $g_{\mu\nu}$ upto a conformal factor. [R. F, P. Karimi, work in progress]

Holographic Anomaly of CCFT

- Use Flat/CCFT correspondence and calculate the anomaly of CCFT.
- Starting point→ asymptotically AdS solutions:

$$ds^2 = \mathcal{A}(u,r,\phi)du^2 - 2e^{2\beta(u,\phi)}dudr + 2\mathcal{B}(u,r,\phi)dud\phi + r^2 d\phi^2. \tag{30}$$

- The equations of motion determine $\mathcal{A}$ and $\mathcal{B}$ as

$$\begin{aligned}\mathcal{A}(u,r,\phi) &= -e^{4\beta(u,\phi)}\frac{r^2}{\ell^2} + \mathcal{M}(u,\phi),\\ \mathcal{B}(u,r,\phi) &= -r\partial_\phi e^{2\beta(u,\phi)} + \mathcal{N}(u,\phi),\end{aligned} \tag{31}$$

where

$$\begin{aligned}&\partial_\phi\mathcal{M} - 2\partial_u\mathcal{N} + 4\mathcal{N}\partial_u\beta = 0\\ &-8\partial_\phi\beta\partial_u\partial_\phi\beta - 4\partial_u\partial_\phi^2\beta + \frac{4}{\ell^2}\mathcal{N}\partial_\phi\beta + \frac{2}{\ell^2}\partial_\phi\mathcal{N} - e^{-4\beta}\partial_u\mathcal{M}\\ &+4\mathcal{M}e^{-4\beta}\partial_u\beta = 0\end{aligned} \tag{32}$$

Holographic Anomaly of CCFT

- (30) has non-flat conformal boundary

$$ds^2|_{C.B} = -\frac{G^2}{\ell^2} e^{4\beta(u,\phi)} du^2 + G^2 d\phi^2 \tag{33}$$

- Components of stress tensor:

$$\begin{aligned} T_{uu} &= \frac{e^{4\beta}\left(\partial_\phi^2\beta + (\partial_\phi\beta)^2\right)}{4\pi\ell G} + \frac{\mathcal{M}}{16\pi\ell G}, \\ T_{u\phi} &= \frac{\mathcal{N}}{8\pi\ell G}, \\ T_{\phi\phi} &= \frac{\ell\mathcal{M}e^{-4\beta}}{16\pi G} - \frac{\ell(\partial_\phi\beta)^2}{4\pi G}. \end{aligned} \tag{34}$$

- Anomoly is computed holographically as

$$T = \frac{C}{24\pi} R_{C.B}, \tag{35}$$

where $C = 3\ell/2G$ is the Brown and Henneaux's central charge.

Holographic Anomaly of CCFT

- Take flat-space limit:

$$\tilde{T}_{++} + \tilde{T}_{--} = \lim_{\frac{G}{\ell}\to 0} \frac{G}{\ell}\left(T_{++} + T_{--}\right)$$

$$\tilde{T}_{++} - \tilde{T}_{--} = \lim_{\frac{G}{\ell}\to 0} \left(T_{++} - T_{--}\right)$$

$$\tilde{T}_{+-} = \lim_{\frac{G}{\ell}\to 0} \frac{G}{\ell} T_{+-} \tag{36}$$

- CCFT is on a spacetime with line-element:

$$d\tilde{s}^2 = -e^{4\beta(u,\phi)}du^2 + G^2 d\phi^2, \tag{37}$$

- This metric is given by an anisotropic scaling from (30)!
- The trace anomaly is

$$\tilde{T} = \tilde{g}^{ij}\tilde{T}_{ij} = \frac{1}{4\pi} c_M \tilde{R} \tag{38}$$

[R. F. , A. Naseh, S. Rouhani, 2015]

Flat-space Holography and Stress Tensor of Kerr Black Hole

- Flat-space limit can be used for proposing a stress tensor for the Kerr black hole.
- The method is similar to three dimensions: Take limit from the holographic calculations of Kerr-AdS black hole:

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 \quad (39)$$

$$+\frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(adt - \frac{r^2+a^2}{\Xi}d\phi\right)^2, \quad (40)$$

$$\Delta_r = (r^2+a^2)\left(1+\frac{r^2}{\ell^2}\right) - 2MGr, \quad \Delta_\theta = 1 - \frac{a^2}{\ell^2}\cos^2\theta,$$

$$\rho^2 = r^2 + a^2\cos^2\theta, \qquad \Xi = 1 - \frac{a^2}{\ell^2}. \quad (41)$$

M is the mass of the black hole and $a = J/M$ where J is the angular momentum of the black hole.

Flat-space Holography and Stress Tensor of Kerr Black Hole

- According to AdS/CFT correspondence, the non-zero components of Kerr-AdS stress tensor are

$$\begin{aligned}
8\pi T_{tt} &= \frac{2M}{r\ell}, \\
8\pi T_{t\phi} &= -\frac{2aM}{r\ell\Xi}\sin^2\theta, \\
8\pi T_{\theta\theta} &= \frac{M\ell}{r\Delta_\theta}, \\
8\pi T_{\phi\phi} &= \frac{M\ell}{r\Xi^2}\sin^2\theta\left(\Xi + \frac{3a^2\sin^2\theta}{\ell^2}\right).
\end{aligned} \tag{42}$$

- $\ell \to \infty$ is not well-defined! However applying proper powers of ℓ to the components of the stress tensor makes the flat limit well defined.

