Dynamics, phase transitions and holography

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with R. A. Janik, H. Soltanpanahi

JHEP **1606**, 047 (2016), Phys. Rev. Lett. **117**, no. 9, 091603 (2016)

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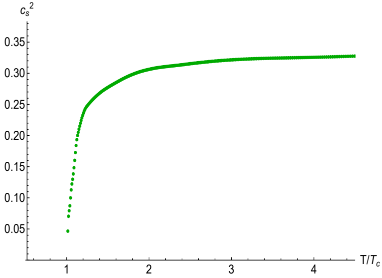
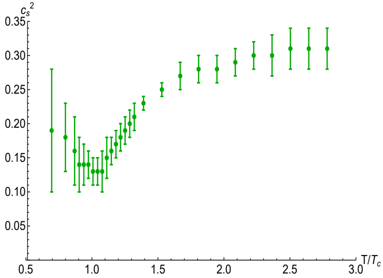
Phase structure at strong coupling

- Systems at strong coupling exhibit various phase structures
- Pure gluon system $\longrightarrow$ 1^{st} order phase transition (left)
- Gluons + quarks $\longrightarrow$ smooth crossover (right)



G. Boyd *et al.* Nucl. Phys. B **469**, 419 (1996)



S. Borsanyi *et al.* JHEP **1009**, 073 (2010)

- Lattice methods do not reach real time dynamics easily
- Use other methods to model strongly coupled phase transitions
- Compute the spectrum of linearized perturbations
- Compute transport coefficients and non-hydrodynamic modes
- Compute the non-linear time evolution
- Investigate the dynamical appearance of diverse phases
- Check linear and non-linear stability

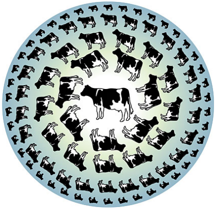
Method:
Use a string theory based approach to formulate models at strong coupling!

Questions

- Does spinodal instability appear for a holographic system with a 1^{st} order phase transition?
- Does the phase separation effect appear dynamically?
- Are there black hole solution with inhomogeneous horizons?
- How do non-hydrodynamic degrees of freedom behave in the critical region?
- Do diffusive modes appear?

Method:
Use a string theory based approach to formulate models at strong coupling!

- Holographic principle
  Quantum gravity in d dimensions must have a number of DOF which scales like that of QFT in $d-1$ dimensions
  't Hooft and Susskind '93



- String Theory realization: *AdS/CFT correspondence*
  Theory is *conformal* and *supersymmetric* Maldacena '97
- Extensions to *non-supersymmetric* and *non-conformal* field theories are possible
- Applications: elementary particle physics and condensed matter physics

Non-conformal holographic plasma

- Top-down construction

  $\mathcal{N} = 4$ broken to $\mathcal{N} = 2^*$ SUSY theory. Known, but complicated dual gravity description

  A. Buchel, S. Deakin, P. Kerner, J. T. Liu, Nucl. Phys. B **784**, 72 (2007)

- Bottom-up construction

  Assuming AdS/CFT dictionary, try to model gravity+matter background to approach as closely as possible to your favourite physics

  U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
  S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

Holographic set-up $\rightarrow$ bottom-up approach

- Boundary: add a source for an operator O_ϕ in a $\mathrm{CFT_d}$

$$\mathcal{L} = \mathcal{L}_{\mathrm{CFT}} + \Lambda^{d-\Delta} O_\phi$$

- Bulk: a gravity-scalar system in $D = d + 1$

$$S = \frac{1}{2\kappa_D^2} \int_{\mathcal{M}} d^D x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] + S_{\mathrm{GH}} + S_{\mathrm{ct}}$$

with the potential

$$V(\phi) = 2\Lambda_C \, (1 + a\,\phi^2)^{1/4} \cosh(\gamma\,\phi) + b_2\,\phi^2 + b_4\,\phi^4 + b_6\,\phi^6$$

- $\Lambda_C = -d(d-1)/2$ is the cosmological constant

U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

Equilibrium configurations

- Metric ansatz for a homogeneous configuration

$$ds^2 = e^{2A(r)}\left(-h(r)dt^2 + d\vec{x}^2\right) - 2e^{A(r)+B(r)}drdt$$

with $\phi(r) = r$ the holographic coordinate

- Solve Einstein+matter equations
- The event horizon: $h(r_H) = 0$
- Entropy and Hawking temperature

$$s = \frac{2\pi}{\kappa_D^2} e^{(d-1)A(r_H)} \qquad\qquad T = \frac{e^{A(r_H)+B(r_H)}|V'(r_H)|}{4\pi}$$

