Real time dynamics and phase separation in a holographic 1^{st} order phase transition

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Physics, IPM

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Janik, Jankowski, HS, 1704.05387
Bajnok, Janik, Jankowski, HS (in progress)

Simple exercise (homework):

$$rS((\partial_r G)^2 + G^2(\partial_r \Phi)^2) + 4G^2(2\partial_r S + r\partial_r^2 S) = 0,$$

$$G^2 S(2r^4 \partial_r A \partial_r S + S(r^4 \partial_r^2 A + 2r^3 \partial_r A - r^2 \partial_t \Phi \partial_r \Phi + V(\Phi)) - 4r^2 \partial_r \partial_t S)$$
$$-r^2(\partial_r G(r^2 B \partial_r B + \partial_t G S^2) - \partial_r B \partial_x G) + r^2(r^2 \partial_r B^2 - \partial_r \partial_x B + rB(2\partial_r B + r\partial_r^2 B))G = 0,$$

$$-GS(S(r^2 \partial_r B \partial_r G - \partial_r \partial_x G) + rB(2r\partial_r G \partial_r S + (2\partial_r G + r\partial_r^2 G)S) - 2\partial_r G \partial_x S)$$
$$+G^2(S^2(r^2 \partial_r^2 B + 2r\partial_r B - \partial_x \Phi \partial_r \Phi) + 2\partial_r S(\partial_x S - r^2 B \partial_r S) - 2S(rB(2\partial_r S + r\partial_r^2 S) + \partial_r \partial_x S))$$
$$+\partial_r G S^2(r^2 B \partial_r G - 2\partial_x G) = 0,$$

$$\partial_x A(r^2(\partial_r BG + B\partial_r G) - \partial_x G) - r^2 \partial_r A \partial_x BG + AG(GS(2r^4 \partial_r A \partial_r S + S(r^4 \partial_r^2 A + 2r^3 \partial_r A + V(\Phi)))$$
$$+r^4 \partial_r B^2) - r^4 \partial_r A B^2 \partial_r G + r^2 \partial_r A B \partial_x G + r^4 \partial_r^2 A B^2 G + 2r^3 \partial_r A B^2 G - 2r^2 \partial_r \partial_x ABG$$
$$-2r^2 \partial_r A G^2 S \partial_t S + 2r^2 \partial_t A G^2 S \partial_r S + \partial_t^2 AG - 2r^2 B \partial_t B \partial_r G + 2\partial_t B \partial_x G + 2r^2 B \partial_r \partial_t BG - 2\partial_t \partial_x BG$$
$$-\partial_t G^2 S^2 - 4G^2 S S^{(0,2,0)}(r,t,x) - G^2 S^2 \partial_t \Phi^2 = 0,$$

$$S^2(G(r^2 \partial_r G(-(\partial_x A + r^2 A \partial_r B - \partial_t B)) + G(r^2(\partial_r \partial_x A + rA(2\partial_r B + r\partial_r^2 B) - \partial_r \partial_t B) - \partial_x \Phi \partial_t \Phi)$$
$$+\partial_t \partial_x G - 2\partial_x G \partial_t G) + B(r^2 G(S^2(r^2 \partial_r A \partial_r G - \partial_r \partial_t G) + r^2 \partial_r B^2 - \partial_r \partial_x B - 2\partial_t G \partial_r SS)$$
$$+G^2(2S(r^4 \partial_r A \partial_r S - r^2 \partial_r \partial_t S) - 2r^2 \partial_t S \partial_r S + S^2 V(\Phi)) + r^2(\partial_r B \partial_x G + \partial_t G \partial_r G S^2)) + 2G^2 \partial_x S \partial_t S$$
$$+r^3 B^2((2\partial_r B + r\partial_r^2 B)G - r\partial_r B \partial_r G) - 2GS(G(r^2(\partial_r B \partial_t S - \partial_t B \partial_r S) + \partial_t \partial_x S) - \partial_t G \partial_x S) = 0,$$

