Brane SUSY Breaking: Old Puzzles and New Results

Augusto Sagnotti
Scuola Normale Superiore and INFN – Pisa

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Istituto Nazionale di Fisica Nucleare

Ten-Dimensional Superstrings

(Briefly)

Ten-Dimensional (Closed) Superstrings

- **Building principles:** spin-statistics *(GSO projections)* and modular invariance

- **Building Blocks:** SO(8) level-one characters ($q = e^{2\pi i \tau}$):

Even(odd) # Fermi osc

$$O_8(V_8) = \frac{\theta^4\left[{0 \atop 0}\right](0|\tau) \pm \theta^4\left[{0 \atop 1/2}\right](0|\tau)}{2\,\eta^4(\tau)} \qquad S_8(C_8) = \frac{\theta^4\left[{1/2 \atop 0}\right](0|\tau) \pm \theta^4\left[{1/2 \atop 1/2}\right](0|\tau)}{2\,\eta^4(\tau)}$$

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty}(1 - q^n) \qquad \theta\left[{\alpha \atop \beta}\right](z|\tau) = \sum_{n\in Z} q^{\frac{1}{2}(n+\alpha)^2} e^{i2\pi(n+\alpha)(z-\beta)}$$

Opposite Weyl proj's

- **Behavior under Modular Transformations:**

$$\tau \to -\frac{1}{\tau} : \; S = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} \qquad \tau \to \tau + 1 : \; T = e^{-1\frac{\pi}{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\tau \to -\frac{1}{\tau} : \; \eta(-1/\tau) = (-i\tau)^{\frac{1}{2}}\eta(\tau) \qquad \tau \to \tau + 1 : \eta(\tau+1) = e^{i\frac{\pi}{12}}\,\eta(\tau)$$

- IIA and IIB:

$$\mathcal{T}_{IIA} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{(V_8 - S_8)(\bar{V}_8 - \bar{C}_8)}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{IIB} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|V_8 - S_8|^2}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8}$$

- 0A and 0B:

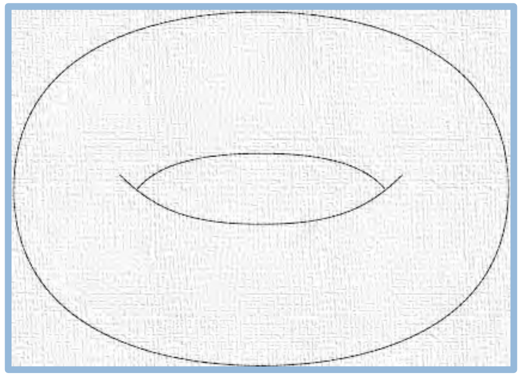
$$\mathcal{T}_{0A} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|O_8|^2 + |V_8|^2 + S_8\bar{C}_8 + C_8\bar{S}_8}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{0B} = \int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\,\eta^8\,\bar{\eta}^8}$$

- HO and HE:

$$\mathcal{T}_{HE} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2} \frac{(V_8 - S_8)(\bar{O}_{16} + \bar{S}_{16})^2}{Im\tau^4\,\eta^8\,\bar{\eta}^8} \qquad \mathcal{T}_{HO} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2} \frac{(V_8 - S_8)(\bar{O}_{32} + \bar{S}_{32})}{Im\tau^4\,\eta^8\,\bar{\eta}^8}$$

- $H_{SO(16)xSO(16)}$:

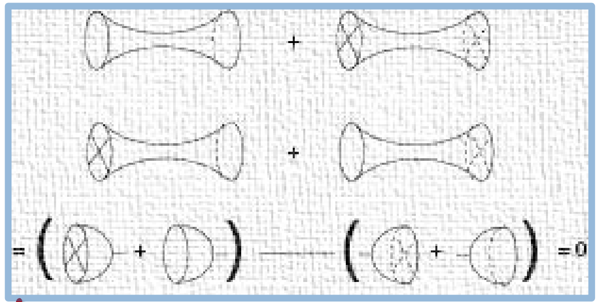
$$\mathcal{T}_{SO(16)\times SO(16)} = \int_{\mathcal{F}} \frac{d^2\tau}{Im\tau^2} \frac{1}{Im\tau^4\,\eta^8\,\bar{\eta}^8} [O_8(\bar{V}_{16}\bar{C}_{16} + \bar{C}_{16}\bar{V}_{16}) + V_8(\bar{O}_{16}\bar{O}_{16} + \bar{S}_{16}\bar{S}_{16}) - S_8(\bar{O}_{16}\bar{S}_{16} + \bar{S}_{16}\bar{O}_{16}) - C_8(\bar{V}_{16}\bar{V}_{16} + \bar{C}_{16}\bar{C}_{16})]$$

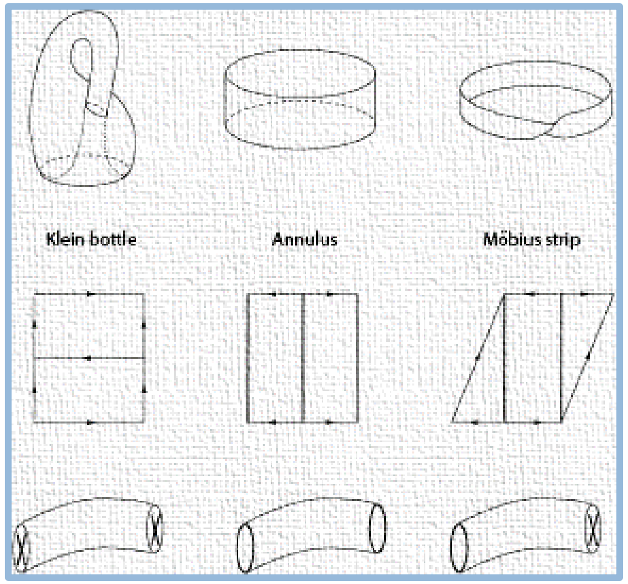


Ten-Dimensional Orientifolds

❖ Follow from the IIB, 0A and 0B models via an orientifold projection *(AS, 1987)*, which fills vacuum with D-branes and O-planes *(Polchinski, 1995)*

❖ RR tadpole condition(s): neutrality conditions





❖ Three 10D NON-TACHYONIC orientifold models:

- Type-I SO(32) superstring *(Green and Schwarz, 1984)*: descends from IIB, vacuum filled with BPS combinations of O- orientifold $(T<0, Q<0)$ and D-branes $(T>0, Q>0)$. Massless Weyl fermions in the adjoint of SO(32)

- U(32) 0'B model *(AS, 1995)*: non-BPS D and O; → Applications to large-N QCD: *Armoni et al*

- USp(32) Sugimoto model (simplest example with "Brane SUSY Breaking"): descends from IIB, vacuum filled with NON BPS combination of O+ orientifold $(T>0, Q>0)$ and anti D-branes $(T>0, Q<0)$. Massless fermions in the (reducible) two-fold antisymmetric of USp(32) → Goldstino

(Sugimoto; Antoniadis, Dudas, AS; Angelantonj, Aldazabal and Uranga, 1999)

10D Tachyon-Free Orientifolds

1. USp(32) type-I: $(e_\mu^a, B_{\mu\nu}, \phi, \psi_\mu, \psi) \oplus (A_\mu^{(ab)}, \lambda^{[ab]})$, USp(32) gauge group [non-linear SUSY, T > 0]

   (Sugimoto, 1999)

$$\mathcal{T}_{Usp(32)} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2} \frac{|V_8 - S_8|^2}{(Im\tau)^4\, \eta^8\, \bar\eta^8}[\tau,\bar\tau] \qquad \mathcal{K}_{USp(32)} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{V_8 - S_8}{(\tau_2)^4\, \eta^8}[2i\tau_2]$$

$$\mathcal{A}_{Usp(32)} = \frac{\mathcal{N}^2}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{V_8 - S_8}{(\tau_2)^4\, \eta^8}[i\tau_2/2] \qquad \mathcal{M}_{USp(32)} = \frac{\mathcal{N}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{(\hat V_8 + \hat S_8)}{(\tau_2)^4\, \hat\eta^8}[i\tau_2/2 + 1/2]$$

$$\mathcal{N} = 32$$

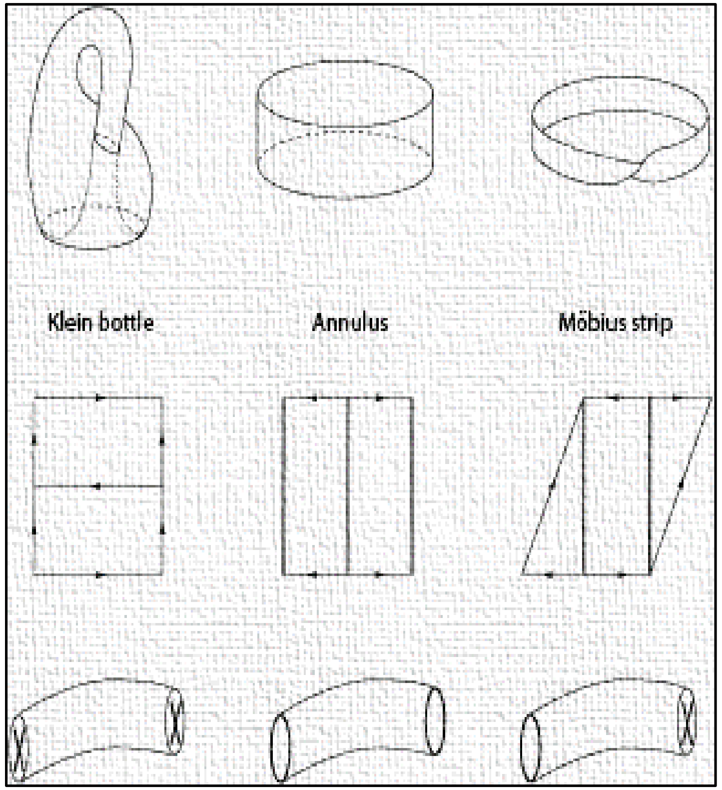
2. U(32) type-0'b: $(e_\mu^a, B_{\mu\nu}, D^+_{\mu\nu\rho\sigma}, \phi, \phi') \oplus (A_{\mu\, a}{}^{b}, \lambda^{[ab]}, \lambda_{[ab]})$, U(32) gauge group [NO SUSY, T > 0]