- The free energy is defined by the action $F = TS_{\mathrm{on-shell}}$

U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

Phase transitions in holography

- Finite T sates correspond to various black hole solutions in the dual spacetime
- Phase structure is determined by the choice of a, γ and $b_2, \ b_4, \ b_6$, coefficients of $V(\phi)$
- With $a \neq 0$ confinig models (IHQCD)
- It is possible to tune parameters to mimic

  $\rightarrow$ crossover e.g. QCD
  $\rightarrow$ 1^{st} order phase transition e.g. pure gluon systems
  $\rightarrow$ 2^{nd} order phase transition

U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

Equation of State (EoS)

- The free energy is defined by the action

$$F = TS_{\mathrm{on-shell}}$$

- Configurations characterized by the horizon radius
- Condition for the 1^{st} order phase transition $F_{\mathrm{BH}_1} = F_{\mathrm{BH}_2}$
- Similar to Hawking-Page transition of pure *AdS*

S. W. Hawking, D. N. Page, Commun. Math. Phys. **87**, 577 (1983)

E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998)

Example I: First order phase transition

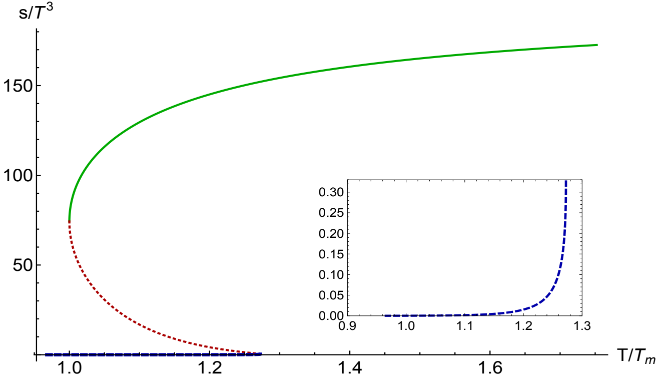
- In $d = 3 + 1$ we choose

  $$V_{1\mathrm{st}}(\phi) = -12\,(1 + a\,\phi^2)^{1/4}\,\cosh(\gamma\,\phi) + b_2\,\phi^2 + b_4\,\phi^4 + b_6\,\phi^6$$

  with $a = 0,\ \gamma = \sqrt{7/12},\ b_2 = 2.5,\ b_4 = b_6 = 0$
- Conformal dimension of the scalar operator is $\Delta = 3.41$
- Transition between two different black hole solutions
- An example of holographic 1^{st} order phase transition
- No known physical counterpart

Example I: First order phase transition

- There exists a critical temperature $T_c \simeq 1.05\,T_m$
- For the unstable region (red dashed line) we have $c_s^2 < 0$



Example II: Confining model IHQCD

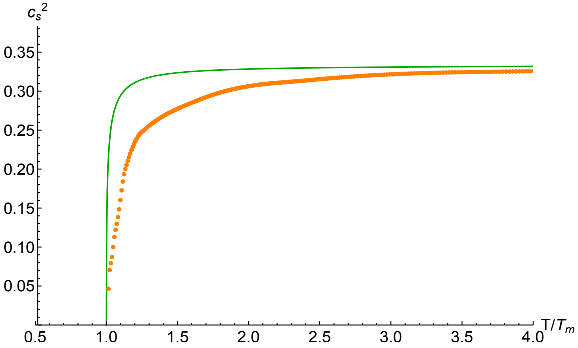
- In $d = 3 + 1$ we choose

$$V_{\mathrm{IHQCD}}(\phi) = -12\,(1+a\,\phi^2)^{1/4}\,\cosh(\gamma\,\phi)+b_2\,\phi^2+b_4\,\phi^4+b_6\,\phi^6$$

  with $a = 1,\ \gamma = \sqrt{2/3},\ b_2 = 6.25,\ b_4 = b_6 = 0$

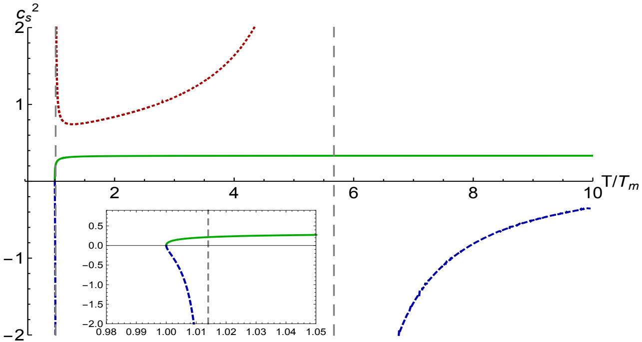
- Conformal dimension of the scalar operator is $\Delta = 3.58$
- Transition between black hole and horizon-less geometry
  S. W. Hawking, D. N. Page, Commun. Math. Phys. **87**, 577 (1983)
- System motivated by the gluon dynamics
- Linear confinement in the meson spectrum, i.e. $m_n^2 \sim n$

