Horizon fluff

A semi-classical approach to (BTZ) black hole microstates

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‘Near Horizon’ collaboration

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Horizon fluff proposal

- Black hole microstates = horizon fluff: subset of near horizon soft hairs not distinguishable by the observers away from the horizon.
- Black hole: a state in the Hilbert space of asymptotic symmetries:

$$[L_n,\, L_m] = (n-m)L_{m+n} + \frac{k}{2}\, n^3\, \delta_{n+m,0}\,.$$

- Soft hairs: states in the Hilbert space of 'near horizon' algebra:

$$[J_n,\, J_m] = \frac{k}{2}\, n\, \delta_{n+m,0}\,.$$

- Duality map between asymptotic Hilbert space (BTZ) to the near horizon Hilbert space provides the required degeneracy (entropy).

Outline

1. Motivation
2. Brown-Henneaux boundary conditions
3. Near horizon boundary conditions
4. Quantization of conical defects
5. Black hole microstates
6. Logarithmic correction to entropy
7. Conclusion

Motivation

Motivation

The **Bekenstein-Hawking** area law for black hole entropy is observer independent and is accessible through semiclassical considerations.

```
S = A/(4G) - q ln A/(4G) +  O(1)

  = ln ( # microstates )
```

Universality of this result suggests that a statistical description of microstates in the thermodynamic limit does not need full knowledge of the underlying quantum theory.

Soft hair

- Diffeomorphic geometries which differ by their boundary behavior can be physically distinct, **asymptotic soft hairs**. The conserved charges associated with the diffeomorphisms relating them is non-zero and form an infinite dimensional algebra.
- A black hole spacetime in particular can carry low-energy quantum excitations, **near horizon soft hairs**, providing a huge degeneracy to their vacuum.

`Could they amount for microstates of black holes?`

`How to identify microstates among them?`

Brown-Henneaux boundary conditions

Bañados geometries

All Locally AdS_3 geometries obeying Brown–Henneaux b.c.:

$$ds^2 = \ell^2 \frac{dr^2}{r^2} - r^2\Big(dx^+ - \frac{\ell^2 L_-(x^-)}{r^2}\, dx^-\Big)\Big(dx^- - \frac{\ell^2 L_+(x^+)}{r^2}\, dx^+\Big)$$

$$L_\pm(x^\pm + 2\pi) = L_\pm(x^\pm)\,, \qquad x^\pm = t/\ell \pm \phi, \qquad \phi \in [0, 2\pi].$$

Constant family:

$$L_+ = L_- = L_0$$

`BTZ black holes:` $L_0 \geq 0$

`Global AdS₃:` $L_0 = -1/4$

`Conical defects:` $-1/4 < L_0 < 0$

Brown-Henneaux symmetries

Bañados geometries fall into representation of asymptotic (simplectic) symmetry algebra which is two copies of Virasoro at Brown-Henneaux central charge $c = 6k$. The infinitesimal action of Virasoro symmetries on functions $L_\pm$ is;

$$\delta_{\epsilon_\pm} L_\pm = 2L_\pm \epsilon'_\pm + \epsilon'_\pm L_\pm - \epsilon'''_\pm / 2 \,.$$

The corresponding geometries with $(L_\pm)$ fall into a one-to-one relation with the coadjoint orbits of these symmetries. They are BTZ black holes, conic spaces and global AdS_3 and their conformal descendants, explicitly:

$$\mathcal{H}_{\mathrm{Vir}} = \mathcal{H}_{\mathrm{BH}} \cup \underbrace{\mathcal{H}_{\mathrm{Conic}} \cup \mathcal{H}_{\mathrm{gAdS}}}_{\mathcal{H}_{\mathrm{CG}}} \,.$$