   (AS, 1995)

$$\mathcal{T}_{0'B} = \frac{1}{2}\int_{\mathcal{F}} \frac{d^2\tau}{(Im\tau)^2}\frac{|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2}{(Im\tau)^4\, \eta^8\, \bar\eta^8}[\tau,\bar\tau] \qquad \mathcal{K}_{0'B} = \frac{1}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{-O_8 + V_8 + S_8 - C_8}{(\tau_2)^4\, \eta^8}[2i\tau_2]$$

$$\mathcal{A}_{0'B} = \int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{\mathcal{N}\bar{\mathcal{N}}\, V_8 - \frac{\mathcal{N}^2 + \bar{\mathcal{N}}^2}{2}\, C_8}{(\tau_2)^4\, \eta^8}[i\tau_2/2] \qquad \mathcal{M}_{0'B} = -\frac{\mathcal{N} + \bar{\mathcal{N}}}{2}\int_0^\infty \frac{d\tau_2}{(\tau_2)^2}\frac{\hat C_8}{(\tau_2)^4\, \hat\eta^8}[i\tau_2/2 + 1/2]$$

$$\mathcal{N} = \bar{\mathcal{N}} = 32$$



3. SO(16) x SO(16) heterotic model: $(e_\mu^a, B_{\mu\nu}, \phi) \oplus (A_\mu^{[ab]}, A_\mu^{[a'b']}, \lambda_L^{aa'}, \lambda_R^{A}, \lambda_R^{A'})$, $SO(16) \times SO(16)$ gauge group [NO SUSY, $\Lambda_{Torus} > 0$]

   (Alvarez-Gaumé, Ginsparg, Moore, Vafa, 1986; Dixon and Harvey, 1986)

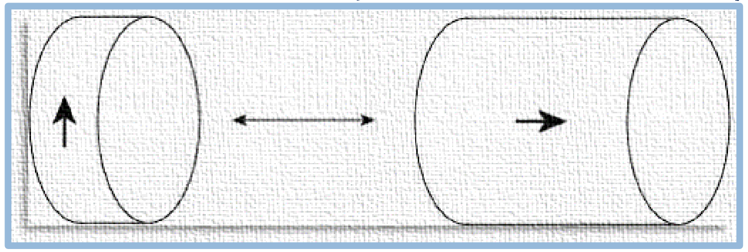
Brane SUSY Breaking

The Problem

- String Theory can be efficiently described around supersymmetric vacua (especially with extended SUSY);

- On the other hand, **broken supersymmetry** is accompanied by **vacuum redefinitions**:

  - In models of **oriented closed strings** these originate from "loop" diagrams (torus and beyond)
  - In **orientifolds** the distinction between tree and loop levels is blurred by **open-closed duality**



  - **"Brane SUSY Breaking":** results from insertion of non-BPS combinations of BPS branes and/or orientifolds *(Sugimoto; Antoniadis, Dudas, AS; Angelantonj, Aldazabal and Uranga, 1999)*

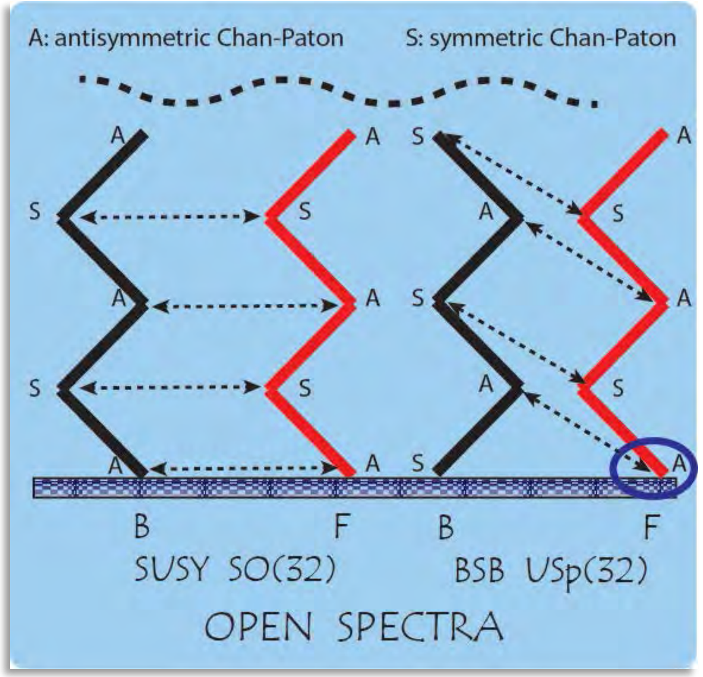
  - **Runaway exponential potential** for dilaton ($\sim e^{-\varphi}$ in string frame, or $\sim e^{\frac{3}{2}\varphi}$ in 10D in Einstein frame)

  - [**Torus (genus-one)** correction in 10D closed-string modes: ~ 1 in string frame, or $\sim e^{\frac{5}{2}\varphi}$ in Einstein frame]

- **In Field Theory** → one can shift fields. **BUT in String Theory** → must start around "wrong vacua"

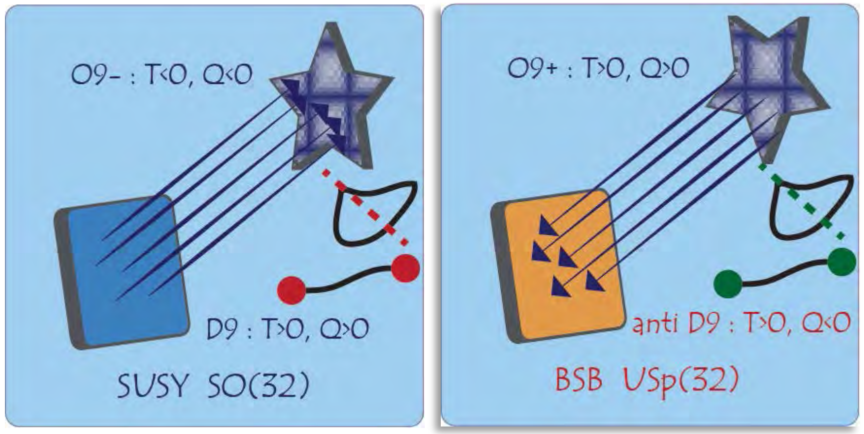
Brane SUSY Breaking (BSB)

(Sugimoto, 1999)
(Antoniadis, Dudas, AS, 1999)
(Angelantonj, 1999)
(Aldazabal, Uranga, 1999)



Tree – level

- SUSY broken at string scale in open sector, exact in closed sector
- Stable vacuum (classically)
- Goldstino in open sector



BSB: Tension unbalance → exponential potential

$$S_{10} = \frac{1}{2k_{10}^2} \int d^{10}x\sqrt{-g}\,\{e^{-2\phi}\left(-R + 4(\partial\phi)^2\right) - Te^{-\phi} + \ldots\}$$

BSB: The Low-Energy Supergravity

(Dudas, and Mourad, 2000, 2001)

$$S = \frac{1}{2\,k_{10}^2} \int d^{10}x \sqrt{-G} \left\{ e^{-2\phi} \left[-R + 4(\partial\phi)^2 \right] - \frac{1}{12}\, \mathcal{H}_3^2 - \frac{1}{4}\, e^{-\phi}\, \mathrm{tr}\, \mathcal{F}^2 - T\, e^{-\phi} \right\}$$

1. **Dudas and Mourad** solved the field equations for a 9D profile that (in string frame) describes a 9D S_1/Z_2 compactification. It leads to **finite 9D Planck mass and gauge coupling**, but there are **singularities** at the ends

$$ds^2 = |\alpha\, y^2|^{\frac{2}{9}}\, e^{\frac{1}{2}\phi_0}\, e^{\frac{1}{4}\alpha\, y^2}\, \eta_{\mu\nu}\, dx^\mu\, dx^\nu + |\alpha\, y^2|^{-\frac{1}{3}}\, e^{-\phi_0}\, e^{-\frac{3}{4}\alpha\, y^2}\, dy^2$$
$$e^\phi = e^{\phi_0}\, |\alpha\, y^2|^{\frac{1}{3}}\, e^{\frac{3}{4}\alpha\, y^2}$$

[similar, albeit more complicated looking, results also obtain for SO(16) x SO(16)]

2. There is an amusing **cosmological counterpart**. The dilaton tadpole lies precisely at the onset of the **"climbing phenomenon"**. The scalar is bound to emerge from the initial singularity **"climbing up" the potential**.

$$ds^2 = |\alpha\, t^2|^{\frac{2}{9}}\, e^{\frac{1}{2}\phi_0}\, e^{-\frac{1}{4}\alpha\, t^2}\, \delta_{ij}\, dx^i\, dx^j - |\alpha\, t^2|^{-\frac{1}{3}}\, e^{-\phi_0}\, e^{\frac{3}{4}\alpha\, t^2}\, dt^2$$
$$e^\phi = e^{\phi_0}\, |\alpha\, t^2|^{\frac{1}{3}}\, e^{-\frac{3}{4}\alpha\, t^2}$$

Halliwell, 1987)
…………
(Dudas, Mourad, 2000)
(Russo, 2004)
(Dudas, Kitazawa, AS, 2010)
…………

String Clues on the Onset of Inflation ?