Example II: Confining model IHQCD



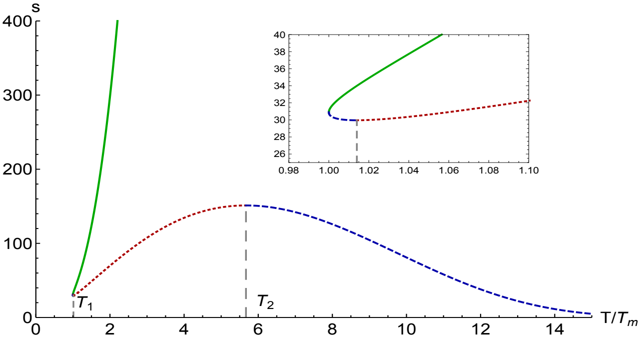
G. Boyd *et.al.* Nucl. Phys. B **469**, 419 (1996)

Example II: Full holographic scan



- Below T_m no black hole solution exists
- Green line - stability region, blue dashed line - spinodal region
- Red dashed line - „dynamically unstable” region

Example II: Entropy density



- The dynamically unstable region for $T_1 < T < T_2$
- The limiting points $T_1 = 1.014\,T_m$ and $T_2 = 5.67\,T_m$

Linear response and Quasinormal modes

- Perturb the system $\mathcal{L} = \mathcal{L}_0 + h_{ij}\delta^3(x)\delta(t)T^{ij}(x)$ the response is the *retarded Green's* function

$$G_R(\omega, k) \propto i \int dtd^3x \; \theta(t)e^{ikx-i\omega t}\langle[T_{ij}(x,t), T_{kl}(0)]\rangle$$

- *Quasinormal modes*, i.e., solutions of linearized fluctuation equations correspond to poles of holographic retarded Green's functions. In general

$$\omega_n(k) = \Omega_n(k) - i\Gamma_n(k)$$

where $n = 1, 2, 3, \ldots$ $\Omega_n(k)$—oscillation frequency, $\Gamma_n(k)$—damping rate. Stable modes have $\Gamma_n(k) > 0$.

P. K. Kovtun, A. O. Starinets, Phys. Rev. D **72**, 086009 (2005)

- Consider perturbations of the $5D$ black hole background

$$g_{ab} = g_{ab}^{\mathrm{BH}} + h_{ab}(r)e^{ikz-i\omega t}, \qquad \Phi = \Phi^{\mathrm{BH}} + \phi(r)e^{ikz-i\omega t}$$

- QNMs are the solutions of linearized fluctuation equations that correspond to poles of holographic retarded Green's functions
- In general

$$\omega_n(k) = \Omega_n(k) - i\Gamma_n(k)$$

where $n = 1, 2, 3, \ldots$ $\Omega_n(k)$—oscillation frequency, $\Gamma_n(k)$—damping rate.

- Stable modes have $\Gamma_n(k) > 0$
- A convenient normalization is: $\quad q = \frac{k}{2\pi T}, \quad \varpi = \frac{\omega}{2\pi T}$

P. K. Kovtun, A. O. Starinets, Phys. Rev. D **72**, 086009 (2005)

Gauge invariant perturbations

- Gauge invariance at linearized level

$$h_{ab} \mapsto h_{ab} - \nabla_a \xi_b - \nabla_b \xi_a \ , \qquad \phi \mapsto \phi - \xi^a \nabla_a \phi$$

- Five independent channels
  $\rightarrow$ sound and non-CFT modes, coupled

$$Z_1(h_{tt}, h_{tz}, h_{zz}, h_{xx} + h_{yy}) \ , \qquad Z_2(\phi, h_{xx} + h_{yy})$$

