Ultra-spinning black holes at EVH limit and their 2d CFT duals

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Recent Trends in String Theory and Related Topics
IPM May 2017

Outline

- Motivation and Introduction
- Introducing two different ultra-spinning limits.
- Generating new ultra-spinning black hole solution.
- Kerr/CFT description for one family of ultra-spinning BHs.
- Extremal vanishing horizon (EVH) black holes and ultra-spinning limit.
- Summery and outlook.

- Attractive structure of black holes motivate us to generate new black hole solutions.

- It is applicable to generate new exact black hole solution by taking some limits on existence solution, such as ultra-spinning limit.

- Large angular momentum black holes (ultra-spinning).

- by employing ultra-spinning methods we can construct new BH solutions with different and unusual horizon structure and boundary.

Kerr-AdS BH

$$ds^2 = -\frac{\Delta_a}{\rho_a^2}\left[dt - \frac{a}{\Xi}\sin^2\theta d\phi\right]^2 + \frac{\rho_a^2}{\Delta_a}dr^2 + \frac{\rho_a^2}{\Sigma_a}d\theta^2,$$
$$+ \frac{\Sigma_a \sin^2\theta}{\rho^2}\left[adt - \frac{r^2+a^2}{\Xi}d\phi\right]^2,$$

where

$$\Delta_a = (r^2+a^2)(1+\frac{r^2}{l^2}) - \frac{2m}{r}, \qquad \Sigma_a = 1 - \frac{a^2}{l^2}\cos^2\theta\,,$$
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$$M = \frac{m}{\Xi}, \qquad J = \frac{ma}{\Xi}.$$

At the limit $a \to l$ metric and charges will diverge !
To have a BH at the maximum value of rotation parameter, it needs to employ some technique (ultra-spinning method)

Ultra-spinning limits

– **Asymptotically flat :** $a \to \infty$

- Myers-perry BHs in the limit of large angular momentum ($a \to \infty$). [*Emparan, Myeres, (2003)*], that yields a static black brane.

– **Asymptotically AdS :** $a \to l$

- $a \to l$ while keeping the physical mass M fixed ($d \geq 6$). [*Caldarelli et al. (2008)*].
- $a, l \to \infty$ while a/l fixed, which is also applicable for ds BH. [*Obers et al. (2008)*].

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- Hyperboloid membrane limit: $a \to l$ while r_+ fixed ($d \geq 4$) [*Caldarelli et al. (2008)*].

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- Hyperboloid membrane limit: $a \to l$ while r_+ fixed ($d \geq 4$) [*Caldarelli et al. (2008)*].
- ARF ultra-spinning limit (super-entropic): $a \to l$ while rescaling the corresponding azimuthal coordinate ϕ [*Mann et al. (2014)*].
  applicable for a metric that is written in an asymptotically rotating frame (ARF).

First method :
ARF ultra-spinning limit

Charged-AdS BH at ultra-spinning limit

ARF ultra-spinning Method [*Mann et al. [arXiv:1411.4309]*]

- i) Transforming metric to an asymptotic rotating frame (ARF) : $\phi = \phi^R + \frac{a}{l^2} dt$
- ii) Change coordinate $\phi \to \varphi = \frac{\phi}{1-a^2/l^2}$,
- iii) taking limit $a \to l$.
- iv) Compactifying the new coordinate : $\varphi \sim \varphi + \mu$

4d $U(1)^4$ gauged supergravity with pairwise equal charge (written in ARF)

$$ds^2 = -\frac{\Delta_r}{W}\big(dt - \frac{a\sin^2\theta}{\Xi}d\phi\big)^2 + W\big(\frac{dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta}\big) + \frac{\Delta_\theta \sin^2\theta}{W}\big(adt - \frac{r_1 r_2 + a^2}{\Xi}d\phi\big)^2,$$

where

$$\Delta_r = r^2 + a^2 - 2mr + \frac{1}{\ell^2} r_1 r_2 (r_1 r_2 + a^2), \qquad \Delta_\theta = 1 - \frac{a^2}{\ell^2}\cos^2\theta,$$

$$W = r_1 r_2 + a^2\cos^2\theta, \qquad r_i = r + 2ms_i^2 = r + q_i, \qquad \Xi = 1 - \frac{a^2}{\ell^2}.$$

Applying ultra-spinning limit : $\varphi = \phi/\Xi$, then $a \to l$

$$ds^2 = -\frac{\tilde{\Delta}_r}{\tilde{W}}\left(dt - \ell\sin^2\theta d\varphi\right)^2 + \tilde{W}\left(\frac{dr^2}{\tilde{\Delta}_r} + \frac{d\theta^2}{\sin^2\theta}\right) + \frac{\sin^4\theta}{\tilde{W}}\left[\ell dt - (r_1 r_2 + \ell^2)d\varphi\right]^2,$$

- The new coordinate φ is non-compact, we compactify it by requiring $\varphi \sim \varphi + \mu$.
- The obtained metric describes a new exact asymptotically AdS BH solution of the 4d $U(1)^4$ gauge supergravity theory.

