On Complexity Growth for $F(R)$ and Critical Gravity

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Based on:

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Outline

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Black holes are nature's extreme memory!

Holographic bound implies amount of information can be stored in a region is limited by area of the region $S \leq \frac{A}{4G}$.
Black holes saturate this bound $S = \frac{A}{4G}$

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For a system with N dof., the scrambling time t_* is a measure of how long it takes for information about a small $\mathcal{O}(1)$ perturbation to spread over $\mathcal{O}(N)$ dof.

Typical system: $t_* \sim N^{\frac{1}{d}}$ Black holes: $t_* = \frac{1}{2\pi T} \log S \sim \log N$

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Are black holes also the fastest computer?

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- Is there any bound on the computational speed?

Lloyd's Conjecture

$$\textbf{max. rate of computation} \leq E$$

Computation and Complexity

- How difficult is it to do a task?
- How difficult is it to prepare a particular state?

Computational Complexity

Minimum number of quantum gates (operations) required to prepare the desired state.

- It grows linearly with time even after thermalization!
- It is bounded by Lloyd,s bound: $\frac{d\mathcal{C}(|\psi(t)>)}{dt} \leq \frac{2Energy}{\pi\hbar}$
  [Brown, Roberts,Susskind, Swingle,Zhao'15]

Holographic Complexity

- There are (at least) two proposals for holographic complexity:

Complexity=Volume (CV) [Standford, Susskind'14]

$$\text{Complexity} \sim \frac{\text{Volume}}{G\ell}$$

Complexity=Action (CA) [Brown, Roberts,Susskind, Swingle,Zhao'15]

$$\text{Complexity} \sim \frac{\text{Action}}{\pi\hbar}$$

See also complexity of reduced state [Alishahiha'15] and complexity=volume version 2.0 [Couch,Fischler,Nguyen'17]

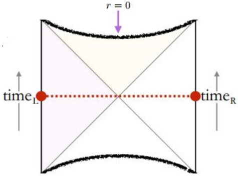
Complexity=Volume (CV) [Susskind, Stanford'14]

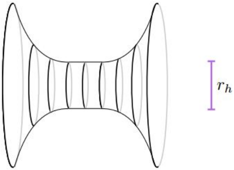
CV- duality

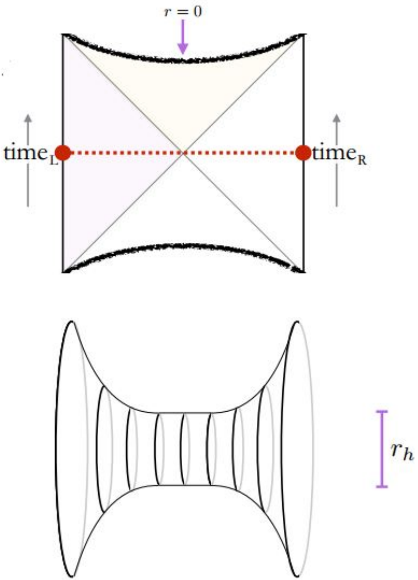
The complexity of the boundary state is proportional to the spatial volume V of a maximal slice behind the horizon $\mathcal{C} \sim \frac{\text{Volume}}{G\ell}$.

ℓ is unfixed length scale that has to be chosen appropriately. It is typically AdS radius of horizon radius.

- It is a criterion of wormhole (Einstein-Rosen bridge) size.
- It grows linearly with time after thermalization: $\mathcal{C}_V \sim TS(t_L + t_R)$
- It passes several tests.
- It has two unpleasant features:
    - Arbitrary scale ℓ
    - Why should the maximal slice play a preferred role?







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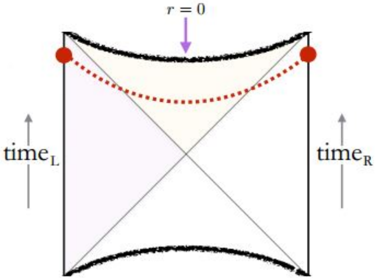