Cosmological Potentials

What potentials lead to slow-roll, and where ?

$$ds^2 = -dt^2 + e^{2A(t)}\, d\mathbf{x}\cdot d\mathbf{x} \quad \Longrightarrow \quad \ddot{\phi} + 3\dot{\phi}\sqrt{\frac{1}{3}\dot{\phi}^2 + \frac{2}{3}V(\phi)} + V' = 0$$

Driving force from V' *vs* friction from V

- **If V does not vanish :** convenient gauge "makes the damping term neater"

$$ds^2 = e^{2\mathcal{B}(t)}\, dt^2 - e^{\frac{2\mathcal{A}(t)}{d-1}}\, d\mathbf{x}\cdot d\mathbf{x}$$
$$V e^{2\mathcal{B}} = V_0$$
$$\tau = t\sqrt{\frac{d-1}{d-2}}, \qquad \varphi = \phi\sqrt{\frac{d-1}{d-2}}$$

$$\dot{\mathcal{A}}^2 - \dot{\varphi}^2 = 1$$
$$\ddot{\varphi} + \dot{\varphi}\sqrt{1+\dot{\varphi}^2} + \frac{V_\varphi}{2V}\left(1+\dot{\varphi}^2\right) = 0$$

- Now driving from logV *vs* O(1) damping

$$V = \varphi^n \longrightarrow \frac{V'}{2V} = \frac{n}{2\varphi}$$

❖ Quadratic potential? Far away from origin *(Linde, 1983)*

❖ Exponential potential? YES or NO

$$V(\varphi) = V_0\, e^{2\gamma\varphi} \longrightarrow \frac{V'}{2V} = \gamma$$

$V = e^{2\gamma\varphi}$: Climbing & Descending Scalars

- $\gamma < 1$? Both signs of speed

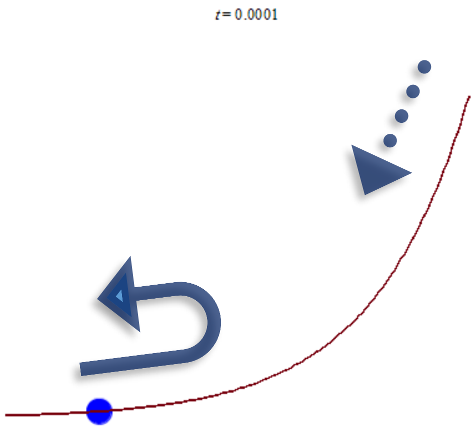
(Halliwell, 1987;…, Dudas and Mourad, 1999; Russo, 2004; Dudas, Kitazawa, AS, 2010)

a. "Climbing" solution (φ climbs, then descends):

$$\dot\varphi = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$

b. "Descending" solution (φ only descends):

$$\dot\varphi = \frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}}\,\tanh\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right) - \sqrt{\frac{1+\gamma}{1-\gamma}}\,\coth\left(\frac{\tau}{2}\sqrt{1-\gamma^2}\right)\right]$$



Limiting τ- speed (LM attractor):

(Lucchin and Matarrese, 1985)

$$v_{lim} = -\frac{\gamma}{\sqrt{1-\gamma^2}}$$

$\gamma = 1$ is "critical": LM attractor & descending solution disappear there and beyond

CLIMBING : in ALL asymptotically exponential potentials with $\gamma \geq 1$!

10D STRING THEORY HAS PRECISELY $\gamma = 1$ (bounded g_s !)

- $\gamma = 1$:

$$\varphi(\tau) = \varphi_0 + \frac{1}{2}\left[\log|\tau-\tau_0| - \frac{1}{2}(\tau-\tau_0)^2\right]$$

$$\mathcal{A}(\tau) = \mathcal{A}_0 + \frac{1}{2}\left[\log|\tau-\tau_0| + \frac{1}{2}(\tau-\tau_0)^2\right]$$

Critical Exponentials and BSB: D < 10

(Dudas, Kitazawa, AS, 2010)
(AS, 2013)
(Fré, AS, Sorin, 2013)

❖ STRING THEORY PREDICTS the exponent in $V = V_0\, e^{2\varphi}$

- D=10 : Polyakov expansion and dilaton tadpole

$$\mathcal{S} = \frac{1}{2\,k_N^2} \int d^{10}x \sqrt{-\det g}\left[R + \frac{1}{2}\, g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi - T\,e^{\frac{3}{2}\phi} + \ldots\right] \quad \longrightarrow \quad \gamma = 1\ (\text{for } \varphi)$$

- D < 10 : two combinations of ϕ and "breathing mode" σ → (Φ_s, Φ_t)
- Φ_t yields a "critical" potential ($\gamma = 1$) if Φ_s is stabilized

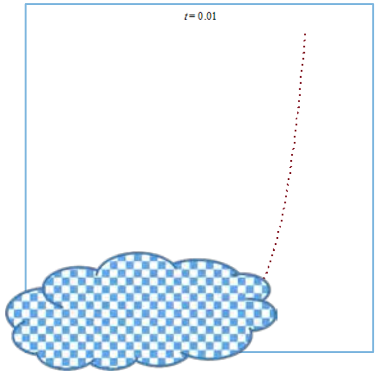
$$S_d = \frac{1}{2\kappa_d^2} \int d^dx\sqrt{-g}\left[R + \frac{1}{2}(\partial\Phi_s)^2 + \frac{1}{2}(\partial\Phi_t)^2 - T_9\, e^{\sqrt{\frac{2(d-1)}{d-2}}\,\Phi_t} + \ldots\right]$$

- If Φ_s is stabilized: a p-brane that couples via $(g_s)^{-\alpha}$ yields :
[the D9-brane we met before had p=9, α=1]

$$\gamma = \frac{1}{12}\,(p + 9 - 6\,\alpha)$$

[NOTE: all multiples of $\frac{1}{12} \simeq 0.08$]

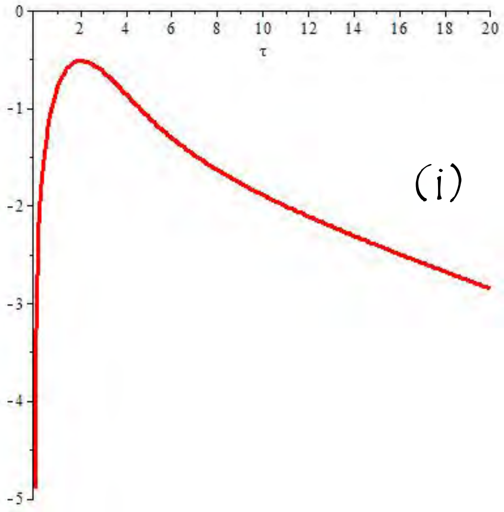
Onset of Inflation via BSB & Climbing?

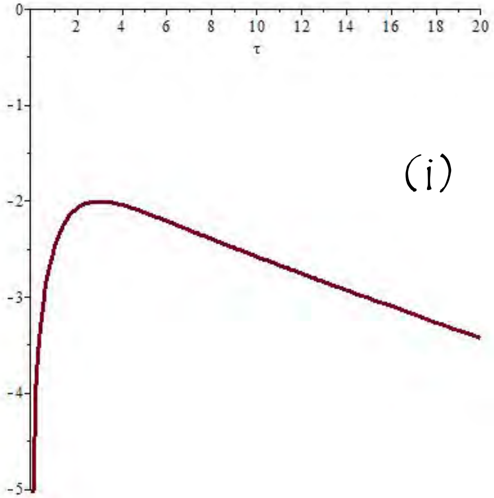


- Critical exponential → CLIMBING
- NOT ENOUGH: need "flat portion" for slow—roll
  [Here we must "guess" (modulo previous slide)]

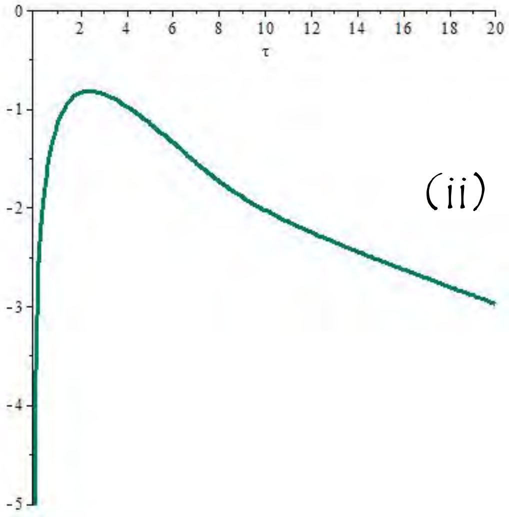
i. Two-exp: $V(\varphi) \;=\; V_0\left(e^{2\varphi} \;+\; e^{2\gamma\varphi}\right)$ $\left[\gamma = \frac{1}{12} \longrightarrow n_s = 0.957\right]$ (PLANCK015 : 0.968 ± 0.06)

ii. More generally : $V(\varphi) \;=\; V_0\left(e^{2\varphi} \;+\; e^{2\gamma\varphi}\right) \;+\; V'(\varphi)$





φ(τ)