  $\rightarrow$ twofold degenerated shear mode

$$Z_3(h_{xz}, h_{xt}), \quad Z_3(h_{yz}, h_{yt})$$

  $\rightarrow$ scalar mode (EOM similar to external scalar field EOM)

$$Z_4(h_{xy})$$

P. K. Kovtun, A. O. Starinets, Phys. Rev. D **72**, 086009 (2005)

Comments on numerics and boundary conditions

- In our coordinate system $\Phi(r) = r$ in the background

  S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

- The near-boundary ($r \to 0$) behaviour

$$Z_1(r) \sim A_1 + B_1\, r^{\frac{d}{d-\Delta}}\,, \quad Z_2(r) \sim A_2\, r + B_2\, r^{\frac{\Delta}{d-\Delta}}$$

  imposes $A_1 = A_2 = 0$ boundary condition

- Other modes have Dirichlet BCs at the conformal boundary
- Ingoing boundary conditions at the horizon
- Chebyshev discretization with high numerical precision
- Newton-Rhapson method for the background
- Generalized eigenvalue problem for QNMs, i.e. $\mathrm{Det}M(\omega) = 0$

- Hydrodynamic mode is defined by

$$\lim_{k\to 0} \omega_{\mathrm{H}}(k) = 0$$

- The sound mode

$$\omega(k) = \pm c_s\, k - \frac{i}{2\,T}\left(\frac{4}{3}\,\frac{\eta}{s} + \frac{\zeta}{s}\right) k^2 + O(k^3)$$

η—shear viscosity, ζ—bulk viscosity, s—entropy density, c_s—speed of sound, T—temperature

- In holographic models also *non-hydrodynamic* modes are present

P. K. Kovtun, A. O. Starinets, Phys. Rev. D **72**, 086009 (2005)
M. P. Heller *et al.* Phys. Rev. Lett. **110**, no. 21, 211602 (2013)

Non-hydrodynamic degrees of freedom (DOF)

- Higher QNM's represent non-hydro DOF in QFT
- Crossing of the hydro and non-hydro modes happens when the hydrodynamic mode is more damped than the higher QNMs
- In the CFT this happens *only* in the shear mode for $k \simeq 1.3(2\pi)T$ I. Amado *et.al.* JHEP **0807**, 133 (2008)
- Non-conformality affects the crossing phenomena in *qualitative* and *quantitative* way
- In $\mathcal{N} = 4$ SYM case the non-hydro QNM's are linked with high order transport coefficients

M. P. Heller, *et al.* Phys. Rev. Lett. **110**, no. 21, 211602 (2013)

- When $c_s^2 < 0$ we have purely damped hydro-modes

$$\omega \approx \pm i|c_s|\,k - \frac{i}{2T}\left(\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s}\right)k^2 = \pm i|c_s|\,k - i\Gamma_s k^2$$

so for small enough k we have $\mathrm{Im}\;\omega > 0$

- This mode is present for a finite range of $0 < k < k_{\mathrm{max}}$
- The maximum momentum for the unstable mode is $k_{\mathrm{max}} = |c_s|/\Gamma_s$
- This appears for systems with a 1^{st} order phase transition; *spinodal* instability

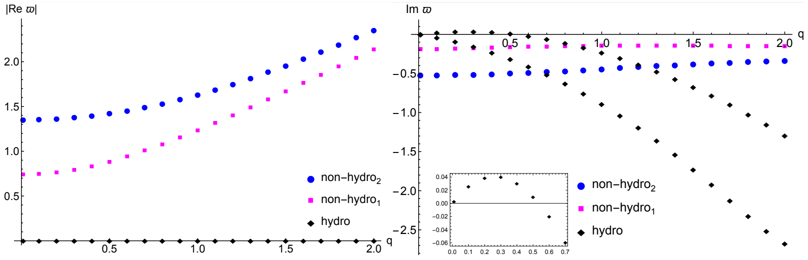
P. Chomaz, M. Colonna, J. Randrup, Phys. Rept. **389**, 263 (2004)

Examples of spinodal instabilities

- Water: superheated liquid and supercooled vapour

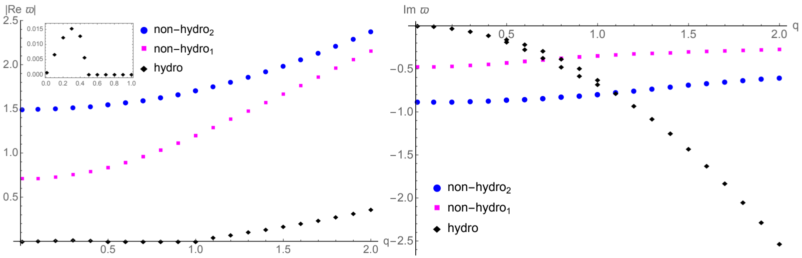
- Spinodal instability in nuclear matter liquid-gas transition