$$G^2(-r^2 S^4(r^2 \partial_r A \partial_r G + rA(2\partial_r G + r\partial_r^2 G) - 2\partial_r \partial_t G) + 2r^2 S^3(\partial_r S(\partial_t G - r^2 A \partial_r G) + \partial_r G \partial_t S)$$
$$-2r^2 \partial_r BS \partial_x S - 2S(r^2(rB^2(2\partial_r S + r\partial_r^2 S) - \partial_x B \partial_r S) + \partial_x^2 S) - S^2(r^4 \partial_r B^2 - 2r^2 \partial_r \partial_x B$$
$$+\partial_x \Phi^2) - 2r^4 B^2 \partial_r S^2 + 2\partial_x S^2) + G^3(-S^2)(2r^2 S(r^2 \partial_r A \partial_r S + rA(2\partial_r S + r\partial_r^2 S) - 2\partial_r \partial_t S)$$
$$+2r^2 \partial_r S(r^2 A \partial_r S - 2\partial_t S) + S^2 V(\Phi)) + GS(r^2 \partial_r G S^3(r^2 A \partial_r G - 2\partial_t G) + \partial_x G(2\partial_x S - r^2(\partial_r BS$$
$$+2B\partial_r S)) + S(\partial_x^2 G - r^2(\partial_x B \partial_r G + rB^2(2\partial_r G + r\partial_r^2 G))) + 2r^2 B \partial_r G(\partial_x S - r^2 B \partial_r S))$$
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$$\begin{aligned}&-GS(S(r^2\partial_r B\partial_r G - \partial_r\partial_x G) + rB(2r\partial_r G\partial_r S + (2\partial_r G + r\partial_r^2 G)S) - 2\partial_r G\partial_x S)\\ &+G^2(S^2(r^2\partial_r^2 B + 2r\partial_r B - \partial_x\Phi\partial_r\Phi) + 2\partial_r S(\partial_x S - r^2 B\partial_r S) - 2S(rB(2\partial_r S + r\partial_r^2 S) + \partial_r\partial_x S))\\ &+\partial_r G S^2(r^2 B\partial_r G - 2\partial_x G) = 0,\end{aligned}$$

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$$\begin{aligned}&S^2(G(r^2\partial_r G(-(\partial_x A + r^2 A\partial_r B - \partial_t B)) + G(r^2(\partial_r\partial_x A + rA(2\partial_r B + r\partial_r^2 B) - \partial_r\partial_t B) - \partial_x\Phi\partial_t\Phi)\\ &+\partial_t\partial_x G - 2\partial_x G\partial_t G) + B(r^2 G(S^2(r^2\partial_r A\partial_r G - \partial_r\partial_t G) + r^2\partial_r B^2 - \partial_r\partial_x B - 2\partial_t G\partial_r SS)\\ &+G^2(2S(r^4\partial_r A\partial_r S - r^2\partial_r\partial_t S) - 2r^2\partial_t S\partial_r S + S^2 V(\Phi)) + r^2(\partial_r B\partial_x G + \partial_t G\partial_r GS^2)) + 2G^2\partial_x S\partial_t S\\ &+r^3 B^2((2\partial_r B + r\partial_r^2 B)G - r\partial_r B\partial_r G) - 2GS(G(r^2(\partial_r B\partial_t S - \partial_t B\partial_r S) + \partial_t\partial_x S) - \partial_t G\partial_x S) = 0,\end{aligned}$$

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Question

How can one describe the dynamics (thermalization) of a (strongly couple) system?

1. Analyzing small excitations of a uniform static system ...
   - Collective modes in CFT/Quasinormal modes in AdS

     Janik, Plewa, Spaliński, HS, PRD '15
     Janik, Jankowski, HS, JHEP '16
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     More in Jakub's talk
2. Full time evolution of a given initial distribution ...
   - It is interesting but rather complicated!