Scalar Bounces and the low − ℓ CMB

- Mukhanov-Sasaki equation :

$$\left(\frac{d^2}{d\eta^2} + k^2 - W_s(\eta)\right) v_k(\eta) = 0$$

- Limiting W_s :

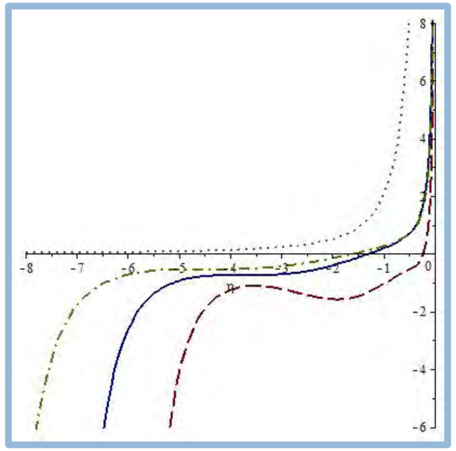
$$W_s \underset{\eta \to -\eta_0}{\sim} -\frac{1}{4}\,\frac{1}{(\eta+\eta_0)^2}\,, \qquad W_s \underset{\eta \to 0^-}{\sim} \frac{\nu^2 - \frac{1}{4}}{\eta^2} \qquad \left(\nu = \frac{3}{2}\,\frac{1-\gamma^2}{1-3\gamma^2}\right)$$

- Power :

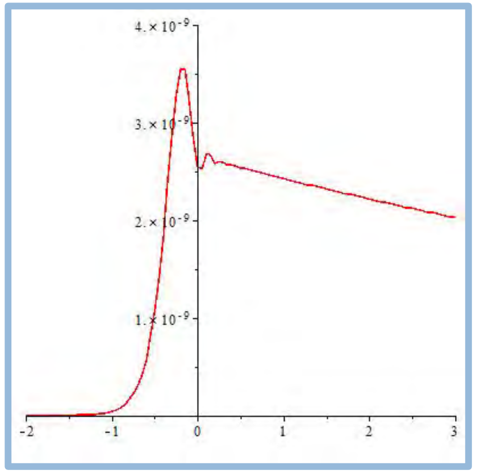
$$P(k) = \frac{k^3}{2\pi^2}\left|\frac{v_k(-\epsilon)}{z(-\epsilon)}\right|^2$$

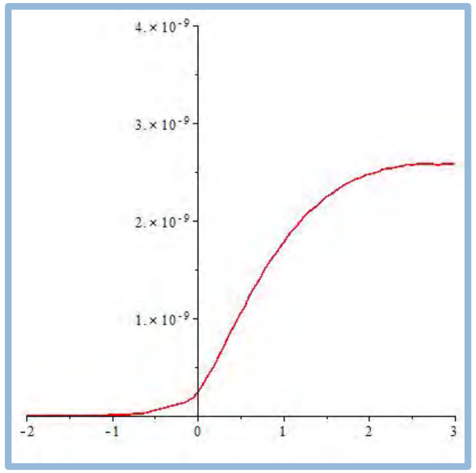
❖ Pre – inflationary fast roll : $P(k) \sim k^3$ vs $P(k) \sim k^{3-2\nu}$
(together with model-dependent fine details near the transition)

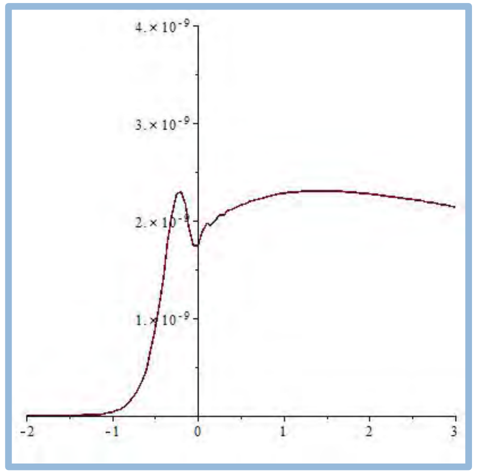
$W_s(\eta)$:

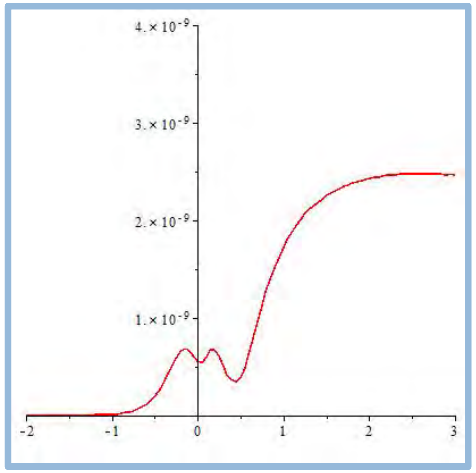


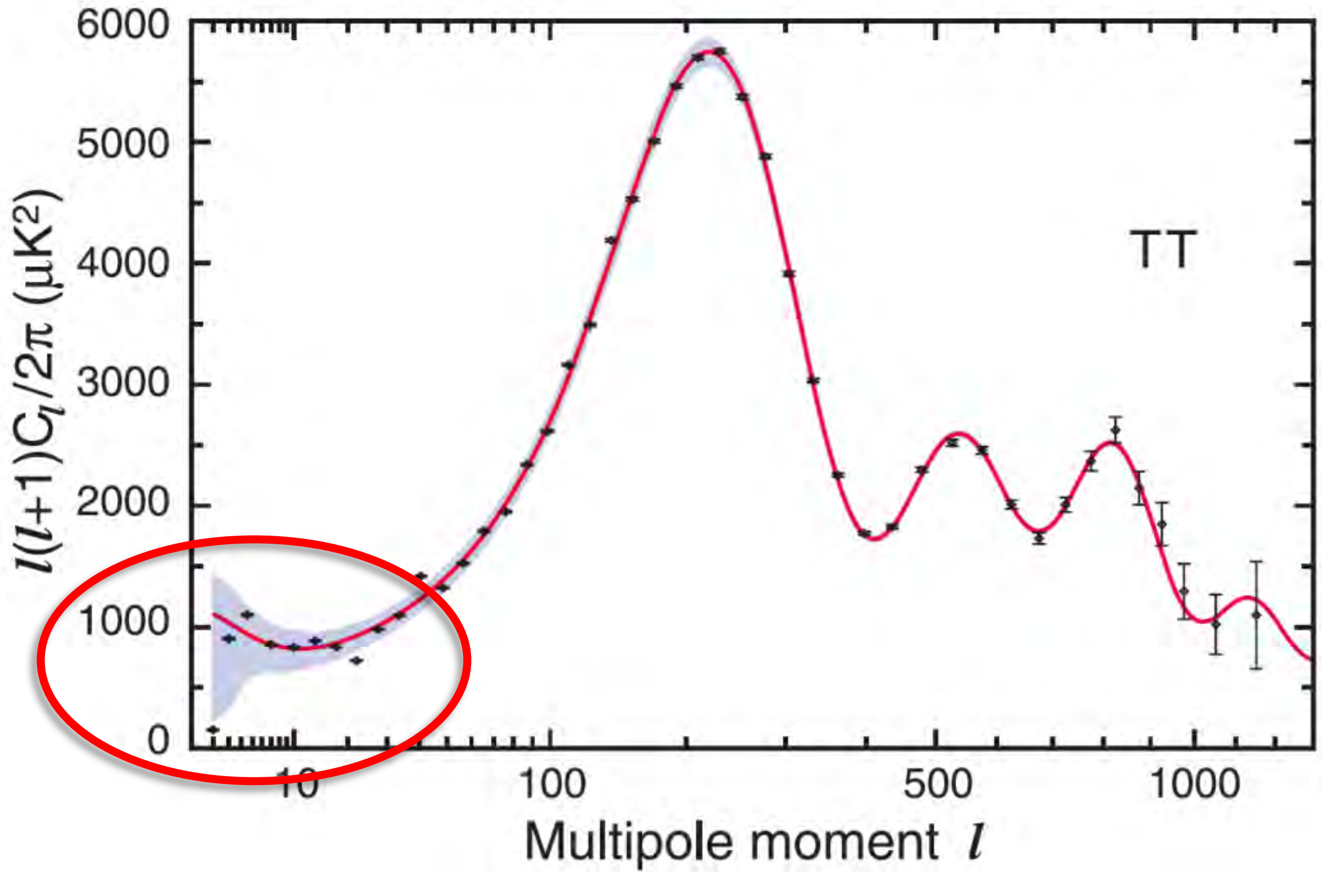
Power spectra with pre-inflationary fast roll (and climbing)