Horizon geometry

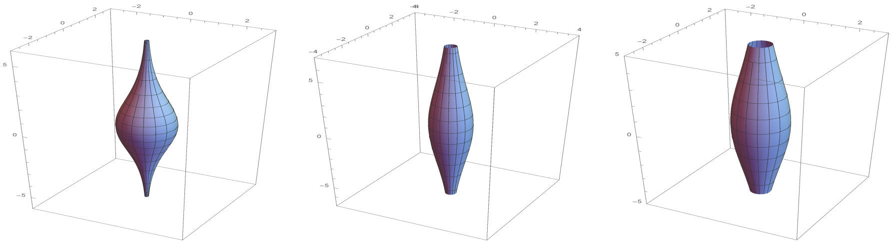
$$ds_h^2 = \frac{\tilde{W}_+}{\sin^2\theta}d\theta^2 + \sin^4\theta\frac{\left((r_+ + q_1)(r_+ + q_2) + \ell^2\right)^2}{\tilde{W}_+}d\varphi^2.$$

- Two poles at $\theta = 0, \pi$, that give rise to a non-compact horizon.
- $k = \ell(1 - \cos\theta)$, for small k, horizon metric becomes

$$ds_h^2 = (r_+ + q_1)(r_+ + q_2)\left[\frac{dk^2}{4k^2} + \frac{4k^2}{\ell^2}d\varphi^2\right],$$

- These poles are not part of the sapcetime. Therefore metric is regular.

4d gauged supergravity BH at US limit [S.M.N, M. Ghominejad, Phys. Rev.D95, 046002 (2017)]



2d non-compact horizons embedded in $\mathbb{R}^3$. with $q_1 = 10$, $q_2 = 0$ (left); $q_1 = 10$, $q_2 = 5$ (middle); $q_1 = 3$, $q_2 = 6$ (right).

Finite entropy : $S = \frac{\mu}{2}\left[(r_+ + q_1)(r_+ + q_2) + \ell^2\right]$

$$E = \frac{2m+q_1+q_2}{2}, \qquad J = \frac{\ell(2m+q_1+q_2)}{2},$$
$$Q_1 = Q_2 = \frac{\sqrt{q_1(2m+q_1)}}{4}, \qquad Q_3 = Q_4 = \frac{\sqrt{q_2(2m+q_2)}}{4}.$$

Higher dimensional charged AdS BHs at US limit

- A general charged solutions of the Einstein-Maxwell Dilaton theory. [*Wu (2011)*]

$$\mathcal{L} = \sqrt{-g}\{R - \frac{(d-1)(d-2)}{4}(\partial\Phi)^2 - \frac{1}{4}e^{-(D-1)\Phi}\mathcal{F}^2 + \frac{1}{l^2}(d-1)[(d-3)e^{\Phi} + e^{-(d-3)\Phi}]\}.$$

$$ds^2 = H^{\frac{1}{d-2}}\left[d\gamma^2 + \frac{U dr^2}{\Delta} + \frac{2m}{UH}\omega^2 + d\Omega^2\right],$$

where $\Xi_i = 1 - a_i^2/l^2$, and

$$d\Omega^2 = \sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{\Xi_i}d\mu_i^2 - \frac{1}{W\rho^2}\left(\sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{\Xi_i}\mu_i d\mu_i\right)^2,$$

$$d\gamma^2 = -\frac{W\rho^2}{l^2}dt^2 + \sum_{i=1}^{N}\frac{r^2+a_i^2}{\Xi_i}\mu_i^2 d\phi_i^2, \qquad \omega = c\,W\,dt - \sum_{i=1}^{N}\frac{a_i\sqrt{\chi_i}}{\Xi_i}\mu_i^2 d\phi_i\,,$$

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- $d = 2N + 1 + \epsilon$, even $\epsilon = 0$, odd $\epsilon = 1$.
- $N = [(d-1)/2]$ rotation parameter a_i corresponding ϕ_i coordinates.
- $[d/2]$ numbers of "direction cosines" μ_i's. $\sum_{i=1}^{N+\epsilon}\mu_i^2 = 1$.
- Gauge field $A = \frac{2ms}{U\,H}\left(c\,W dt - \sum_{i=1}^{N}\frac{a_i\sqrt{\chi_i}}{\Xi_i}\mu_i^2 d\phi\right)$.

Ultra-spinning steps

- We choose ϕ_j to be ultra-spinning direction
- Switch to an asymptotically rotating frame (ARF), by setting $\phi_j = \phi_j^R + \frac{a_j}{l^2} t$.
- Replacing new coordinate $\varphi_j = \frac{\phi_j^R}{\Xi_j}$, then taking the limit $a_j \to l$,
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Resulting BH solution: $ds^2 = \hat{H}^{\frac{1}{D-2}} \left[d\gamma_s^2 + \frac{\hat{U} dr^2}{\hat{\Delta}} + \frac{2m}{\hat{U}\hat{H}} \omega_s^2 \right] + d\Omega_s^2.$

$$d\gamma_s^2 = -\left(\rho^2(\hat{W} + \mu_j^2) + \mu_j^2 l^2\right) \frac{dt^2}{l^2} + \rho^2 \mu_j^2 d\varphi_j^2 + \frac{2\rho^2 \mu_j^2 dt d\varphi}{l} + \sum_{i \neq j} \frac{r^2 + a_i^2}{\Xi_i} \mu_i^2 d\phi_i^2,$$

$$d\Omega_s^2 = \sum_{i \neq j}^{N+\epsilon} \frac{r^2 + a_i^2}{\Xi_i} d\mu_i^2 - 2 \frac{d\mu_j}{\mu_j} \left(\sum_{i \neq j}^{N} \frac{r^2 + a_i^2}{\Xi_i} \mu_i d\mu_i \right) + \frac{d\mu_j^2}{\mu_j^2} (\rho^2 \hat{W} + l^2 \mu_j^2),$$

