The 4d Cardy formula and its violation through an asymptotic Higgs mechanism

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Based on

High-Temperature Asymptotics of the 4d Superconformal Index, Ph.D. thesis, University of Michigan, 2016

Cardy formulas in 2d and 4d

- Cardy formulas in 2d CFT:

  $i)\ Z_{S^1}(\beta) \sim e^{\frac{c}{\beta}}$ Cardy ✓

  $ii)\ \chi(\beta) \sim e^{\frac{c}{\beta}}$ $(2,2)$ SUSY ✓

- Cardy(-like) formulas in 4d CFT:

  $i)\ Z_{S^3}(\beta) \sim e^{\frac{?}{\beta^3}}$ No universal formula **x**

  $ii)\ \mathcal{I}(\beta) \sim e^{\frac{(c-a)}{\beta}}$ $\mathcal{N}=1$ SUSY **?**
  (Di Pietro-Komargodski '14)

Papers on the 4d Cardy formula

- 3d dualities from 4d dualities (Aharony et al '13)
- $c - a$ from the $\mathcal{N} = 1$ superconformal index (AAA, Liu, Szepietowski '14)
- Cardy Formulae for SUSY Theories in d=4 and d=6 (Di Pietro and Komargoski '14)
- High-temperature asymptotics of supersymmetric partition functions (AAA '15)

- *High-Temperature Asymptotics of the 4d Superconformal Index* (AAA '16)

Context: 4d Lagrangian SuperConformalFieldTheory (SCFT)

A quantum theory of 4d gauge and matter fields (grouped into vector and chiral multiplets).

Supersymmetry means there exists a fermionic "supercharge" operator Q acting on the Hilbert space of the theory, such that

$$\{Q, Q^\dagger\} = H, \qquad Q^2 = 0.$$

Note: Q pairs the states it doesn't annihilate!

Conformal symmetry implies an additional $\mathrm{U}(1)_R$ symmetry.

The local operators of such a Quantum Field Theory are charged under the maximal compact bosonic subgroup of the superconformal group:

$$\Delta, j_1, j_2, r$$

Motivation

- Holography: Cardy limit of CFT_2 encodes Blackhole physics; Cardy limit of CFT_4 seems to encode giant gravitons.

- Gauge dynamics: Cardy limit of CFT_4 seems to encode the crossed channel gauge dynamics on $R^3 \times S^1$.

What is the 4d superconformal index? (1/2)

It is a combinatorial partition function that counts certain gauge-invariant local operators in 4d SCFTs.

$$\mathcal{I}(\beta) \equiv \sum_{\text{gauge inv. ops.}} (-1)^F e^{-\beta(\Delta - \frac{r}{2})}$$
$$= \sum_{\text{BPS operators}} (-1)^F e^{-\beta(\Delta - \frac{r}{2})}$$

The index receives contributions from the supersymmetric (protected) operators only: *the sum* **localizes** *over BPS operators*.

It often depends on β (inverse temperature $\in]0, \infty[$) in a complicated manner.

What is the 4d superconformal index? (2/2)

The index of a theory with a semi-simple gauge group G can be computed via certain well-understood combinatorial procedures (plethystic exponentiation, and projecting onto the gauge singlet sector).

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$$\mathcal{I}(\beta) = \frac{(q;q)^{2r_G}}{|W|} \int \mathrm{d}^{r_G}x \, \frac{\prod_\chi \prod_{\rho^\chi \in \Delta_\chi} \Gamma_e(q^{r_\chi} z^{\rho^\chi})}{\prod_{\alpha_+} \Gamma_e(z^{\pm\alpha_+})},$$

with $q = e^{-\beta}$, $z = e^{2\pi i x}$. The integral is over $-\frac{1}{2} \leq x_j \leq \frac{1}{2}$; the x_j parameterize the gauge group ($j = 1, \ldots, r_G$).

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The index could have been obtained (Assel, Cassani, Martelli) via path-integration on $S^3 \times S^1_\beta$. The path-integral localizes on the moduli space of the holonomies around S^1_β—or essentially the Polyakov loops!

What's there in the high-temperature limit? (1/5)

The index is given by an Elliptic Hypergeometric Integral:

$$\mathcal{I}(\beta) = \int \mathrm{d}^{r_G} x \; F(\mathbf{x}; \beta).$$

The high-temperature ($\beta \to 0$) limit corresponds to the **hyperbolic limit** of the Elliptic Hypergeometric Integral.

The machinery for analyzing the hyperbolic limit of EHIs is developed in *Limits of Elliptic Hypergeometric Integrals* (Rains '06).