Near horizon boundary conditions

Near horizon boundary conditions

All locally AdS_3 geometries with **horizon** at $r = 0$ are parametrized by 4 **real** functions

$$\mathrm{d}s^2 \;=\; \mathrm{d}r^2 - \ell^2 \sinh^2 \frac{r}{\ell} \left[a\, dt - \omega d\varphi\right]^2 + \cosh^2 \frac{r}{\ell} \left[\Omega\, dt + \gamma d\varphi\right]^2$$

$$\partial_t J^\pm = \pm \partial_\varphi \zeta^\pm ; \qquad 2\zeta^\pm \equiv -a \pm \tfrac{\Omega}{\ell} \quad \text{and} \quad 2J^\pm \equiv \tfrac{\gamma}{\ell} \pm \omega \,.$$

Constant family:

$$J_+ = J_- = \pm J_0 \,, \quad L_0 = {J_0}^2$$

`BTZ black holes:` $J_0 \geq 0$

`Global AdS`$_3$`:` $J_0 = \frac{i}{2}$

`Conical defects:` $J_0 = \frac{i\nu}{2} \,, \quad \nu \in (0,1)$

Near horizon boundary conditions

$$\begin{aligned} ds^2 &= dr^2 - \ell^2 \sinh^2 \frac{r}{\ell} \left[a\, dt - \omega d\varphi\right]^2 + \cosh^2 \frac{r}{\ell} \left[\Omega\, dt + \gamma d\varphi\right]^2 \\ &= dr^2 - (ar)^2\, dt^2 + \gamma^2\, d\varphi^2 + \mathcal{O}(r^2). \qquad \varphi \sim \varphi + 2\pi \end{aligned}$$

- Rindler space: Universal near horizon to any non-extremal horizon.

Near horizon boundary conditions

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&= \mathrm{d}r^2 - (ar)^2\,\mathrm{d}t^2 + \gamma^2\,\mathrm{d}\varphi^2 + \mathcal{O}(r^2). \qquad \varphi \sim \varphi + 2\pi
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$$

- Rindler space: Universal near horizon to any non-extremal horizon.
- In the canonical description the Rindler acceleration a is fixed.
- AdS radius ℓ drops out of the near horizon line-element.
- All solutions have a regular horizon, regardless of the value of γ, ω as long as $a/(2\pi)$ is identified with the Unruh temperature.

Symmetries of the Near-Horizon

The most general transformation that preserves this boundary condition and also preserves the the field equation, with $\delta\zeta = 0$, transforms J's as;

$$\delta_\eta J = \eta' \,.$$

$u(1)_k$-algebra

$$[J_n,\, J_m] = \frac{k}{2}\, n\, \delta_{n+m,0}\,.$$

`How are these two geometries related?` Check how J is transformed under conformal transformaition acting on the metric:

$$\delta_\epsilon J = (\epsilon J)' - \epsilon''/2 \quad \to \quad L = J^2 + J' \,.$$

$$\boldsymbol{L}_n \equiv \frac{6}{c} \sum_{p\in\mathbb{Z}} \boldsymbol{J}_{n-p} \boldsymbol{J}_p + in\boldsymbol{J}_n$$

Quantization of conical defects

Wilson lines as primary fields

- The fields $J(\phi)$ are primary fields if the anomalous term ϵ'' can be ignored that is for **black hole** sector ($J_0^\star = J_0$) with J being **large**;

$$\boxed{L = J' + J^2 \quad \to \quad \delta_\epsilon J = (\epsilon J)' - \epsilon''/2}$$

- We can construct the primary field $\mathcal{W}$;

$$\boxed{\mathcal{W}(\phi) = e^{-2\int^\phi J} \quad \to \quad \delta_\epsilon \mathcal{W} = (\epsilon \mathcal{W})'}$$

with the **periodicity** property:

$$\mathcal{W}^\pm(\phi + 2\pi) = e^{\mp 4\pi J_0} \mathcal{W}^\pm(\phi)$$

which is a good description for **conic spaces** ($J_0^\star = -J_0$).