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Outline

1. *AdS/non-CFT setup*
   - How to model the gravity?
   - Numerical setup
   - 1st order phase transition at linear level
2. *Full time evolution and 1st phase transition*
   - Gravity setup
   - Time evolution
   - Final state
3. *Summary and future directions*

AdS/non-CFT setup

How to model the gravity?

1. Top-down approach: Deform $\mathcal{N}=4$ SYM (to $\mathcal{N}=2^*$) explicitly known (but rather complicated) gravity
2. Bottom-up approach: Assume AdS/CFT but try to model the gravity+matter background to approach as closely as possible to QCD physics (or other physics of interest)
   - The simplest nontrivial model

$$S=\frac{1}{2\kappa_{d+1}^2}\int d^{d+1}x\,\sqrt{|g|}\left[R-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right]+S_{\mathrm{GH}}+S_{\mathrm{ct}}$$

Gursoy et. al '07; Gubser et. al '08

   - We choose $V(\phi)$ such that it reproduces the physics of interest (like lQCD equation of state, 1^{st} or 2^{nd} order transition)

$$V(\phi)\sim -d(d-1)+\frac{m^2}{2}\phi^2+\mathcal{O}(\phi^4)\,,\qquad m^2=\Delta(\Delta-d)$$

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Numerical set up for static, homogenous solutions

EoS: (Mathematica)

- Einstein equations: nonlinear coupled second order diff eqs
- Numerical approach: Newton method, spectral Chebyshev polynomials with 80 grid points, numerical precision=200 and accuracy $\sim 10^{-50}$
- Proper boundary conditions for asymptotically AdS black hole solutions in Eddington-Finkelstein coordinates
- The only free parameter is ϕ_H
- EoS:
  $\epsilon(T)$ *or* $s(T)$ *or* $c_s^2(T) = \frac{d\ln(T)}{d\ln(s)}$

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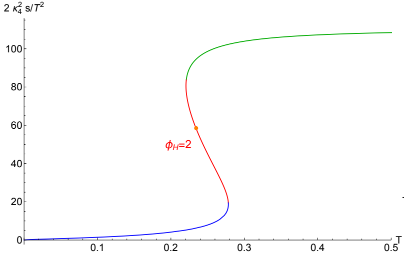
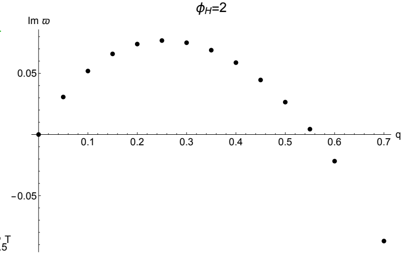
1^{st} order phase transition

Janik, Jankowski, HS '17

EoS



hydro QNM



Thermodynamic instability: $C = T\partial s/\partial T < 0$

Dynamical instability: in large λ for hydro sound/bulk mode

$$\delta g_{\mu\nu} \& \delta\phi \sim e^{-i\omega t + ikx}, \qquad \varpi := \frac{\omega}{2\pi T}, \quad q = \frac{k}{2\pi T}$$

1^{st} order phase transition

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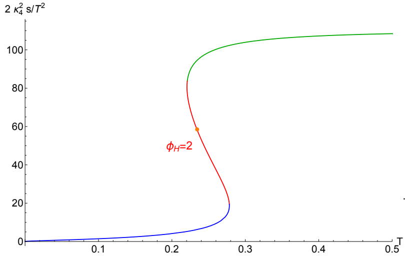

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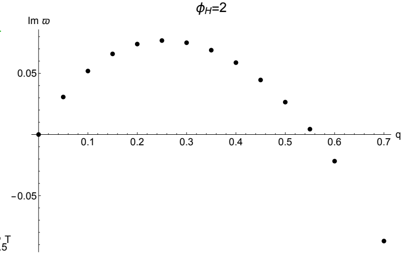