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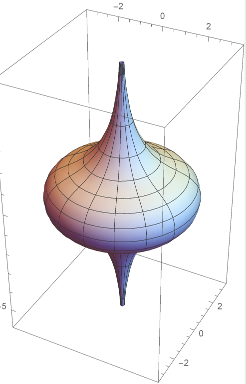
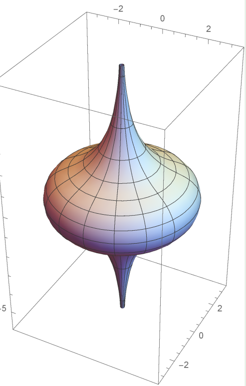
There are some poles at $\mu_j = 0$,
Indicating a ***non-compact horizon*** *(topologically sphere with some punctures).*

It is ***impossible*** to generate a **multi ultra-spinning** solution.



- New exact BH solutions of EMDΛ theory in all higher dimensions with regular horizon.
- Having a non-compact horizon, topologically is sphere with some punctures ($\mu_j = 0$).



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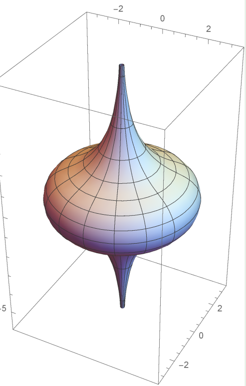
- The poles $\mu_j = 0$ are removed from the spacetime and can be viewed as a sort of boundary.
- They possess a finite entropy as $S = \frac{\mathcal{V}_{D-2}\, r_+^{\epsilon-1}\, c}{4\sqrt{f(r_+)}} \prod_{i\neq j}^{N} \frac{r_+^2+a_i^2}{\Xi_i}$.



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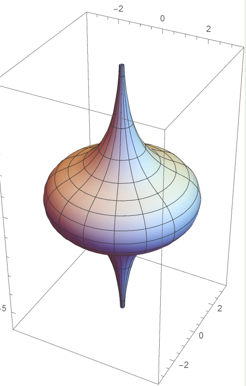
- The asymptotic boundary rotate with the speed of light.

Charged general ultra-spinning BHs [*S.M.N, M. Ghominejad [ariv:1702.03448]*]



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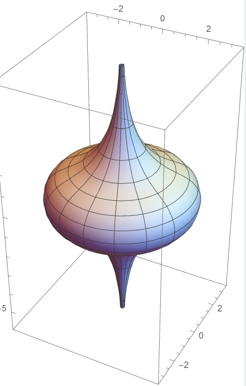

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Second ultra-spinning method :
Hyperboloid membrane limit (HM)

Singly spinning Kerr-AdS in d dimensions

$$ds^2 = -\frac{\Delta_a}{\rho_a^2}\left[dt - \frac{a}{\Xi}\sin^2\theta d\phi\right]^2 + \frac{\rho_a^2}{\Delta_a}dr^2 + \frac{\rho_a^2}{\Sigma_a}d\theta^2,$$
$$+ \frac{\Sigma_a \sin^2\theta}{\rho^2}\left[adt - \frac{r^2+a^2}{\Xi}d\phi\right]^2 + r^2\cos^2\theta\, d\Omega_{d-4}^2.$$

$\Xi = 1 - a^2/l^2$, $\qquad \Delta_a = (r^2+a^2)(1+\frac{r^2}{l^2}) - \frac{2m}{r}$

Hyperboloid membrane limit method:

i) Defining new coordinate σ $\sinh(\sigma/2) = \sin\theta/\sqrt{\Xi}$, $\qquad \sigma \in [0,\infty)$,

ii) Taking the limit $a \to l$ while keeping fixed σ.

Resulting metric :

$$ds^2 = -f\left(dt - \ell\sinh^2(\sigma/2)d\phi\right)^2 + \frac{dr^2}{f} + \frac{r^2+l^2}{4}\left(d\sigma^2 + \sinh^2\sigma d\phi^2\right) + r^2 d\Omega_{d-4}^2,$$
$$f = 1 + \frac{r^2}{l^2} - \frac{2mr^{d-5}}{r^2+l^2}.$$

Horizon has topology $\mathbb{H}^2 \times S^{d-4}$

Singly spinning Kerr-AdS at HM limit [*Caldarelli et al. [arXiv:0806.1954]*]

$$ds^2 = -f\left(dt - l\sinh^2(\sigma/2)d\phi\right)^2 + \frac{dr^2}{f} + \frac{r^2+l^2}{4}\left(d\sigma^2 + \sinh^2\sigma d\phi^2\right) + r^2 d\Omega^2_{d-4},$$