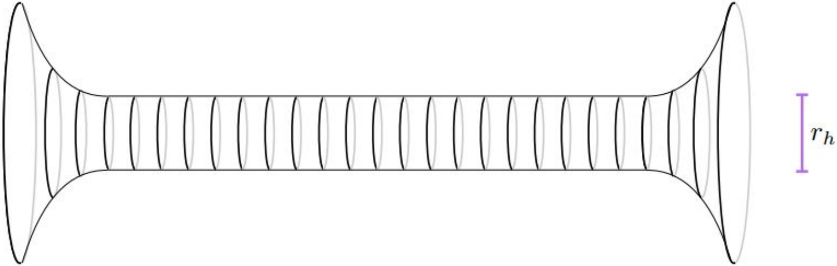

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Complexity=Action (CA) [Brown, Roberts,Susskind, Swingle,Zhao'15]

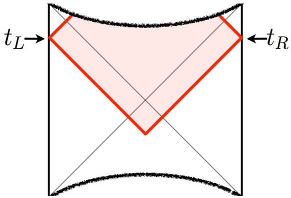
CA- duality

The complexity of the boundary state is equal to the classical action of Wheeler-DeWitt patch.

Wheeler-DeWitt (WDW) patch is defined as the bulk domain of dependence of a Cauchy slice anchored at the boundary state.

$$\mathcal{C}(|\psi(t_R, t, L)\rangle) = \frac{\text{Action}}{\pi\hbar}$$

$$\frac{d}{dt}\mathcal{C}(|\psi(t_R, t_L)\rangle) \geq \frac{2E}{\pi\hbar}$$



Outline

$F(R)$ gravity

$$I = \frac{1}{16\pi G}\int_{\mathcal{M}} d^{d+2}x\sqrt{-g}\, F(R) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^{d+1}x\, \sqrt{-\gamma}\, F'(R)K,$$

where $F'(R) = \frac{\partial F(R)}{\partial R}$.

$$F'(R)R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} - (\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)F'(R) = 0.$$

- AdS-Schwarzschild is a solution to this equation.

$$ds^2 = -fdt^2 + f^{-1}dr^2 + r^2 d\Omega_d^2, \quad f(r) = 1 + \frac{r^2}{\ell^2} + \frac{M_0}{r^{d-1}}$$

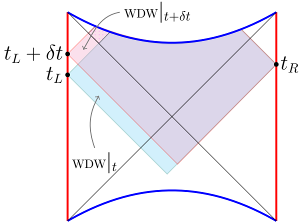
ADM mass of this solution is given as

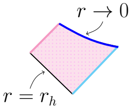
$$\mathcal{M} = \frac{\Omega_d d}{16\pi G\ell^2}(\ell^2 + r_h^2)F'(R)r_h^{d-1}$$

Complexity/Action Growth in $F(R)$

- To evaluate the on-shell action on the WDW patch, besides the generalized Gibbons-Hawking terms, one also needs boundary terms for null boundaries and intersections.
- In general it is very hard to find them. However, they are known for Einstein gravity [Parattu,Chakraborty, Majhi Padmanabhan], [Lehner, Myers ,Poisson ,Sorkin'16].
- As far as the late time behavior of the complexity growth is concerned one my circumvent this challenge by following the approach considered in [Complexity=Action (CA)[Brown, Roberts,Susskind, Swingle,Zhao'15]].
- It matches with more rigorous calculation based on all boundary terms (it has been shown for Einstein theory [Lehner, Myers ,Poisson ,Sorkin'16])
- I will calculate the rate of increase of action of a WDW patch of the two-sided black hole (which could be dual to the rate of growth of complexity of the boundary state).

Complexity/Action Growth in $F(R)$ cont.





- **Bulk term**

$$\begin{aligned}\delta I_{\mathcal{M}} &= I_{\mathcal{M}}\left[\mathrm{WDW}|_{t+\delta t}\right] - I_{\mathcal{M}}\left[\mathrm{WDW}|_t\right] \\ &= \frac{1}{16\pi G}\int_{r\to 0}^{r_h}\int_t^{t+\delta t}\int_{S^d} \mathcal{L}_{F(R)}\, d\Omega_d\, dt\, dr \qquad \text{(at late time)}\end{aligned}$$

- **Boundary term**

$$\delta I_{\partial\mathcal{M}} = \frac{1}{8\pi G}\int_t^{t+\delta t}\int_{\Omega^d} dt\, d\Omega_d\, KF'(R)\Big|_{r\to 0}^{r_h}$$

Complexity/Action Growth in $F(R)$ cont.