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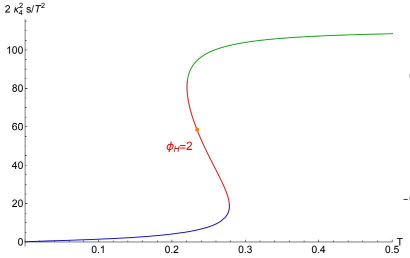

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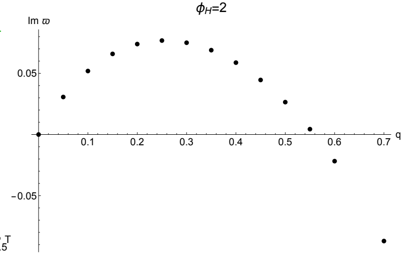

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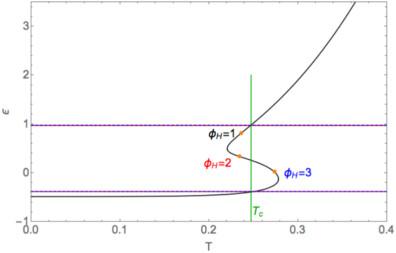
Nonlinear regime

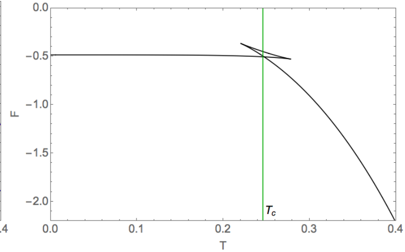
Dynamical system and 1st phase transition

4D gravity

EoS



Free energy



Potential: $V(\phi) = -6\cosh(\frac{\phi}{\sqrt{3}}) - \frac{2}{10}\phi^4$, $\Delta = 2$

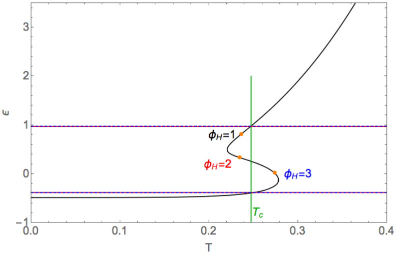
Ansatz (EF coordinate):

$$ds^2 = -A\,dt^2 - \frac{2\,dt\,dr}{r^2} + S^2\,(G\,dx^2 + G^{-1}\,dy^2) - 2Bdtdx,$$

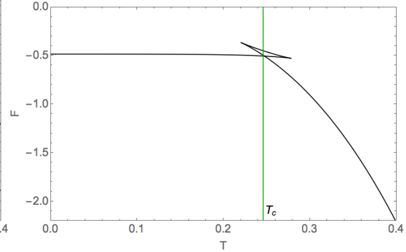
A, S, G, ϕ are functions of r, t, x. x is periodic.

4D gravity

EoS



Free energy



Potential: $V(\phi) = -6\cosh(\frac{\phi}{\sqrt{3}}) - \frac{2}{10}\phi^4$, $\Delta = 2$

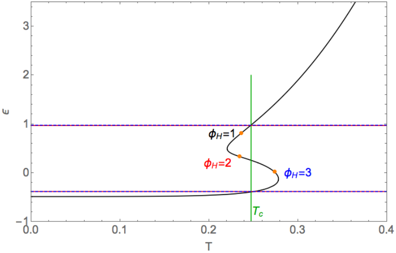
Ansatz (EF coordinate):

$$ds^2 = -A\,dt^2 - \frac{2\,dt\,dr}{r^2} + S^2\,(G\,dx^2 + G^{-1}\,dy^2) - 2Bdtdx,$$

A, S, G, ϕ are functions of r, t, x. x is periodic.