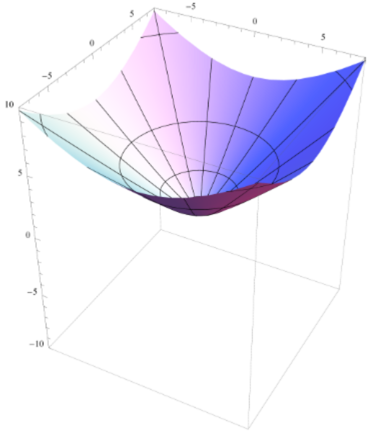
$$f = 1 + \frac{r^2}{l^2} - \frac{2mr^{d-5}}{r^2+l^2}.$$

- The obtained metric is a new exact solution of Einstein-Λ theory.
- The horizon has topology $\mathbb{H}^2 \times S^{d-4}$.
- These solutions represent asymptotically **AdS** rotating black membranes.
- Its Conformal boundary :

  $ds^2_{bdry} = -\left(dt - l\sinh^2(\sigma/2)d\phi\right)^2 + \frac{l^2}{4}\left(d\sigma^2 + \sinh^2\sigma d\phi^2\right) + l^2 d\Omega^2_{d-4}.$

  - 4d case : AdS_3.
  - In $d > 4$ boundary is $\mathrm{AdS}_3 \times S^{d-4}$.
- These classes of solutions are different with "rotating topological black holes" (Horizon topology $\mathbb{H}^2 \times \mathbb{H}^{d-4}$).

Horizon topology



Horizon embedding for 4d case.

Represents an asymptotically AdS rotating black hyperboloid membranes.

5d Myers-Perry AdS BH

Multi-spinning BHs (MP black hole in 5d):

$$\begin{aligned} ds^2 &= -\frac{\Delta}{\rho^2}\left(dt - \frac{a\sin^2\theta}{\Xi_a}d\phi_1 - \frac{b\cos^2\theta}{\Xi_b}d\phi_2\right)^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2}\left(adt - \frac{(r^2+a^2)}{\Xi_a}d\phi_1\right)^2 \\ &+ \frac{\Delta_\theta\cos^2\theta}{\rho^2}\left(bdt - \frac{(r^2+b^2)}{\Xi_a}d\phi_2\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 \\ &+ \frac{l^2+r^2}{l^2r^2\rho^2}\left(abdt - \frac{b(r^2+a^2)\sin^2\theta}{\Xi_a}d\phi_1 - \frac{a(r^2+b^2)\cos^2\theta}{\Xi_a}d\phi_2\right)^2 \end{aligned}$$

where

$$\begin{aligned} \Delta &= \frac{1}{r^2}(r^2+a^2)(r^2+b^2)(1+\frac{r^2}{l^2}) - 2m, \quad \Delta_\theta = 1 - \frac{a^2}{l^2}\cos^2\theta - \frac{b^2}{l^2}\sin^2\theta, \\ \rho^2 &= r^2 + a^2\cos^2\theta + b^2\sin^2\theta, \qquad \Xi_a = 1 - \frac{a^2}{l^2}, \qquad \Xi_b = 1 - \frac{b^2}{l^2} \end{aligned}$$

Hyperboloid membrane limit at a direction :

- Replacing $\sinh(\sigma/2) = \frac{1}{\Xi_a}\sin\theta, \qquad \sigma \in [0,\infty).$
- Taking limit $a \to l$

$$
\begin{aligned}
ds^2 &= \Big[\frac{-\hat{\Delta}}{\hat{\rho}^2}\Big(dt - l\sinh^2(\sigma/2)d\phi_1 - b\frac{d\phi_2}{\Xi_b}\Big)^2 + \frac{r^2+l^2-1}{r^2\rho^2}\Big(dt - \frac{(b^2+r^2)}{\Xi_b}d\phi_2\Big)^2 \\
&+ \frac{1}{l^2r^2}\Big(bldt - b\rho^2\sinh^2(\sigma/2)d\phi_1 - \frac{l(b^2+r^2)}{\Xi_b}d\phi_2\Big)^2 \\
&+ \frac{\rho^2}{2}\Big(\frac{b^2}{l^2}+1+\Xi_b\cosh\sigma\Big)\sinh^2(\sigma/2)d\phi_1^2 + \frac{\hat{\rho}^2}{\hat{\Delta}}dr^2 + \frac{\hat{\rho}^2\cosh^2(\sigma/2)}{1+\Xi_b\sinh^2(\sigma/2)}\frac{d\sigma^2}{4}.
\end{aligned}
$$

Represents a new exact solution of the Einstein-Λ theory in $5d$.

- No additional HM limit can be taken in the b-direction.
- Asymptotically AdS black membrane.
- Horizon is a non-compact manifold : constant negative curvature.
- The conformal boundary is AdS_4.
- It is compatible to apply ARF ultra-spinning limit at b-direction ($\varphi_2 = \phi_2/\Xi_b, \quad b \to l$), resulting geometry is ultra-spinning in both directions simultaneously.

