Remarks on exotic theories of gravity in 6 dimensions

Marc Henneaux

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(where the little group (algebra) is $su(2) \oplus su(2)$).

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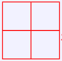
(chiral tensor of mixed Young symmetry , chiral two-forms, scalars - 128 physical degrees of freedom).

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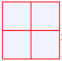
(chiral tensor of mixed Young symmetry , chiral two-forms, scalars - 128 physical degrees of freedom).

The theory is expected to have $E_{6,6}$-symmetry (like maximal supergravity in 5 dimensions), the chiral 2-forms being in the **27** and the scalars parametrizing the coset $E_{6,6}/USp(8)$, which has dimension $78-36=42$.

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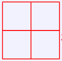
(chiral tensor of mixed Young symmetry , chiral two-forms, scalars - 128 physical degrees of freedom).

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However, only the equations of motion of that theory are known, and only in the free case. Furthermore, there is no known Lagrangian, even in the absence of interactions.

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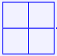
- Explain why a chiral tensor of mixed  -symmetry contains gravity ;

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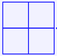
- Explain why a chiral tensor of mixed -symmetry contains gravity ;
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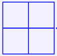
- Explain why a chiral tensor of mixed -symmetry contains gravity ;
- and explain the construction of an explicit Lagrangian (Work in collaboration with Victor Lekeu and Amaury Leonard).

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There are, however, subtleties with respect to the 2-form case.

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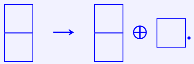

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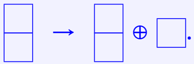

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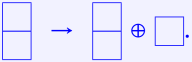

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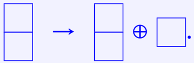

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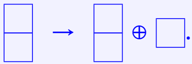

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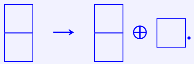

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Thus, a chiral 2-form in six dimensions gives *a single* vector in 5 dimensions.

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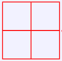
The gauge symmetries for a -field ("$(2,2)$-field") are

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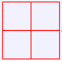
The gauge symmetries for a -field ("(2,2)-field") are

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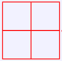
The gauge symmetries for a -field ("$(2,2)$-field") are

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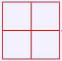
The gauge symmetries for a -field ("$(2,2)$-field") are $\delta T_{\alpha_1\alpha_2\beta_1\beta_2} = \mathbb{P}_{(2,2)}\left(\partial_{\alpha_1}\eta_{\beta_1\beta_2\alpha_2}\right)$ where $\eta_{\beta_1\beta_2\alpha_2}$ is an arbitrary $(2,1)$-tensor. Here, $\mathbb{P}_{(2,2)}$ is the projector on the $(2,2)$ symmetry.

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Note that there is a mismatch between the number of self-duality conditions, namely 175, and the number of components of the $(2,2)$-tensor field, namely 105.

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Note that there is a mismatch between the number of self-duality conditions, namely 175, and the number of components of the $(2,2)$-tensor field, namely 105.

But the self-duality conditions are not all independent.

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[One recovers $\mathscr{E}^{ijrs} - \mathscr{B}^{ijrs} = 0$ up to a term that can be absorbed in a redefinition of T_{0jrs}.]

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Three dimensions is very special from the point of view of conformal geometry.

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What plays the role of the Weyl tensor is the Cotton tensor, which contains 3 derivatives of the spin-2 field.

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- (This can be generalized to higher spins, M.H.-Hörtner-Leonard 2016)

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The prepotential is determined up to diffeomorphisms and Weyl transformations.

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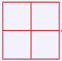
In the particular case of a -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions !

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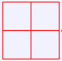
In the particular case of a -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions !

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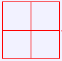
In the particular case of a  -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions !
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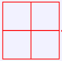
In the particular case of a  -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions !

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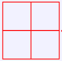
In the particular case of a  -tensor, the critical dimension turns out to be 5, i.e., 6 spacetime dimensions !
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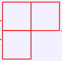
( -diffeos +  -Weyl)

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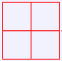
The Cotton tensor is a  -tensor,

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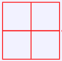
The Cotton tensor is a  -tensor,

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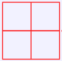
The Cotton tensor is a  -tensor,

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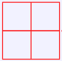
The Cotton tensor is a -tensor,

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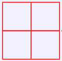
The Cotton tensor is a -tensor,

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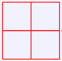
One can show that conversely, any -tensor which is transverse and traceless

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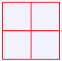
The Cotton tensor is a -tensor,

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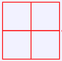
One can show that conversely, any -tensor which is transverse and traceless

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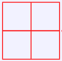
The Cotton tensor is a  -tensor,

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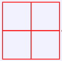
One can show that conversely, any  -tensor which is transverse and traceless

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In particular, the constraint on the electric field of the $(2,2)$-gauge field T implies that one can express T in terms of a prepotential Z so that $\mathcal{E}^{ijrs}[T[Z]] \equiv G^{ijrs}[T[Z]] = D^{ijrs}[Z]$. The relationship $T = T[Z] =$ "∂Z" can be easily written down.

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This is the searched-for action for a chiral (2,2)-tensor in six dimensions.

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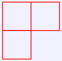
and of the Curtright action for a  -field

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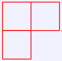
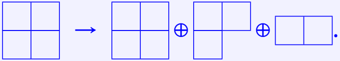
and of the Curtright action for a -field



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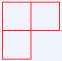
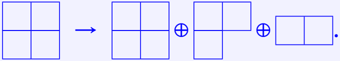
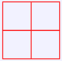
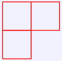
and of the Curtright action for a  -field



But  is trivial in 5 dimensions while  describes another graviton in dual form.

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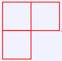
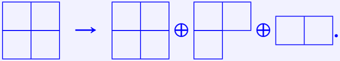
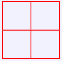
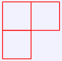
and of the Curtright action for a -field



But  is trivial in 5 dimensions while  describes another graviton in dual form.

The duality condition equates the two types of gravitons.

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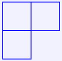
A similar construction can be achieved for the $(3, 1)$ theory, which involves a  tensor with self-dual field strength.