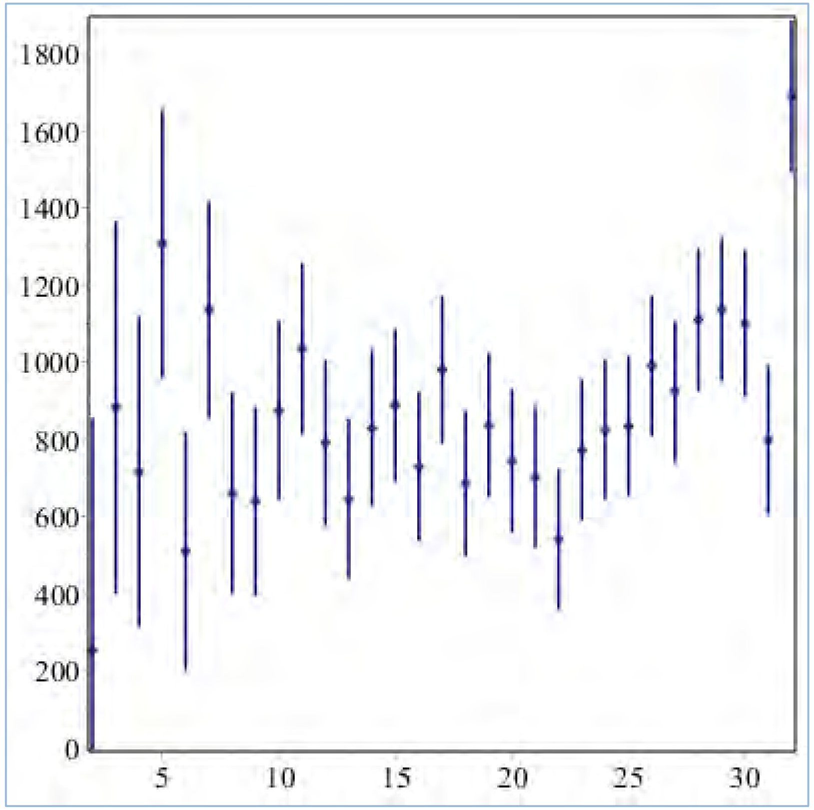












PLANCK 015

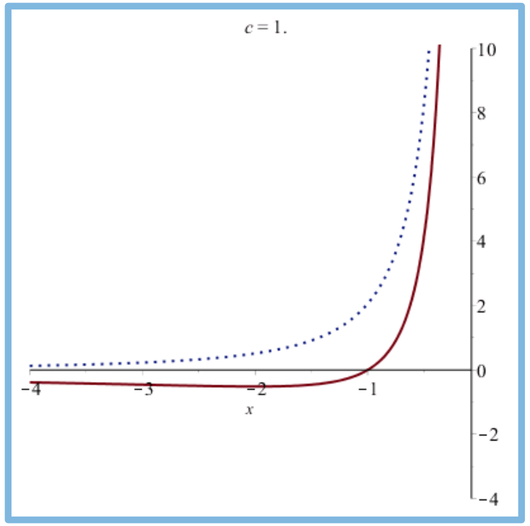
$+$: $A_\ell \sim \ell(\ell+1) \int \frac{dk}{k} P_R(k)\ j_\ell(k\Delta\eta)^2$ ~ $P_R\left(k = \frac{\ell}{\Delta\eta}\right)$

$-$: Cosmic Variance

Are we seeing signs of the onset of inflation ?

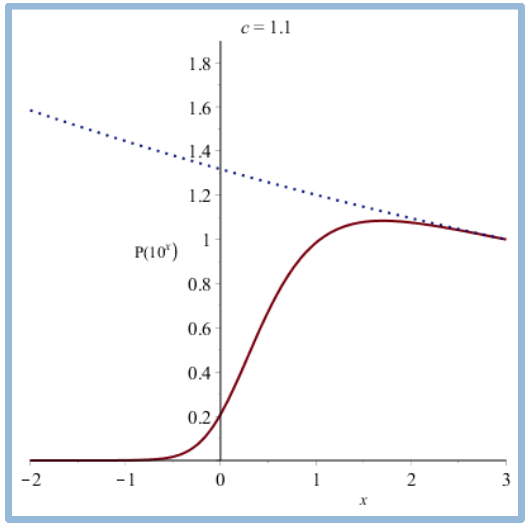
Analytic Mukhanov-Sasaki Power Spectra

(Dudas, Kitazawa, Patil, AS, 2012)



$$\frac{d^2 v_k(\eta)}{d\eta^2} + \left[k^2 + \Delta^2 - W_s(\eta)\right] v_k(\eta) = 0$$

$$W_s' = \frac{\nu^2 - \frac{1}{4}}{\eta^2} - \Delta^2 \longrightarrow P(k) \sim \frac{k^3}{\left[k^2 + \Delta^2\right]^\nu}$$

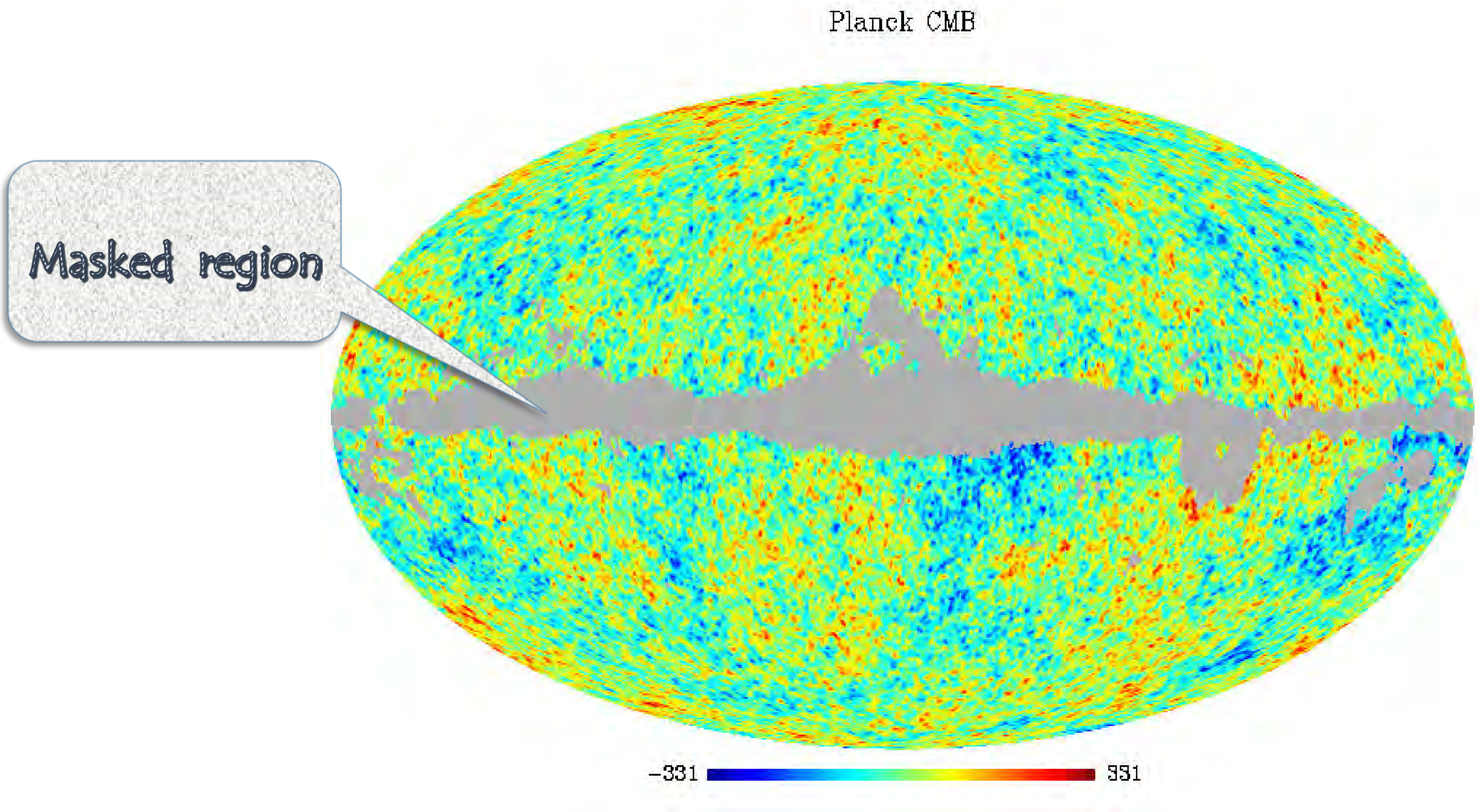


- If W_s crosses the real axis → Power cutoff
- One can also produce a "caricature" pre-inflationary peak

$$W_S = \frac{\nu^2 - \frac{1}{4}}{\eta^2}\left[c\left(1 + \frac{\eta}{\eta_0}\right) + (1-c)\left(1 + \frac{\eta}{\eta_0}\right)\right]^2$$

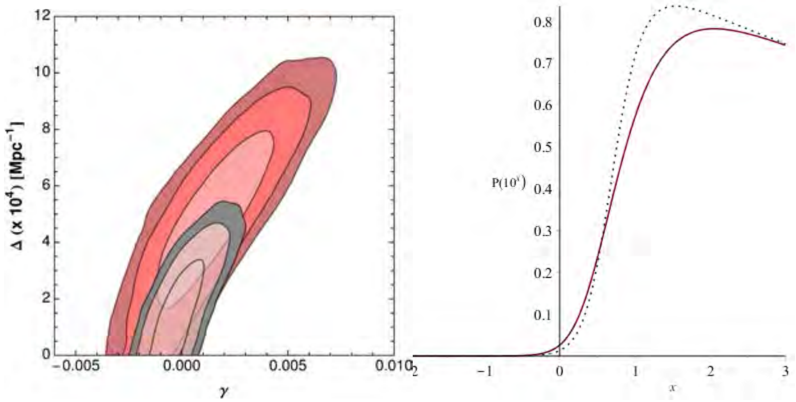
$$P_{\mathcal{R}}(k) \sim \frac{(k\,\eta_0)^3 \exp\left(\frac{\pi\left(\frac{c}{2}-1\right)\left(\nu^2-\frac{1}{4}\right)}{\sqrt{(k\,\eta_0)^2 + (c-1)\left(\nu^2-\frac{1}{4}\right)}}\right)}{\left|\Gamma\left(\nu + \frac{1}{2} + \frac{i\left(\frac{c}{2}-1\right)\left(\nu^2-\frac{1}{4}\right)}{\sqrt{(k\,\eta_0)^2 + (c-1)\left(\nu^2-\frac{1}{4}\right)}}\right)\right|^2 \left[(k\,\eta_0)^2 + (c-1)\left(\nu^2 - \frac{1}{4}\right)\right]^\nu}$$

Pre-Inflationary Relics in the CMB?