Combination of HM limit and SE limit [*Hennigar et al. [arXiv:1512.02293]*]

HM limit at ϕ_1-direction, and ARF ultra-spinning limit in ϕ_2-direction.

$$
\begin{aligned}
ds^2 &= \left[\frac{-\hat{\Delta}}{\hat{\rho}^2}\left(dt - l\sinh^2(\sigma/2)d\phi_1 - bd\varphi_2\right)^2 + \frac{r^2+l^2-1}{r^2\rho^2}\left(dt - (b^2+r^2)d\varphi_2\right)^2\right. \\
&+ \frac{1}{l^2r^2}\left(bldt - b\rho^2\sinh^2(\sigma/2)d\phi_1 - l(b^2+r^2)d\varphi_2\right)^2 \\
&+ \frac{\rho^2}{2}\left(\frac{b^2}{l^2}+1\right)\sinh^2(\sigma/2)d\phi_1^2 + \frac{\hat{\rho}^2}{\hat{\Delta}}dr^2 + \hat{\rho}^2\cosh^2(\sigma/2)\frac{d\sigma^2}{4}.
\end{aligned}
$$

The new obtained BH solution is again asymptotically AdS.

Interestingly, there is no punctures in the horizon metric.

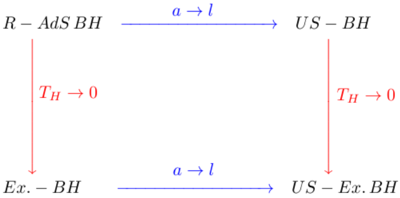
Higher dimensional Kerr-AdS BHs at HM limit+SE

Dimensions	Possible US limits	Horizon topology	Conf. bdry.
4d(ϕ , θ)	1-HM	$\mathbb{H}^2$	AdS$_3$
	1-US	sphere with punctures	AdS$_3$
	1-HM	$\mathbb{H}^2 \times S^1$	AdS$_3 \times S^1$
$5d(\phi, \psi, \theta)$	1-US	sphere with punctures	AdS$_4$
	1-HM + 1-US	$\mathbb{H}^2 \times S^1$ no punctures	flat
	1-HM	$\mathbb{H}^2 \times S^2$	AdS$_3 \times S^2$
6d($\phi, \psi, \theta_1, \theta_2$)	2-HM	negative constant curvature	AdS$_5$
	1-US	sphere with punctures	
	1-HM + 1-US	$\mathbb{H}^2 \times S^2$ with punctures	flat
	1-HM	$\mathbb{H}^2 \times S^3$	AdS$_3 \times S^2$
	2-HM	negative constant curvature	AdS$_6$
7d($\phi, \psi, \xi, \theta_1, \theta_2$)	1-US	sphere with punctures	
	1-HM + 1-US	$\mathbb{H}^3 \times S^2$ with punctures	
	2-HM + 1-US	$\mathbb{H}^4 \times S^1$, no punctures	flat

- ***Describe novel exact asymptotically AdS black membrane/hole solutions in the Einstein-Λ theory.***
- It is possible to perform the HM limit as many times as there are polar angles.
- **ARF US limit and HM limits are commutative with each otherr.**

Extrimality under both ultra-spinning limits

- There are extremal BHs in both US limit cases.
- We show extremality preserves under both US limits (for a large class of solutions).



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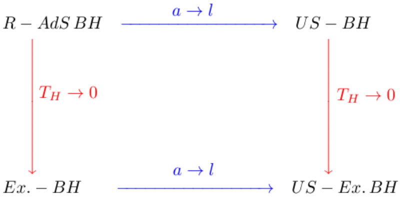

NHEG commutes with both ultra-spinning (SE and HM).

i) Obtaining ultra-spinning BH solution $(a \to l)$, then finding the extremality conditions $T_H = 0 \implies C_1(q_i, m, l, r_0) = 0$

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$$
\begin{array}{ccc}
R - AdS\,BH & \xrightarrow{\quad a \to l \quad} & US - BH \\
\Big\downarrow {\scriptstyle T_H \to 0} & & \Big\downarrow {\scriptstyle T_H \to 0} \\
Ex. - BH & \xrightarrow{\quad a \to l \quad} & US - Ex.\,BH
\end{array}
$$

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ii) Finding extremal version $(T_H = 0) \implies C(q_1, q_2, m, a, l, r_0) = 0$ then performing ultra-spinning $(a \to l)$ limit $\implies C_2(q_i, m, l, r_0) = 0$

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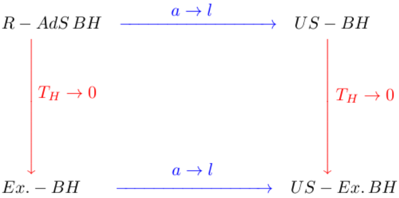

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$$\boxed{C_1(q_i, m, l, r_0) = C_2(q_i, m, l, r_0).}$$

Kerr/CFT Correspondence

Quantum states in the near horizon region of an ***extremal rotating black hole*** are holographically dual to a 2d chiral CFT.

Quantum states in the near horizon region of an ***extremal rotating black hole*** are holographically dual to a 2d chiral CFT.

Ingredients :

- Asymptotic symmetry group (ASG) of a near horizon extremal Kerr geometry (NHEG) that obey suitably chosen boundary conditions.
- The Lie brackets of the generators close on a ***centreless Virasoro algebra***.
- The Dirac brackets of the associated charges lead to a Virasoro algebra with a **central extension**.