Quantization of $\mathcal{W}$ fields

From $\delta_\eta J = \eta'/2$ we get;

$$\delta_{\eta_\pm}\mathcal{W}^\pm = -\eta_\pm \mathcal{W}^\pm\,, \qquad \eta_\pm = \int^\phi \eta' \pm i\eta_0$$

and using the appropriate mode expansion

$$\mathcal{W}(\varphi) = \sum_{n\in\mathbb{Z}} \mathcal{W}^\nu_n e^{i(n\pm\nu)\varphi}, \qquad \mathcal{W}^\nu_n = \tfrac{6}{c}\langle \boldsymbol{\mathcal{W}}^\nu_n\rangle\,,$$

we get;

$$[\boldsymbol{J}_n, \boldsymbol{\mathcal{W}}^\pm_m] = -\,i\boldsymbol{\mathcal{W}}^\pm_{n+m}\,, (\forall n \neq 0), \qquad [\boldsymbol{J}_0, \boldsymbol{\mathcal{W}}^\pm_n] = \mp\, i\tfrac{c}{6}\boldsymbol{\mathcal{W}}^\pm_n \delta_{n,0}\,.$$

The $\boldsymbol{\mathcal{W}}$ operators are like ladder operators of $\boldsymbol{J}_0$.

Free field realization of $\mathcal{W}$-fields

The "interaction terms" $\langle \mathcal{W}\mathcal{W}J \rangle$ and $\langle \mathcal{W}JJ \rangle$ are suppressed by factors of $1/c$. In the large c regime, the algebra can be closed as:

$$[\boldsymbol{\mathcal{W}}_n^{\pm\nu}, \boldsymbol{\mathcal{W}}_m^{\mp\nu'}] = \frac{c}{12}(n \pm \nu)\ \delta_{n,-m}\delta(\nu - \nu')\ , \qquad \nu, \nu' \in (0,1),$$

This gives a free field realization for $\mathcal{W}$-fields.

Near horizon soft hairs

Bohr-type quantization condition

$$\nu = \frac{r}{c}, \qquad r = 1, \cdots, c\,, \qquad c \in \mathbb{Z}$$

- To construct the Hilbert space $\mathcal{H}_{\mathrm{CG}}$, we use the Fourier modes $\mathcal{J}_n$:

$$\mathcal{J}_{cn+r} \equiv \sqrt{6}\mathcal{W}^r_n, \qquad r = \nu c = 1, 2, \cdots, c\,.$$

- The commutation relation of $\mathcal{W}_n$'s takes a very simple form

$$[\mathcal{J}_n, \mathcal{J}_m] = \frac{n}{2}\delta_{n,-m}\,,$$

- We define the vacuum state as global AdS_3 by

$$\mathcal{J}_n|0\rangle = 0, \quad \forall n \geq 0.$$

Virasoro algebra of near horizon

In parallel to the black hole sector we can have a Virasoro generator;

$$\boldsymbol{L}_n^r = \tfrac{6}{c} \sum_{p \in \mathbb{Z}} :\mathcal{W}_{n-p}^{-r} \mathcal{W}_p^r: + \tfrac{1}{2} \left(\tfrac{1}{24} - \tfrac{1}{8} (\tfrac{2r}{c} - 1)^2 \right) \delta_{n,0}$$

The generators $L_n = \sum_{r=1}^c L_n^r$, can be written in terms of $\mathcal{J}_n$ modes;

$$\boldsymbol{L}_n = \frac{1}{c} \sum_{p \in \mathbb{Z}} :\mathcal{J}_{nc-p} \mathcal{J}_p: - \frac{1}{24c} \delta_{n,0}$$

They satisfy a Virasoro algebra at Brown-Henneaux central charge c.