Pre-Inflationary Relics in the CMB?

(Gruppuso, Kitazawa, Mandolesi, Natoli, AS, 2015)

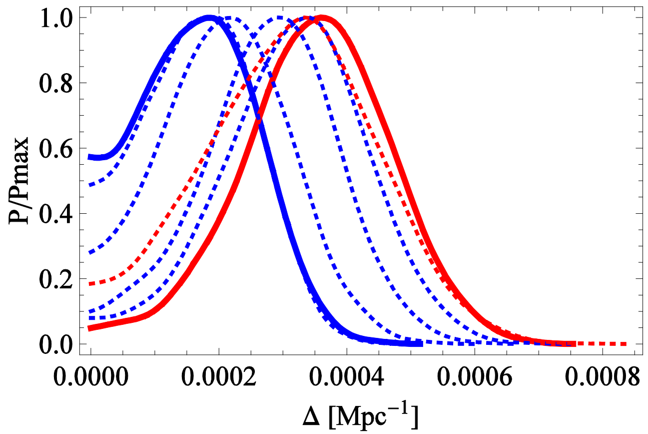


- **Extend ΛCDM** to allow for low-ℓ suppression:

(Kitazawa, AS, 2014)

$$\mathcal{P}(k) \;=\; A\;(k/k_0)^{n_s-1} \;\to\; \frac{A\;(k/k_0)^3}{\left[(k/k_0)^2+(\Delta/k_0)^2\right]^{\nu}}$$

- ❖ NO effects on standard ΛCDM parameters (6+16 nuisance)
- ❖ A new scale Δ. Preferred value? Depends on Galactic masking.



$$\Delta = (0.351 \pm 0.114) \times 10^{-3}\,\mathrm{Mpc}^{-1}$$

RED : + 30−degree extended mask

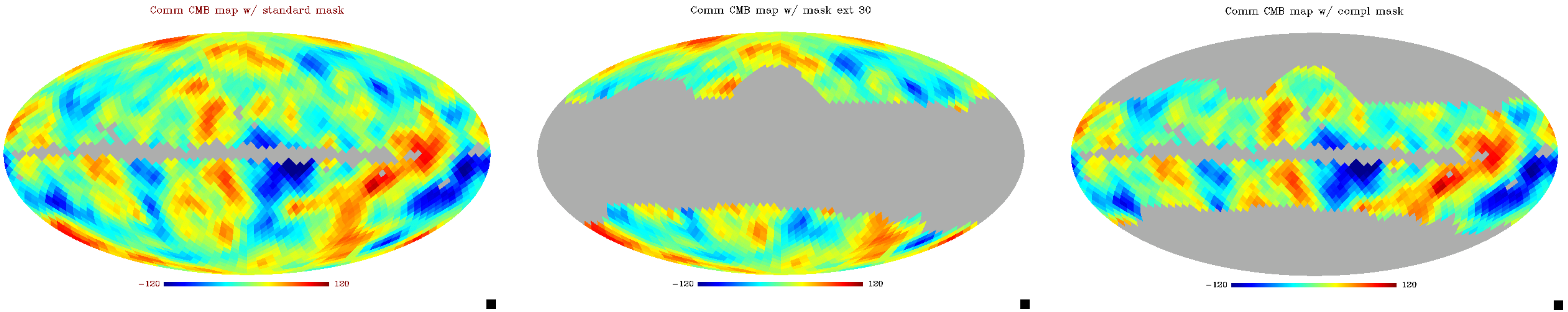
$> 99\%$ confidence level

- What is the corresponding energy scale at onset of inflation?

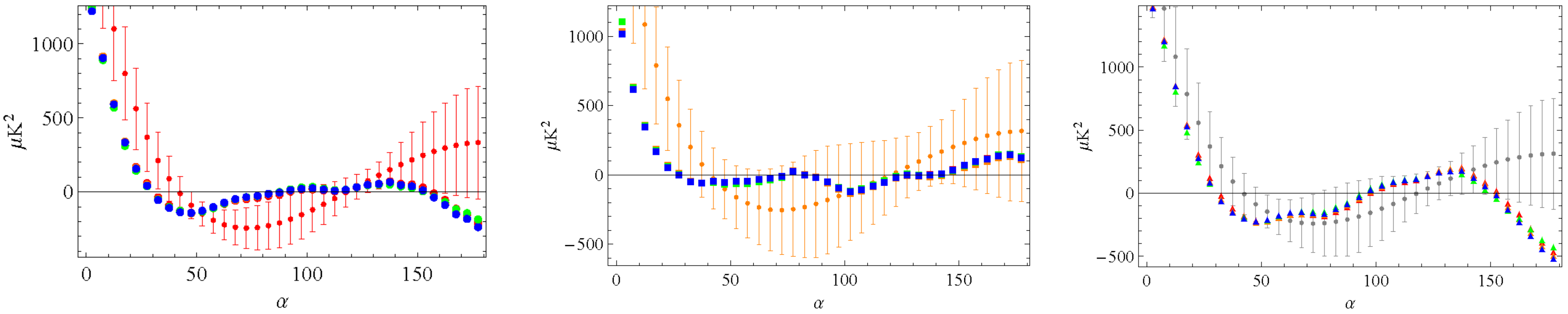
$$\Delta^{Infl} \;\sim\; 2.4\times 10^{12}\; e^{N-60}\;\mathrm{GeV} \;\sim\; 10^{12}-10^{14}\mathrm{GeV}\;\;\mathrm{for}\;\;N\;\sim\;60-65$$

More in Detail

(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, in progress)

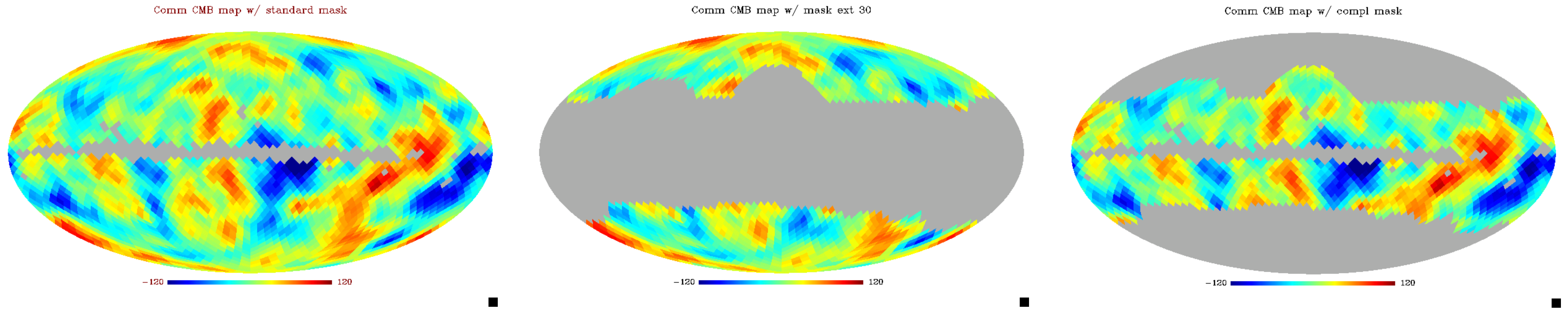


2 - Point : Distinct Behavior in Complementary Regions

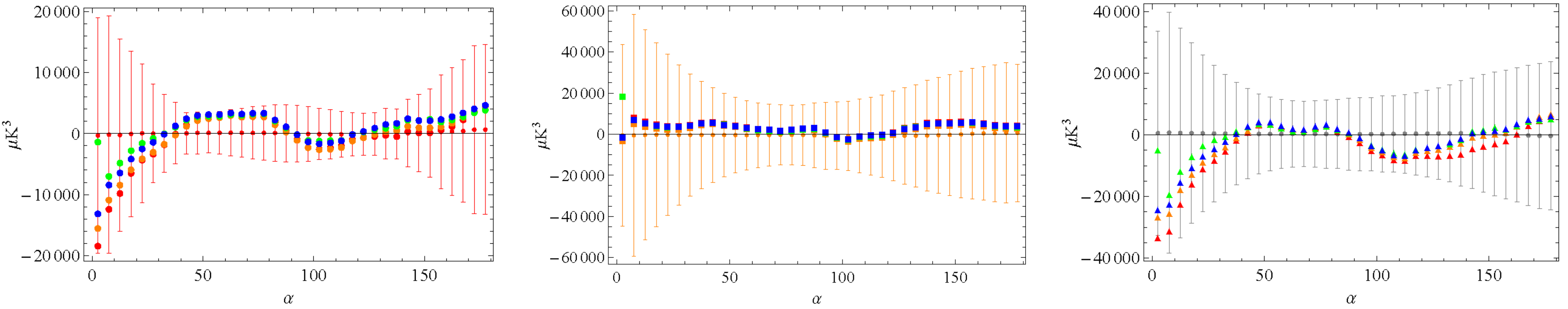


More in Detail

(Gruppuso, Kitazawa, Lattanzi, Mandolesi, Natoli, AS, in progress)



3 – Point (collapsed): Everywhere Gaussian



AdS Vacua from Dilaton Tadpoles and Form Fluxes

Low-Energy Lagrangians

In string frame (T→ Λ for heterotic):

$$\mathcal{S} = \frac{1}{2\,k_{10}^2} \int d^{10}x\,\sqrt{-G}\left\{e^{-2\phi}\left[-R + 4(\partial\phi)^2\right] - \frac{1}{2(p+2)!}\,e^{-2\,\beta_S\,\phi}\,\mathcal{H}_{p+2}^2 - \frac{1}{4}\,e^{-2\,\alpha_S\,\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{\gamma_S\,\phi}\right\}$$