4d ultra-spinning $U(1)^4$ gauged SUGRA BH

$$ds^2 = -\frac{\tilde{\Delta}_r}{\tilde{W}}\left(dt - \ell \sin^2\theta d\varphi\right)^2 + \tilde{W}\left(\frac{dr^2}{\tilde{\Delta}_r} + \frac{d\theta^2}{\sin^2\theta}\right) + \frac{\sin^4\theta}{\tilde{W}}\left[\ell dt - (r_1 r_2 + \ell^2) d\varphi\right]^2,$$

Extremality conditions

- $\tilde{\Delta}_r|_{r=r_0} = 0, \qquad T_H|_{r=r_0} = 0 \Longrightarrow \qquad C(r_0, q_1, q_2, m, l) = 0$

Near Horizon Limit

$\Delta_r = X(r - r_0)^2 + \mathcal{O}(r - r_0)^3,$

$$r = r_0(1 + \lambda\hat{r}), \qquad \varphi = \hat{\varphi} + \Omega_H^0 \hat{t}, \qquad t = \frac{\hat{t}}{2\pi T'_H r_0 \lambda}, \qquad \hat{\theta} = \theta.$$

Taking limit $\lambda \to 0$

$$
\begin{aligned}
ds^2 &= \frac{\tilde{W}_0}{X}\Big(-\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2}\Big) + \frac{\tilde{W}_0}{\sin^2\theta} d\hat{\theta}^2 \\
&+ \frac{\sin^4\theta}{\ell^2 \tilde{W}_0}\Big[(r_0+q_1)(r_0+q_2)+\ell^2\Big]^2 \Big(d\hat{\varphi} + k\,\hat{r} d\hat{t}\Big)^2 , \\
k &= \frac{\ell(2r_0+q_1+q_2)}{X\Big[(r_0+q_1)(r_0+q_2)+\ell^2\Big]},
\end{aligned}
$$

$$
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\end{aligned}
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$AdS_2 \times S^{d-2}$; $\mathcal{L}_\xi g_{\mu\nu} = 0 \implies SL(2,\mathbb{R})_L \times U(1)_R$ Isometry group

Boundary conditions : [*Strominger (2008)*]

$$
h_{\mu\nu} = \begin{pmatrix} \mathcal{O}(r^2) & \mathcal{O}(1) & \mathcal{O}(1/r) & \mathcal{O}(1/r^2) \\ & \mathcal{O}(1) & \mathcal{O}(1/r) & \mathcal{O}(1/r) \\ & & \mathcal{O}(1/r) & \mathcal{O}(1/r^2) \\ & & & \mathcal{O}(1/r^3) \end{pmatrix}, \quad a_\mu = \mathcal{O}(r, \frac{1}{r}, 1, 1/r^2).
$$

Near Horizon Extremal Geometry (NHEG)

$$
\begin{aligned}
ds^2 &= \frac{\tilde{W}_0}{X}\Big(-\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2}\Big) + \frac{\tilde{W}_0}{\sin^2\theta} d\hat{\theta}^2 \\
&+ \frac{\sin^4\theta}{\ell^2 \tilde{W}_0}\left[(r_0+q_1)(r_0+q_2)+\ell^2\right]^2 \left(d\hat{\varphi} + k\,\hat{r} d\hat{t}\right)^2 , \\
k &= \frac{\ell(2r_0+q_1+q_2)}{X\left[(r_0+q_1)(r_0+q_2)+\ell^2\right]},
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$$

$\mathcal{L}_\zeta g_{\mu\nu} = h_{\mu\nu}, \implies i[\zeta_m, \zeta_n] = (m-n)\zeta_{m+n}.$ $SL(2,\mathbb{R}) \times Vir_L$

$$
\begin{aligned}
ds^2 &= \frac{\tilde{W}_0}{X}\Big(- \hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2}\Big) + \frac{\tilde{W}_0}{\sin^2\theta} d\hat{\theta}^2 \\
&+ \frac{\sin^4\theta}{\ell^2 \tilde{W}_0}\Big[(r_0+q_1)(r_0+q_2)+\ell^2\Big]^2 \Big(d\hat{\varphi} + k\, \hat{r} d\hat{t}\Big)^2 ,
\end{aligned}
$$

Central charge $c = 3\frac{\mu}{\pi}\frac{\ell(2r_0+q_1+q_2)}{X}$.

Kerr/CFT for 4d $U(1)^4$ gauged supergravity BH at US limit [SMN, M. Ghominejad, Phys.Rev.D 95,046002 (2017)]

$$ds^2 = \frac{\tilde{W}_0}{X}\left(-\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2}\right) + \frac{\tilde{W}_0}{\sin^2\theta} d\hat{\theta}^2 + \frac{\sin^4\theta}{\ell^2 \tilde{W}_0}\left[(r_0+q_1)(r_0+q_2)+\ell^2\right]^2 \left(d\hat{\varphi} + k\,\hat{r} d\hat{t}\right)^2,$$

Central charge $c = 3\frac{\mu}{\pi}\frac{\ell(2r_0+q_1+q_2)}{X}$.

Frolov-Thorne temperature $T_L = -\frac{\partial T_H/\partial r_+}{\partial \Omega_H/\partial r_+}\Big|_{r_+=r_0}$, $T_R = \frac{r_0}{\lambda} T_H\Big|_{r_+=r_0}$.

$$T_L = \frac{1}{2\pi k} = \frac{X\left[(r_0+q_1)(r_0+q_2)+\ell^2\right]}{2\pi\ell(2r_0+q_1+q_2)}, \qquad T_R = 0.$$

Entropy matching!