Expectation: (AAA, JT Liu, and P Szepietowski '14, '15; Di Pietro-Komargodski '14)

$$\mathcal{I}(\beta) \approx e^{\frac{A}{\beta} + B \ln \beta + C + D\beta + \ldots}$$

What's there in the high-temperature limit? (2/5)

Following Rains' approach we find that the index (of theories with non-chiral matter content) simplifies at high temperatures as

$$\mathcal{I}(\beta) = \int_{\mathfrak{h}_{cl}} F(\mathbf{x};\beta) \xrightarrow{\beta\to 0} \int_{\mathfrak{h}_{cl}} \exp\left[-\left(\mathcal{E}_0^{DK}(\beta) + V^{\text{eff}}(\mathbf{x};\beta)\right)\right],$$

where $\mathfrak{h}_{cl}$ stands for $-\frac{1}{2} \leq x_i \leq \frac{1}{2}$, while

$$\mathcal{E}_0^{DK}(\beta) = -\frac{16\pi^2}{3\beta}(c-a)$$

and

$$V^{\text{eff}} = \frac{4\pi^2}{\beta} L_h(\mathbf{x})$$

The star of our show, the function $L_h(\mathbf{x})$, is real, continuous, and piecewise linear. Also $L_h(0) = 0$.

What's there in the high-temperature limit? (3/5)

We can write

$$\mathcal{I}(\beta) \approx \int_{\mathfrak{h}_{qu}} e^{-[\mathcal{E}_0^{DK}(\beta) + V^{\mathrm{eff}}(\mathbf{x};\beta)]} \approx e^{-[\mathcal{E}_0^{DK}(\beta) + V^{\mathrm{eff}}_{\mathrm{min}}(\beta)]},$$

with $\mathfrak{h}_{qu}$ the locus of minima of $L_h(\mathbf{x})$, and $V^{\mathrm{eff}}_{\mathrm{min}}(\beta)$ the minimum of $V^{\mathrm{eff}}(\mathbf{x};\beta)$.

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The end result looks like

$$\ln \mathcal{I}(\beta) = \frac{16\pi^2}{3\beta}(c - a - \frac{3}{4}L_{h\ \text{min}}) + \dim\mathfrak{h}_{qu} \ln(\frac{2\pi}{\beta}) + O(\beta^0).$$

The Di Pietro-Komargodski formula receives a correction if $L_h(\mathbf{x})$ is not positive semi-definite.

What's there in the high-temperature limit? (4/5)

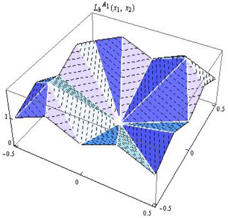
$$\ln \mathcal{I}(\beta) = \frac{16\pi^2}{3\beta}(c - a - \frac{3}{4}L_{h\ \mathrm{min}}) + \mathrm{dim}\mathfrak{h}_{qu} \ln(\frac{2\pi}{\beta}) + O(\beta^0).$$

For example, for the $\mathcal{N} = 4$ theory we have $L_h(\mathbf{x}) = 0$. Therefore $\mathrm{dim}\mathfrak{h}_{qu} = \mathrm{dim}\mathfrak{h}_{cl} = r_G$. Since for this theory $c - a = 0$, we get

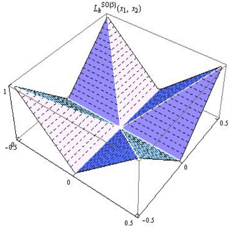
$$\ln \mathcal{I}_{\mathcal{N}=4}(\beta) = r_G \ln(\frac{2\pi}{\beta}) + O(\beta^0).$$

What's there in the high-temperature limit? (5/5)

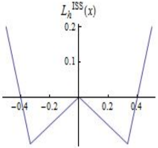
More examples:



SU(3) SQCD fixed points



SO(5) SQCD fixed points



The ISS SCFT (has $c < a$)

The 4d superconformal index has a Cardy-like asymptotics, unless the theory exhibits an asymptotic Higgs mechanism!

Thanks for your attention!

Taking the large-N limit first

A field theory computation shows

$$\mathcal{I}(\beta) \xrightarrow{N\to\infty} I^{\mathrm{m.t.}}(\beta)$$

Hence the holographic computation of the large-N index involves only Kaluza-Klein particles.

It turns out that a simple differential operator acting on the large-N index gives the bulk loop correction to the Weyl anomaly of the SCFT.

It can be checked, in a large class of holographic SCFTs, that the operator acting on the index does indeed produce the boundary subleading central charges. This addresses the Holographic Weyl Anomaly problem quite generally, validating AdS/CFT beyond the leading order in large-N, and in infinitely many cases.

Thanks for your attention!