In Einstein frame (T→ Λ for heterotic):

$$\mathcal{S} = \frac{1}{2\,k_{10}^2} \int d^{10}x\sqrt{-g}\left\{-R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2\,(p+2)!}\,e^{-2\,\beta_E^{(p)}\,\phi}\,\mathcal{H}_{p+2}^2 - \frac{1}{4}\,e^{-2\,\alpha_E\,\phi}\,\mathrm{tr}\,\mathcal{F}^2 - T\,e^{\gamma_E\,\phi}\right\}$$

Here:

Orientifolds:

$$\gamma_E = \frac{3}{2}\ (\gamma_S = -1)\ ,\ \alpha_E = -\frac{1}{4}\ (\alpha_S = \frac{1}{2})\ ,\ \beta_E^{(1)} = -\frac{1}{2}\ ,\ \beta_E^{(5)} = \frac{1}{2}\ (\beta_S = 0)$$

Heterotic:

$$\gamma_E = \frac{5}{2}\ (\gamma_S = 0)\ ,\ \alpha_E = \frac{1}{4}\ (\alpha_S = 1)\ ,\ \beta_E^{(1)} = \frac{1}{2}\ ,\ \beta_E^{(5)} = -\frac{1}{2}\ (\beta_S^{(1)} = 1)$$

Brane Profiles

1. The starting point is a class of metrics of the type (whose structure is familiar from the SUSY case):

$$ds^2 = e^{2A(r)}\, g(\mathrm{L_k}) + e^{2B(r)} dr^2 + e^{2C(r)}\, g(\mathrm{E_{k'}})$$

2. There exists a CFT analysis of (charged and uncharged) brane configurations for this type of orientifold systems. *(Dudas, Mourad, AS, 2001)*

How will the CFT analysis, which is set up around the flat space vacuum, and thus ignoring the dilaton tadpole, connect to the actual deformed backgrounds?

Radial Dynamical System

Second-order equations: $\left(F = (p+1)A - B + (D-p-2)C \right)$

$$\mathcal{E}_A \equiv A'' + A'\,F' = -\frac{T}{D-2}\,e^{2B+\gamma\phi} + k\,p\,e^{2(B-A)}$$
$$+\frac{(D-p-3)}{2(D-2)}\,e^{-2\beta_E\phi-2(p+1)A}\,(b')^2 + 4\,\frac{(D-p-2)}{(D-2)}\,e^{2C-2\alpha_E\phi}\,(a')^2 + 8\,\frac{(D-p-2)(D-p-3)}{(D-2)}\,e^{2B-4C-2\alpha_E\phi}\,a^2\,(1-a)^2$$
$$\mathcal{E}_C \equiv C'' + C'\,F' = -\frac{T}{D-2}\,e^{2B+\gamma\phi} + k'(D-p-3)\,e^{2(B-C)}$$
$$-\frac{(p+1)}{2(D-2)}\,e^{-2\beta_E\phi-2(p+1)A}\,(b')^2 - \frac{4p}{D-2}\,e^{-2C-2\alpha_E\phi}\,(a')^2 - 8\,\frac{(D+p-2)(D-p-3)}{(D-2)}\,e^{2B-4C-2\alpha_E\phi}\,a^2\,(1-a)^2$$
$$\mathcal{E}_\phi \equiv \phi'' + \phi'\,F' = \frac{T\,\gamma\,(D-2)}{8}\,e^{2B+\gamma\phi} + \frac{\beta_E\,(D-2)}{8}\,e^{-2\beta_E\phi-2(p+1)A}\,(b')^2$$
$$-\alpha_E\,(D-2)\,(D-p-2)\,(a')^2\,e^{-2C-2\alpha_E\phi} - 2\,\alpha_E\,(D-2)\,(D-p-2)\,(D-p-3)\,a^2\,(1-a)^2\,e^{2B-4C-2\alpha_E\phi}$$
$$\mathcal{E}_a \equiv a'' + a'\,(F' - 2C' - 2\alpha_E\,\phi') - 2\,e^{2(B-C)}\,(D-p-3)\,a\,(1-a)\,(1-2a) = 0$$
$$\mathcal{E}_b \equiv \left(e^{-2\beta_E\phi-(p+1)A-B+(D-p-2)C}\,b'\right)' = 0$$

First-order constraint:

$$(p+1)A'[p\,A' + (D-p-2)C'] + (D-p-2)C'[(D-p-3)C' + (p+1)A']$$
$$-\frac{4\,(\phi')^2}{D-2} + T\,e^{2B+\gamma\phi} - k\,p\,(p+1)\,e^{2(B-A)} - k'(D-p-3)(D-p-2)\,e^{2(B-C)}$$
$$+\frac{1}{2}\,e^{-2\beta_E\phi-2(p+1)A}\,(b')^2 - 4\,e^{-2C-2\alpha_E\phi}\,(D-p-2)\,(a')^2$$
$$+8\,e^{2B-4C-2\alpha_E\phi}\,(D-p-2)(D-p-3)\,a^2\,(1-a)^2 = 0$$

Example: Supersymmetric Branes

Gauge choice:
(k=0,k′=1)

$$C \;=\; B \;+\; \log r$$

$$ds^2 \;=\; e^{2A(r)}\, g(\mathrm{L_k}) \;+\; e^{2B(r)} dr^2 \;+\; e^{2C(r)}\, g(\mathrm{E_{k'}})$$

$A \sim B \sim \phi$:

$$(p+1)A \;+\; (D-p-3)B \;=\; 0$$

$$\frac{\beta_E\,(D-2)^2}{4}\; B \;+ (p+1)\,\phi \;=\; 0$$

If $\beta_E \neq 0$:

$$\lambda \;=\; \beta_E \;+\; \frac{4\,(p+1)(D-p-3)}{\beta_E\,(D-2)^2}$$

$$e^{-\lambda\,\phi} \;=\; e^{-\lambda\,\phi_0}\left[\left(\frac{r_0}{r}\right)^{D-p-3} \;+\; 1\right]$$

$$e^{2\,A} \;=\; e^{\frac{8\,\phi_0\,(D-p-3)}{\beta_E\,(D-2)^2}}\left[\left(\frac{r_0}{r}\right)^{D-p-3} \;+\; 1\right]^{-\,\frac{8(D-p-3)}{\beta_E\,\lambda\,(D-2)^2}}$$

$$e^{2\,B} \;=\; e^{-\,\frac{8\,\phi_0\,(p+1)}{\beta_E\,(D-2)^2}}\left[\left(\frac{r_0}{r}\right)^{D-p-3} \;+\; 1\right]^{\frac{8(p+1)}{\beta_E\,\lambda\,(D-2)^2}}$$

Non-Singular Vacuum Configurations

1. The class of **metrics**: $$ds^2 = e^{2A(r)}\, g(\mathrm{L_k}) + dr^2 + e^{2C(r)}\, g(\mathrm{E_{k'}})$$

2. **Constant dilaton profiles :** aim at "fixing" the dilaton, despite the runaway potentials

3. Vacuum configurations for $\mathcal{H}_{p+2}$: $$\mathcal{H}_{p+2} = h\, e^{(p+1)\,A(r) + 2\,\beta_E^{(p)}\,\phi - (8-p)\,C}\, \epsilon(p+1)\, dr$$

4. Allow also for **non-trivial internal gauge fields**, identifying subgroups of the gauge groups with the tangent space to internal spheres ($\tilde{y}^T\, \tilde{y} = 1$) : the simplest option is for SU(2): *(Wu and Yang, 1969)*

$$\mathcal{A} = i\, a(r)\, \left(\tilde{y}\, d\tilde{y}^T - d\tilde{y}\, \tilde{y}^T\right) \longrightarrow \mathcal{F} = i\, \xi\, d\tilde{y}\, d\tilde{y}^T \qquad \left(\text{or } a = \frac{\xi}{2}\right)$$

Orientifold Vacuum Configurations

In this fashion the field equations reduce to

$$(\star):\ T\,e^{\gamma_E\,\phi} = \frac{\xi\,\alpha_E}{\gamma_E}\,(8-p)\,(7-p)\,e^{-4C-2\alpha_E\phi} - \frac{\beta_E^{(p)}\,h^2}{\gamma_E}\,e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

$$16\,k'\,e^{-2C} = \xi\left[8+p+\frac{2\,\alpha_E}{\gamma_E}\,(8-p)\right]e^{-4C-2\alpha_E\phi} + \frac{h^2\left(p+1-\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\,e^{-2A} + \xi\,\frac{(8-p)(7-p)}{16(p+1)}\left(1-\frac{2\alpha_E}{\gamma_E}\right)e^{-4C-2\alpha_E\phi} + \frac{h^2}{16(p+1)}\left(7-p+\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

(*): dilaton eq: strong constraints due to **positivity of *l.h.s.*** (α_E, $\beta_E < 0$ for orientifolds & $T>0$, **NEED** H_3 fluxes)

First two eqs: determine $k'=1$ (internal sphere), and (implicitly) its radius e^C and ϕ in terms of h and ξ

Third eq: determines for $k=0$ $A \sim r$, and thus AdS in Poincaré coordinates (or in other slicings for $k \neq 0$)