Cardy formula : $S = \frac{\pi^2}{3} c_L T_L$.

$$S_{CFT} = \frac{\mu}{2}\left[(r_+ + q_1)(r_+ + q_2) + \ell^2\right] = S_{BH}$$

NHEG :

- Extremal version by imposing $T|_{r=r_0} = 0$ and $\hat{\Delta}|_{r=r_0} = 0$
- Coordinate transformation

$$r = r_0(1+\lambda\hat{r}), \quad \varphi_j = \hat{\varphi}_j + \Omega_j^0 \hat{t}, \quad \phi_i = \hat{\phi}_i + \Omega_i^0 \hat{t}, \quad t = \frac{2\hat{Y}_0}{r_0 \Delta_0'' \lambda}\hat{t}, \quad \mu_i = \hat{\mu}_i.$$

- $\lambda \to 0$

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- Extremal version by imposing $T|_{r=r_0} = 0$ and $\hat{\Delta}|_{r=r_0} = 0$
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- $\lambda \to 0$

$$\begin{aligned} ds^2 &= \hat{H}_0^{1/(d-2)} \Bigg[\frac{2\hat{U}_0}{\Delta_0''}\Big(-\hat{r}^2 d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2}\Big) + \sum_{i,k\neq j}^{N} \tilde{g}_{ik}(d\hat{\phi}_i + k_i\,\hat{r}\,d\hat{t})(d\hat{\phi}_k + k_k\,\hat{r}\,d\hat{t}) \\ &+ \sum_i^N g_{ij}(d\hat{\phi}_i + k_i\,\hat{r}\,d\hat{t})(d\hat{\varphi}_j + k_j\hat{r}\,d\hat{t}) + d\hat{\Omega}_0^2 \Bigg], \end{aligned}$$

This class of NHEG admit $N = [(d-1)/2]$ commuting copies of the Virasoro algebra.

$$c_i = \frac{3\,\mu}{4\pi^2} k_i \int d^{n-1} y_\alpha \left(det\tilde{g}_{ij} \prod_{\alpha=1}^{n-1} F_\alpha \right)^{1/2} \int d\phi_1 \ldots d\phi_i, \qquad i = 1 \ldots n-1, \quad i \neq j.$$

$$c_i = \frac{3\mu\, k_i}{4\pi^2} Area|_{r0}.$$

Kerr/CFT for general charged AdS super-entropic BHs [*S.M.N, M. Ghominejad [ariv:1702.03448]*]

$$c_i = \frac{3\,\mu}{4\pi^2} k_i \int d^{n-1} y_\alpha \Big(det\tilde{g}_{ij} \prod_{\alpha=1}^{n-1} F_\alpha \Big)^{1/2} \int d\phi_1 \dots d\phi_i, \qquad i = 1 \dots n-1, \quad i \neq j.$$

$$c_i = \frac{3\mu\, k_i}{4\pi^2} Area|_{r0}.$$

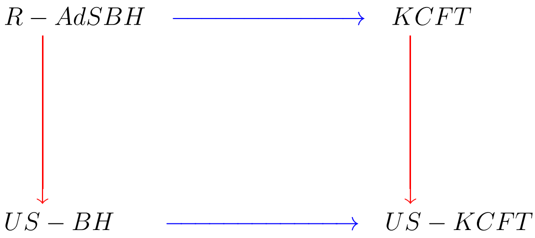
The $N = [(d-1)/2]$ Frolov–Thorne temperatures associated to each copy of CFTs

$$T_{L\phi_i} = -\frac{\partial T_H/\partial_r}{\partial \Omega_{\phi_i}/\partial_r}|_{r=r_0} = \frac{1}{2\pi\, k_i}, \qquad T_{L\varphi_j} = -\frac{\partial T_H/\partial_r}{\partial \Omega_{\varphi_j}/\partial_r}|_{r=r_0} = \frac{1}{2\pi\, k_j}.$$

$$S_{CFT} = \frac{\pi^2}{3} c_1 T_{\phi_1} = \frac{\pi^2}{3} c_2 T_{\phi_2} = \cdots = \frac{\pi^2}{3} c_j T_{\phi_j} = \frac{\pi^2}{3} c_j T_{\varphi_j} = \frac{A|_{r=r_0}}{4} = S_{BH}(r_+ = r_0).$$

The microscopic entropy of each CFT is the same as S_{BH}.

NHEG limit under ultra-spinning limit



Two different order limits for a general rotating AdS black hole (R-AdS BH). Horizontal arrows (blue) represent the near horizon (NH) limit. Also the vertical ones (red) show the ultra-spinning (US) limit.

We show that in both paths the resulting limit (US-KCFT) are exactly the same. Namely the NHEG and US limits commute with each other.

Ultra-spinning BHs at EVH limit

Extremal Vanishing Horizon (EVH) BHs

- A particular class of black holes with vanishing T and A_H, but with $A_H/T = fixed$.
- Examples of EVH black holes/rings (stationary and static)
  - massless BTZ
  - 5d Kerr with one vanishing angualr momentum
  - two and three-charges 5d $U(1^3)$ gauged supergravity
  - ...
- Near horizon geometry of any EVH black hole has a pinching AdS_3 throat. [*Sheikh-jabbari et al(2011)*]
- One can define Near EVH black holes.
- Near horizon limit of Near EVH black hole contains a (pinching) BTZ geometry. [*Sheikh-jabbari et al(2015)*]

Ultra-spinning black holes at EVH limit!