  P. Chomaz, M. Colonna, J. Randrup, Phys. Rept. **389**, 263 (2004)



- Modes for $T \simeq 1.06\, T_m$ where $c_s^2 \simeq -0.1$
- The hydrodynamic mode follows the thermodynamic instability
- Scale of the bubble $=$ k for which $\mathrm{Im}\ \omega$ is maximal
- The maximal value of $\mathrm{Im}\ \omega$ is called the growth rate
- Non-hydrodynamic modes have weak k-dependence

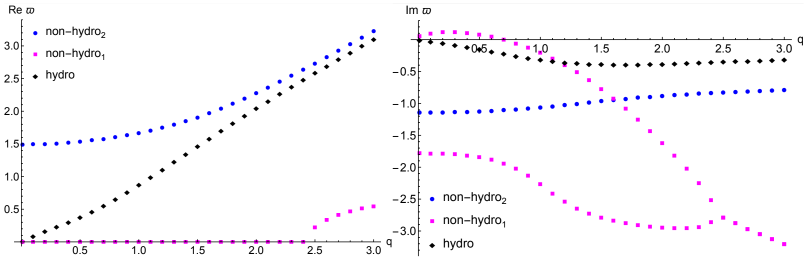
Example I: Diffusive modes



- Modes for $T \simeq 1.00004\, T_m$
- $\mathrm{Re}\ \omega = 0$ for a range of momenta (here $0.5 < q < 1$)
- The sound mode becomes nonpropagating for this range of q
- The onset of such a behaviour was also seen in

  U. Gürsoy, S. Lin, E. Shuryak, Phys. Rev. D **88**, no. 10, 105021 (2013)
- Crossing between hydro sound mode and non-hydro mode

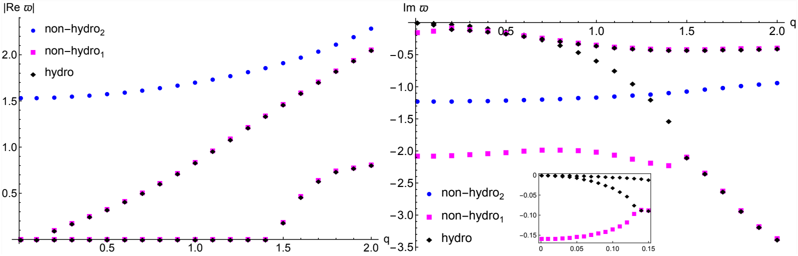
Example II: Dynamical instability



- Quasinormal modes at $T = 1.027\,T_m$
- System displays dynamical instability in spite of thermodynamical stability!
- The system is unstable against uniform $(k = 0)$ perturbations
- Possible implications for thermalization time

U. Gursoy, *et al.* Phys. Rev. D **94**, no. 6, 061901 (2016)

Example II: Ultralocality violation at $T = T_m$



- Small gap between hydro and non-hydro DOF at low q
- Hydro and non hydro joining at $q_J \simeq 0.14$ and $q_J \simeq 1.5$
- At q_J the real part develops
- This structure is unique for $T = T_m$

potential	sound q_c	shear q_c	c_s^2	ζ/s
$V_{\rm QCD}$	0.8	1.1	0.124	0.041
$V_{\rm 2nd}$	0.55	0.9	0.0	0.061
$V_{\rm 1st}$	0.8	1.15	0.0	0.060
$V_{\rm IHQCD}$	0.14	1.25	0.0	0.512

- The crossing momentum q_c at $T = T_c$ ($V_{\rm QCD}$, $V_{\rm 2nd}$) and $T = T_m$ ($V_{\rm 1st}$, $V_{\rm IHQCD}$)
- In contrast to the CFT case crossing happens in both channels
- Applicability of hydro is restricted near the transition (especially in the IHQCD model)

- Thermodynamic instability $\rightarrow$ dynamical instability
- Converse doesn't seem to be true!

  U. Gursoy, *et al.* Phys. Rev. D **94**, no. 6, 061901 (2016)
- Non-trivial phase structure limits the applicability of hydrodynamics
- In most cases non-hydro degrees of freedom have very weak dependence on k $\rightarrow$ *„ultralocality"*
- Non-linear time evolution $\longrightarrow$ the phase transition concept beyond the notion of thermodynamic equilibrium
- A conjecture that for a class of systems $c_s^2 \leq 1/3$

  P. M. Hohler, M. A. Stephanov, Phys. Rev. D **80**, 066002 (2009)
  A. Cherman, T. D. Cohen, A. Nellore, Phys. Rev. D **80**, 066003 (2009)

  C. Hoyos, *et al.* Phys. Rev. D **94**, no. 10, 106008 (2016)
- For more see talk by Hesam!