AdS Slicings

Let us take a closer look at the last equation:

$$(A')^2 = k\, e^{-2A} + \frac{1}{R^2} \qquad (k = \pm 1, 0)$$

It is solved by:

$$ds^2 = R^2 \sinh^2\left(\frac{r}{R}\right) \left(ds^2\right)_{k=1} + dr^2$$
$$ds^2 = R^2 \cosh^2\left(\frac{r}{R}\right) \left(ds^2\right)_{k=-1} + dr^2$$
$$ds^2 = R^2\, e^{2\frac{r}{R}} \left(ds^2\right)_{k=0} + dr^2$$

These metrics emerge from **three different slicing of the same AdS space**, for which

$$X_{-1}^2 + X_0^2 - \sum_{i=1}^{p+1} X_i^2 = R^2$$

$$ds^2 = -\,dX_{-1}^2 - dX_0^2 + \sum_{i=1}^{p+1} dX_i^2$$

1. **K = 1 slicing:** $-\eta_{\mu\nu}\, X^\mu\, X^\nu - X_{p+1}^2 = R^2\,, \quad X^\mu = R\, y^\mu \cosh\xi\,, X_{p+1} = R\sinh\xi\,, \quad -\eta_{\mu\nu}\, y^\mu\, y^\nu = 1$
2. **K=-1 slicing:** $X_{-1}^2 - \eta_{\mu\nu}\, X^\mu\, X^\nu = R^2\,, \quad X^\mu = R\, y^\mu \sinh\xi\,, X_{-1} = R\cosh\xi\,, \quad \eta_{\mu\nu}\, y^\mu\, y^\nu = 1$
3. **K=0 slicing:** $X_{-1}^2 - X_{p+1}^2 - \eta_{\mu\nu}\, X^\mu\, X^\nu = R^2\,, \ U, V = X_{-1} \pm X_{p+1}\,, \quad V = \frac{R^2}{U} + \eta_{\mu\nu}\, \frac{U}{R^2}\, y^\mu\, y^\nu$

$AdS_3 \times S_7$ Orientifold Vacua $\left(T = \frac{16}{\pi^2}\,(BSB), \frac{8}{\pi^2}\,(0'B) \right)$

Concentrate on the **first two equations** [Here $\alpha_E < 0$, $\beta_E < 0$, and therefore we **NEED** a 3-form flux, with p=1]:

$$(\star):\ T\, e^{\gamma_E \phi} = \frac{\xi\, \alpha_E}{\gamma_E}\,(8-p)\,(7-p)\, e^{-4C - 2\alpha_E \phi} - \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \xi \left[8 + p + \frac{2\,\alpha_E}{\gamma_E}\,(8-p) \right] e^{-4C - 2\alpha_E \phi} + \frac{h^2 \left(p + 1 - \frac{2\,\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

Combining them leads to two branches of solutions :

$$e^{2C} = \frac{2\,\xi\, e^{\frac{\phi}{2}}}{1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}}$$

$$\frac{h^2}{32} = \frac{\xi^7\, e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}\right)^7} \left[\frac{42}{\xi} \left(1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}} \right) + 5\, T\, e^{2\phi} \right]$$

The **lower sign** yields an asymptotically weak-coupling solution (small coupling for large sizes)

$$g_s \equiv e^{\phi} \sim \frac{12}{(2\,h\,T^3)^{\frac{1}{4}}}\,, \qquad R^4\, g_s^3 \sim \frac{144}{T^2}\,, \qquad (A')^2 \sim k\, e^{-2A} + \frac{6}{R^2}$$

$AdS_3 \times S_7$ Orientifold Vacua $\left(T = \frac{16}{\pi^2}\,(BSB), \frac{8}{\pi^2}\,(0'B) \right)$

$$
\begin{aligned}
e^{2C} &= \frac{2\,\xi\, e^{\frac{\phi}{2}}}{1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}} \\
\frac{h^2}{32} &= \frac{\xi^7\, e^{4\phi}}{\left(1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}}\right)^7} \left[\frac{42}{\xi} \left(1 \pm \sqrt{1 - \frac{\xi T}{3}\, e^{2\phi}} \right) + 5\, T\, e^{2\phi} \right]
\end{aligned}
$$

- The solution corresponding to the **lower sign** exists for ξ=0 (with residual unbroken gauge group Usp(32) or U(32)), and for ξ=1 (with residual unbroken gauge group Usp(24) or U(24))

- The solution corresponding to the **upper sign** only exists for ξ=1, associates **large radii** to **strong couplings**, and **survives in the limit of vanishing T**, so that it is a solution also for the type-I superstring. It lies outside the perturbative reach, but it has an interesting heterotic counterpart, as we shall see shortly.

Heterotic Vacuum Configurations

In this fashion the field equations reduce to

$$(\star):\ \Lambda\, e^{\gamma_E \phi} = \frac{\xi\,\alpha_E}{\gamma_E}\,(8-p)\,(7-p)\,e^{-4C-2\alpha_E\phi} - \frac{\beta_E^{(p)}\,h^2}{\gamma_E}\,e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

$$16\,k'\,e^{-2C} = \xi\left[8+p+\frac{2\,\alpha_E}{\gamma_E}\,(8-p)\right]e^{-4C-2\alpha_E\phi} + \frac{h^2\left(p+1-\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\,e^{-2A} + \xi\,\frac{(8-p)(7-p)}{16(p+1)}\left(1-\frac{2\alpha_E}{\gamma_E}\right)e^{-4C-2\alpha_E\phi} + \frac{h^2}{16(p+1)}\left(7-p+\frac{2\,\beta_E^{(p)}}{\gamma_E}\right)e^{-2(8-p)C+2\beta_E^{(p)}\phi}$$

(*): dilaton eq: now $\alpha_E > 0$ for heterotic & $\Lambda > 0$, but we can allow, surprisingly, unbounded H_3 or H_7 fluxes ($\beta_E >$ &< 0). Actually, one can also eliminate these fluxes altogether, and then the solutions, supported by the gauge fields alone, exist for all p. Only for p=3,7, however, can they have unbounded radii and small string coupling.

First two eqs: determine again k′=1 (internal sphere), and (implicitly) its radius e^C and ϕ in terms of h and ξ

Third eq: determines again for k=0 A ~r, and thus AdS in Poincaré coordinates (or in other slicings for $k \neq 0$)

$AdS_3 \times S_7$ Heterotic Vacua $\left(\Lambda \approx \frac{4\pi^2}{25}\right)$

Let us concentrate on the first two equations (here $\alpha_E > 0$, but we want to allow for large h fluxes, with p=1):

$$(\star):\ T\, e^{\gamma_E \phi} = \frac{\xi\, \alpha_E}{\gamma_E}\,(8-p)\,(7-p)\, e^{-4C - 2\alpha_E \phi} - \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \xi\left[8 + p + \frac{2\alpha_E}{\gamma_E}(8-p)\right] e^{-4C - 2\alpha_E\phi} + \frac{h^2\left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

Combining them leads to one branch of solutions :

$$e^{2C} = \frac{2\,\xi\, e^{-\frac{\phi}{2}}}{1 + \sqrt{1 + \frac{\xi\Lambda}{3}\, e^{2\phi}}}$$

$$\frac{h^2}{32} = \frac{e^{-4\phi}}{\left[1 + \sqrt{1 + \frac{\Lambda}{3}\, e^{2\phi}}\right]^7}\left[42\left(1 + \sqrt{1 + \frac{\Lambda}{3}\, e^{2\phi}}\right) - 13\,\Lambda\, e^{2\phi}\right]$$

This solution affords again a weak-coupling limit, and asymptotically

$$g_s \equiv e^{\phi} \sim \left(\frac{21}{h^2}\right)^{\frac{1}{4}}, \qquad g_s\, R^4 \sim 1\,, \qquad (A')^2 \sim k\, e^{-2A} + \frac{21}{4\,R^2}$$

BUT: it continues to exist even if $\Lambda = 0$ (strong-weak coupling dual of second orientifold one in the limit)

$AdS_7 \times S_3$ Heterotic Vacua ($\Lambda \approx \frac{4\pi^2}{25}$)

Let us concentrate on the first two equations (here $\alpha_E > 0$, but we want to allow for large h fluxes, with p=5):

$$(\star): \; T\, e^{\gamma_E \phi} = \frac{\xi\, \alpha_E}{\gamma_E}\,(8-p)\,(7-p)\, e^{-4C - 2\alpha_E \phi} - \frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \xi\left[8 + p + \frac{2\,\alpha_E}{\gamma_E}\,(8-p)\right] e^{-4C - 2\alpha_E\phi} + \frac{h^2\left(p + 1 - \frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

Combining them leads to one branch of solutions :

$$e^{2C} = \frac{\xi}{2}\,\frac{e^{-\frac{\phi}{2}}}{1 - \sqrt{1 - \xi\,\Lambda\, e^{2\phi}}}\,,$$

$$\frac{h^2}{3} = \xi^3\,\frac{\left[\frac{17\,\Lambda}{24}\, e^{2\phi} - \frac{1}{\xi}\left(1 - \sqrt{1 - \xi\,\Lambda\, e^{2\phi}}\right)\right]}{\left(1 - \sqrt{1 - \xi\,\Lambda\, e^{2\phi}}\right)^3}\,;$$

This solution also affords again a weak-coupling limit, and asymptotically

$$g_s \equiv e^{\phi} \sim \left(\frac{5}{h^2\,\Lambda^2}\right)^{\frac{1}{4}}, \qquad g_s^5\, R^4 \sim \frac{1}{\Lambda^2}\,, \qquad (A')^2 \sim k\, e^{-2A} + \frac{1}{4\,R^2}$$

BUT: this branch exists only if $\Lambda \neq 0$

Thank You

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