- ARF ultra-spinning BHs at EVH limit
- EVH BHs at ARF ultra-spinning limit

We consider general multi-spinning Kerr-AdS black holes. The temperature and entropy

$$T = \frac{1}{2\pi}\left[r_+\left(\frac{r_+^2}{l^2}+1\right)\sum_{i=1}^{N}\frac{1}{a_i^2+r_+^2}-\frac{1}{r_+}\left(\frac{1}{2}-\frac{r_+^2}{2l^2}\right)^{\delta}\right],$$

$$S = \frac{\mathcal{A}_{d-2}}{4r_+^{1-\delta}}\prod_{i=1}^{N}\frac{a_i^2+r_+^2}{\Xi_i},$$

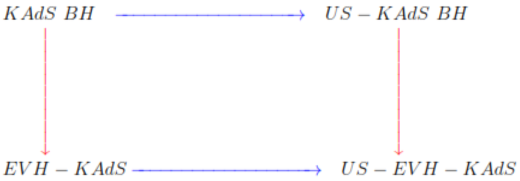
EVH limit: $A, T_H \sim \epsilon \to 0, \qquad \frac{T_H}{A} = fixed.$

We find an EVH rotating solution in odd dimensions by imposing the limits

$$a_p = r_+ = 0, \quad a_i \neq 0, \quad (i = 1 \dots N = [(d-1)/2],\ i \neq p).$$

and an important constraint between the parameters of the solution

$$m = \prod_{i\neq p}^{N} a_i^2$$



Two different order limits: Horizontal arrows (blue) represent the ARF ultra-spinning (US) limit. The vertical ones (red) show the EVH limit .

We show that in both paths the resulting limit (US-EVH-KAdS) are exactly the same.

- The ultra-spinning direction : $\varphi_j = \phi_j/\Xi_j, \quad a_j \to l$
- The EVH limit impose : $a_p, r_+ \to 0, \quad p \neq j.$
- The Near horizon limit of EVH limit also commute with ultra-spinning limit.
- The near horizon of ultra-spinning BHs at EVH limit (EVH limit of ultra-spinning BHs) contains an AdS_3.

- Performing HM limit onto EVH 5d Meyrs-Perry BH.
- EVH black holes require $r_+ = 0$, and $ab = 0$. We choose $b = r_+ = \epsilon^\alpha = 0$, and $a \neq 0$.
- For the obtained EVH metric we study the effect of Hyperboloid membrane limit in the ϕ_1 direction. We need to the following scaling

$$\sin\theta = \sqrt{\Xi_a}\sinh\frac{\sigma}{2}$$

then we take $a \to \ell$, The resulting geometry is

$$\begin{aligned} ds^2 &= \Big[\big(dt - \ell\sinh^2(\sigma/2)\,d\phi_1\big)^2 + \frac{1+\cosh(\sigma/2)}{2}\sinh^2(\sigma/2)d\phi_1^2 + \\ &+ \frac{r^2}{l^2+r^2}d\phi_2^2\Big]\Big[\frac{l^2(l^2+r^2)}{r^2(2l^2+r^2)}dr^2 + \hat{\rho}^2\cosh^2(\sigma/2)\frac{d\sigma^2}{4}\Big] \end{aligned}$$

- The conformal boundary is a flat spacetime.
- Angular velocities on the horizon for both direction are vanishing, but boundary rotates in ϕ_1 direction with the speed of light.

- The Near horizon limit of EVH-HM solution is also contains an AdS_3.

$$
\begin{aligned}
ds^2 &= \hat{H}_0^{2/3}\Big[\frac{\ell^4}{2(\ell^2+q)}\big(-\hat{r}^2 d\hat{t}^2+\frac{d\hat{r}^2}{\hat{r}^2}+\hat{r}^2\,d\phi_2^2\big)+\frac{(\ell^2+q)^2}{4\hat{H}_0^2\ell^2}\,\sinh^2\sigma d\phi_1^2 \\
&+ \hat{\rho}^2\cosh^2(\hat{\sigma}/2)\frac{d\hat{\sigma}^2}{4}\Big],
\end{aligned}
$$

which contains a pinching AdS_3 throat.

The hyperboloid membrane limit and EVH limit do not commute with each other.

Concluding remarks

- By employing two different ultra-spinning techniques we generate new classes of charged rotating BH solutions.
- Obtained geometries in both cases possess a non-compact horizon.
- In the hyperboloid membrane limit case the area is not finite, one can define an entropy density.
- In the super-entropic case the noncompact horizon has a finite area.
- It is compatible to apply both different ultra-spinning limits simultaneously.
- In both US techniques the extremality conditions commute with the ultra-spinning limit.

- NHEG of all new BHs (both cases), possess an AdS_2 sector.

- We show that ARF ultra-spinning BHs have well-defined Kerr/CFT correspondence description.
- One can construct EVH solutions from each of ultra-spinning BHs. Their NHEG contains an AdS_3 sector.
- EVH limit and ARF ultra-spinning limits commute with each other, but HM and EVH limits are not commutative.

- Thermodynamics of hyperboloid black membrane.
- More on EVH solutions from HM and mixing HM+US limits
- Study on Kerr/CFT duality for HM black hole solutions

Thank you for your attention