4D gravity

EoS



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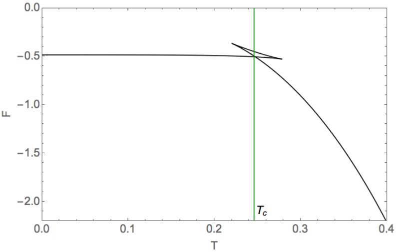

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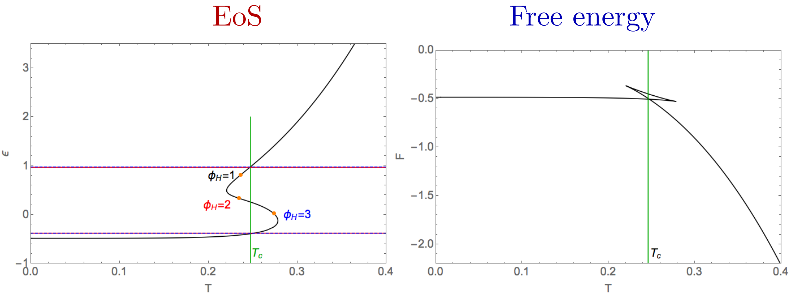

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Numerical setup for dynamical, inhomogenous solutions

Setup: (Python)

- Einstein equations: $2+1$ pde's (using $d_+ := \partial_t + \frac{A}{2}\partial_r$)
- spectral Chebyshev with #80 in radial: $0 \leq r \leq 1$
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- Smooth/Sharp filtering in x direction: we use 0.6 as filter parameter
- Initial configuration: unstable BH $+\delta S(r,x)$
- Proper boundary conditions for asymptotically AdS solutions with fixed sources (Microcanonical ensemble)
- Apparent horizon boundary condition at $r_H = 1$
- Holographic renormalization:
  $$\langle T_{i\,j}(x,t)\rangle \qquad and \qquad \langle O_\phi(x,t)\rangle$$
- Ward identities:
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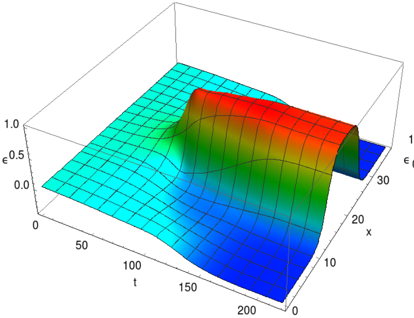
Time evolution

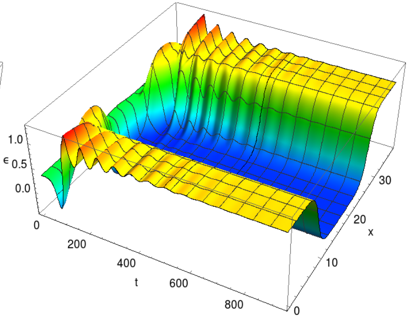
Janik, Jankowski, HS '17

Perturbations with 12π period: Periodic solution

$\delta S \propto \cos(k\,x)$ $\qquad$ $\delta S \propto \exp(-w_0 \cos(k\,x)^2)$