Flat-space Holography and Stress Tensor of Kerr Black Hole

- Our proposal for the Kerr stress tensor:

$$
\begin{aligned}
8\pi\tau_{tt} &= \frac{2M}{r\sqrt{G}}, \\
8\pi\tau_{t\varphi} &= -\frac{3aM}{r\sqrt{G}}\sin^2\theta, \\
8\pi\tau_{\theta\theta} &= \frac{M\sqrt{G}}{r}, \\
8\pi\tau_{\varphi\varphi} &= \frac{M\sqrt{G}}{r}\sin^2\theta.
\end{aligned} \tag{43}
$$

[R. F. , Y. Izadi (2016)]

- The anisotropic conformal boundary is given by

$$
d\tilde{s}^2 = \frac{r^2}{G}\left[-dt^2 + Gd\theta^2 + G\sin^2\theta d\varphi^2\right]. \tag{44}
$$

- Using (43) and (43) in the Brown and York formula results in the correct charges of Kerr!

Rindler-space Holography

- Rindler spacetime is the flat limit of Rindler-AdS spacetime,

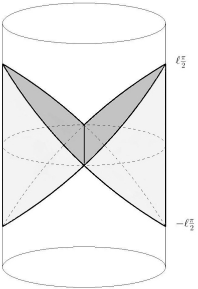
$$ds^2 = -\alpha^2 r^2 d\tau^2 + \frac{dr^2}{1+\frac{r^2}{\ell^2}} + \left(1+\frac{r^2}{\ell^2}\right) d\chi^2, \qquad (45)$$

- Observer at $r = r_0$ perceive a constant acceleration $a_{(3)}^2 = \frac{1}{r_0^2} + \frac{1}{\ell^2}$.
- Proper time of the observer is given by $\alpha r_0 \tau$.
- τ in metric (45) is the proper time of an observer located at $r = r_0 = \frac{1}{\alpha}$.
- Temperature is

$$T = \frac{a_{(4)}}{2\pi} = \frac{1}{2\pi r_0} = \frac{\sqrt{a_{(3)}^2 - \frac{1}{\ell^2}}}{2\pi} \qquad (46)$$

Rindler-space Holography

- Rindler-AdS covers a portion of global AdS which consists of two wedges



- The physics inside the two wedges of Rindler-AdS has a holographic description as entangled states of a pair of CFTs which live on the boundary of Rindler-AdS wedges. [Van Raamsdonk,et. al. (2012)]

Rindler-space Holography

Our proposal:

- Dual theory of Rindler is the contracted CFT resulted in from Rindler-AdS/CFT correspondence.
  [R. F. , A. Naseh (2014)]
- The symmetries of two dimensional CCFT predict the same two-point functions which one may find by taking the flat-space limit of three dimensional Rindler-AdS holographic results.
- It is possible to find an energy-momentum tensor for Rindler Gravity by using Flat/CCFT correspondence:

$$\tilde{T}_{\tau\tau} = \frac{\alpha^2}{16\pi}, \qquad \tilde{T}_{\chi\chi} = \frac{1}{16\pi G^2}. \tag{47}$$

Rindler-space Holography

Application in the black hole physics:

- Near horizon geometry of non-extreme black holes has a Rindler part.
- The dual theory at the horizon of non-extreme black holes is CCFT.
- Non-rotating BTZ:

$$ds^2 = -f(\rho)d\tau^2 + f(\rho)^{-1}d\rho^2 + \rho^2 d\phi^2 \tag{48}$$

  where $f(\rho) = \frac{\rho^2}{\ell^2} - 8GM$ and M is the mass of black hole.
- Defining a new coordinate $y = \rho - \rho_h$ and considering the region given by $y \ll \rho_h$ results in

$$ds^2_{NH} = -f'(\rho_h)yd\tau^2 + (f'(\rho_h)y)^{-1}dy^2 + \rho_h^2 d\phi^2 \tag{49}$$

Rindler-space Holography

- Defining new coordinate r by $dr = dy/\sqrt{f'(\rho_h)y}$ results in

$$ds^2_{NH} = -\frac{f'^2(\rho_h)}{4} r^2 d\tau^2 + dr^2 + \rho_h^2 d\phi^2 \tag{50}$$

- The above metric is Rindler with $\alpha = f'(\rho_h)/2 = \sqrt{8GM}/\ell$ and a compact χ coordinate given by $\chi = \rho_h \phi$.
- Line element (50) is the flat-space limit of a Rindler-AdS metric which has the same α and ϕ.
- Dual of Rindler-AdS is a CFT. Demanding that Cardy formula gives the same result as Bekenstein-Hawking entropy of Rindler-AdS results in

$$\begin{aligned} h &= \frac{\rho_h^2}{16\ell G}\left(2 - \frac{\alpha\ell^2}{\rho_h} + 2\sqrt{1 - \frac{\alpha\ell^2}{\rho_h}}\right), \\ \bar{h} &= \frac{\rho_h^2}{16\ell G}\left(2 - \frac{\alpha\ell^2}{\rho_h} - 2\sqrt{1 - \frac{\alpha\ell^2}{\rho_h}}\right) \end{aligned} \tag{51}$$

Rindler-space Holography

- h_L and h_M are well-defined in the $G/\ell \to 0$ limit:

$$h_L = \frac{\rho_h^2}{4G}\sqrt{-\frac{\alpha}{\rho_h}}, \qquad h_M = -\frac{\alpha\rho_h}{8}. \tag{52}$$

- The Cardy-like formula of CCFT precisely result in Bekenstein-Hawking entropy of non-rotating BTZ!

Thank you