Black hole microstates

State of a BTZ

Duality ($\mathcal{L}_{nc} = c\boldsymbol{L}_n$)

$$\frac{1}{c}\sum_{p\in\mathbb{Z}}\mathcal{J}_{nc-p}\mathcal{J}_p = in\boldsymbol{J}_n + \frac{6}{c}\sum_{p\in\mathbb{Z}}\boldsymbol{J}_{n-p}\boldsymbol{J}_p.$$

The two Hilbert spaces in $\mathcal{H}_{\rm Vir} = \mathcal{H}_{\rm BH} \cup \mathcal{H}_{\rm CG}$ are related.

$$\mathcal{H}_{\rm CG} \longleftrightarrow \mathcal{H}_{\rm BH}$$

A given AdS_3 black hole state:

$$|\ {\rm BTZ} > \in \mathcal{H}_{\rm BH} \longleftrightarrow \texttt{micro-states} = |\Psi > \in \mathcal{H}_{\rm CG}$$

$$\langle \boldsymbol{L}_n \rangle_{\rm BTZ} = \tfrac{1}{2}(\ell M \pm J)\delta_{n,0}\,.$$

Horizon fluffs = Microstates $\in \mathcal{H}_{\text{CG}}$

$$\boxed{[\mathcal{J}_m, \mathcal{J}_n] = \frac{m}{2}\delta_{m+n,0}}$$

$$\boxed{|\Psi >= \mathcal{J}_{-m_i} \cdots \mathcal{J}_{-m_2}\mathcal{J}_{-m_1}|0\rangle\,, \quad \forall m_i > 0}$$

So $|\Psi\rangle$ describes a blackhole state if;

$$\boxed{\tfrac{1}{2}(\ell M \pm J) = \langle \boldsymbol{L}_0^{\pm}\rangle = \frac{1}{c}\langle \boldsymbol{\mathcal{L}}_0^{\pm}\rangle = \frac{1}{c}\sum_i n_i}$$

Mathematically, this reduces to Hardy and Ramanujan combinatorial problem: the number p(N) of ways a positive integer N can be partitioned into non-negative integers in the limit of large N;

$$p(N) \simeq \frac{1}{4N\sqrt{3}} exp\left(2\pi\sqrt{\frac{N}{6}}\right)\,, \qquad N \gg 1\,.$$

Entropy

Microcanonical entropy as logarithm of the number of states:

$$S_0 = \ln p(N^+) + \ln p(N^-) = 2\pi \left(\sqrt{\frac{cL_0^+}{6}} + \sqrt{\frac{cL_0^-}{6}} \right)$$

where $cL_0^{\pm} = c(\ell M \pm J) = \sum_i n_i = N^{\pm}$.

Reminding ${L_0}^{\pm} = ({J_0}^{\pm})^2$, the entropy is;

$$S_0 = \frac{\pi c}{3} \left(J_0^+ + J_0^- \right)$$

Logarithmic correction to entropy

Micro-canonical entropy

$$\boxed{S_0 = \frac{\pi c}{3}\left(J_0^+ + J_0^-\right)}$$

The logarithmic correction:

$$S = S_0 - 2\,\ln S_0 + \ldots,$$

We should map the J fields onto the microcanonical fields in terms of h,

$$J(\phi) = J_0 h'(\phi) - \frac{1}{2}\frac{h''(\phi)}{h'(\phi)}\,.$$

The microcanonical entropy is obtained through replacing J_0 with

$$J_0 = \frac{k}{2\pi}\left\langle \int_0^{2\pi} \mathrm{d}\phi\, J(\phi)\right\rangle_{\text{BTZ}}.$$

Micro-canonical entropy

Using the replica trick:

$$\left\langle \ln(h'(\phi))\big|_0^{2\pi} \right\rangle_{\rm BTZ} = -\ln J_0 + \ J_0 \text{ -independent terms.}$$

Consequently we find the exact match for the log correction in the mic-canonical ensemble for BTZ black holes;

$$S_{\rm mic} == S_{\rm BH} - \frac{3}{2}\ \ln S + \ldots .$$

Conclusion

Summary

The horizon fluff proposal is a semi-classical proposal for constructing all BTZ microstates. Remarkably, within this semiclassical setting we not only derived the Bekenstein-Hawking entropy but also the logarithmic corrections. We just used semi-classical results about symmetries and some basic “Bohr-type” quantization on central charge and the dificit angle and the fact that the near horizon and asymptotic energies differ by a factor of $1/c$ and do not need any detailed UV completed quantum gravity description.

Questions?