$\phi_H = 3$



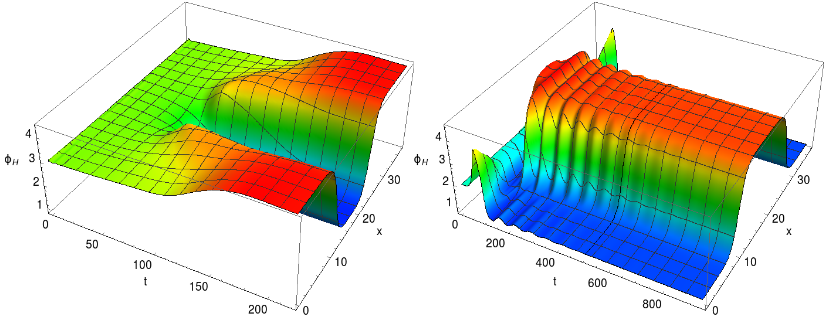
$\phi_H = 2$

Time evolution

Janik, Jankowski, HS '17

Perturbations with 12π period: Inhomogenous horizon

$\delta S \propto \cos(k\, x)$ $\delta S \propto \exp(-w_0 \cos(k\, x)^2)$



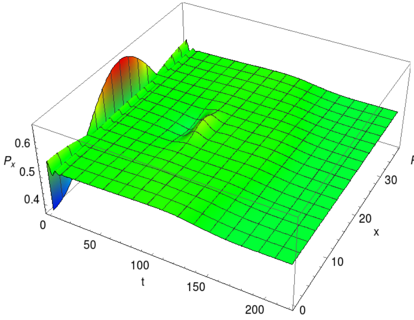
$\phi_H = 3$ $\phi_H = 2$

Time evolution

Janik, Jankowski, HS ’17

Perturbations with 12π period: No gradient along x at late time

$\delta S \propto \cos(k\,x)$ $\delta S \propto \exp(-w_0 \cos(k\,x)^2)$



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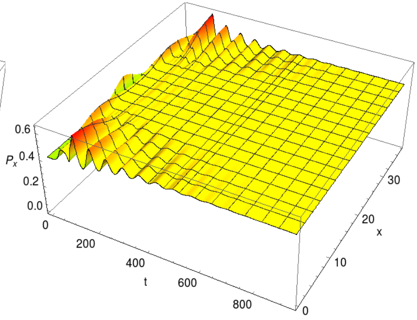

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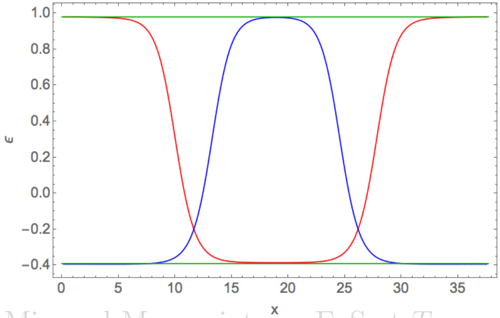
Final state

Janik, Jankowski, HS '17

Final state seats at $T = T_c$:



Green lines: Min and Max points on EoS at T_c

Blue line: $\epsilon(x, t_f)$ for the single mode perturbation

Red line: $\epsilon(x, t_f)$ for mixed modes perturbation

Hydrodynamics is applicable with 2nd-order trans. coef. $c_{x/y}, f_{x/y}$:

$$P_{x/y} = P_{eq}(\epsilon) + c_{x/y}(\epsilon)\,(\partial_x \epsilon)^2 + f_{x/y}(\epsilon)\,(\partial_x^2 \epsilon)$$

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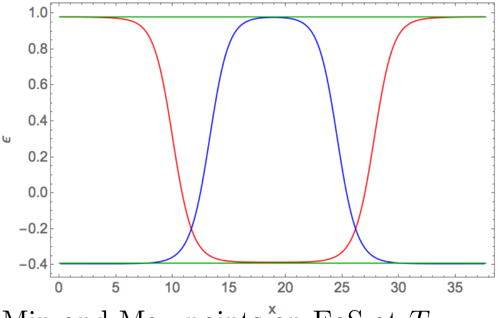

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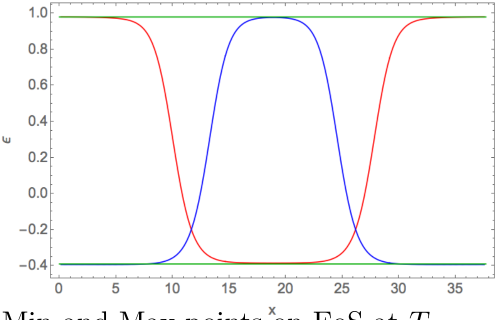

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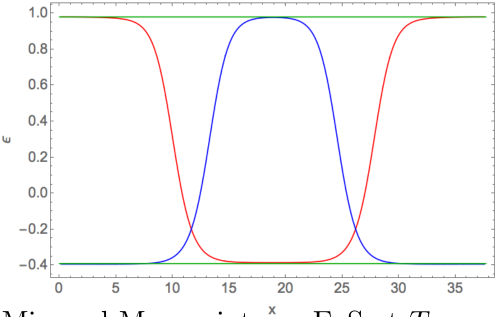

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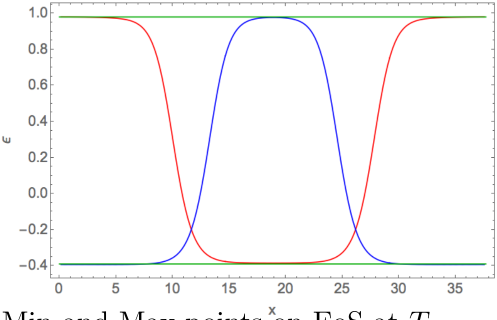

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Summary and future directions

1. There are new stationary BHs with inhomogenous horizon
2. There are new stationary domain wall BHs
3. Hydrodynamics is applicable to corresponding strongly coupled FTs
4. *Study the inhomogenous stationary solutions (final states), e.g. using known probes ...*
5. *Investigating the nonequilibrium physics is on the way, e.g. introducing well-defined probes ...*

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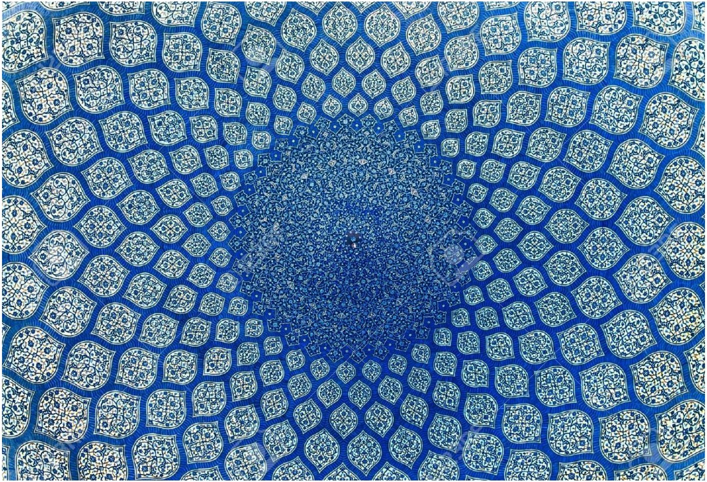
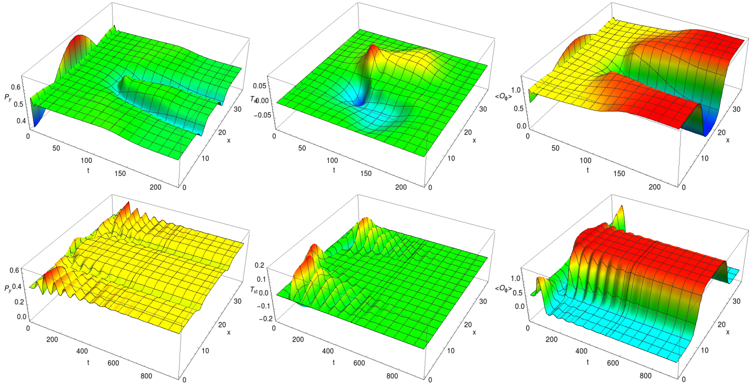


Thank you for your attention

Backups


P_y
T_{xt}
$<O_\phi>$
t
x
0
50
100
150
200
400
600
800
10
20
30
0.6
0.5
0.4
0.05
0.00
−0.05
1.0
0.5
0.0
0.2
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0.1
−0.1
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