- Bulk Term

$$\delta I_{\mathcal{M}} = \frac{\Omega_d F(R)}{16\pi G(d+1)} r_h^{d+1}\,\delta t$$

- Boundary Term

$$\delta I_{\partial\mathcal{M}} = -\frac{\Omega_d F(R)}{16\pi G(d+1)}((d+1)r_h^2 + d\,\ell^2)r_h^{d-1}\delta t.$$

- ADM mass

$$\mathcal{M} = \frac{\Omega_d d}{16\pi G\ell^2}(\ell^2 + r_h^2)F'(R)r_h^{d-1}$$

- **Bulk + Boundary**

$$\delta(I_{\mathcal{M}} + I_{\partial\mathcal{M}}) = 2\mathcal{M}\delta t$$

Complexity growth

It saturates complexity growth bound $\dot{\mathcal{C}} = \frac{\delta I}{\delta t} = 2\mathcal{M}$.

Critical Gravity: Bulk Action

$$I_{\mathcal{M}} = \frac{1}{16\pi G} \int_{\mathcal{M}} d^{d+2}x \sqrt{-g} \left[R - 2\Lambda - \frac{1}{m^2} \left(R^{\mu\nu} R_{\mu\nu} - \frac{d+2}{4(d+1)} R^2 \right) \right],$$

- m is a dimensionful parameter.

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- It admits AdS and AdS black holes with radius ℓ.

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- m is a dimensionful parameter.
- It admits AdS and AdS black holes with radius ℓ.
- It is known that at the critical point $m^2 = \frac{d^2}{2\ell^2}$ the model degenerates yielding to a log-gravity [Alishahiha,Fareghbal'11]

Critical Gravity: Boundary Term

bulk term

$$I_{\mathcal{M}} = \frac{1}{16\pi G}\int_{\mathcal{M}} d^{d+2}x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{d+2}{4(d+1)}R^2\right)\right],$$

boundary term [Hohm,Tonni'10]

$$I_{\partial\mathcal{M}} = \frac{1}{16\pi G}\int_{\partial\mathcal{M}} d^{d+1}x\sqrt{-\gamma}\left(-2K - \hat{f}^{ij}K_{ij} + \hat{f}K\right).$$

where

$$\hat{f}^{ij} = f^{ij} + 2h^{(i}N^{j)} + sN^iN^j,$$

$$f^{\mu\nu} = \begin{pmatrix} s & h^i \\ h^i & f^{ij} \end{pmatrix}. \quad f_{\mu\nu} = -\frac{2}{m^2}\left(R_{\mu\nu} - \frac{1}{2(d+1)}Rg_{\mu\nu}\right).$$

and N^i is shift function in ADM decomposition.

Critical Gravity: Static Solution

- AdS-Schwarzschild is a solution to this theory.

$$ds^2 = -fdt^2 + f^{-1}dr^2 + r^2 d\Omega_d^2, \quad f(r) = 1 + \frac{r^2}{\ell^2} + \frac{16\pi G}{\Omega_d d}\frac{M_0}{r^{d-1}}$$

$$\frac{2\Lambda}{d(d+1)}\ell^4 + \ell^2 - \frac{d(d-2)}{4m^2} = 0.$$

- The ADM mass/energy of this solution is

$$\mathcal{M} = M_0(1 - \frac{d^2}{2\ell^2 m^2})$$

Complexity growth in critical gravity: Static

- bulk term

$$\delta I_{\mathcal{M}} = -\frac{\Omega_d r_h^{d+1}}{8\pi G \ell^2}\left(1 - \frac{d^2}{2\ell^2 m^2}\right)\delta t$$

- boundary term term

$$\delta I_{\partial\mathcal{M}} = \frac{\Omega_d}{8\pi G}\left(d\, r_h^{d-1} + \frac{(d+1) r_h^{d+1}}{\ell^2}\right)\left(1 - \frac{d^2}{2\ell^2 m^2}\right)\delta t$$

- bulk + boundary $\delta(I_{\mathcal{M}} + I_{\partial\mathcal{M}}) = 2\mathcal{M}\delta t$

Complexity growth

It saturates complexity growth bound $\dot{\mathcal{C}} = 2\mathcal{M} = 2M_0(1 - \frac{d^2}{2\ell^2 m^2})$.

at the critical point $m^2 = \frac{d^2}{2\ell^2}$ where the model develops a log gravity, the rate of growth vanishes!

3D Critical Gravity (New Massive Gravity): Stationary

- 3d critical gravity is known as New Massive Gravity (NMG).
- BTZ balck hole is a solution to this theory

$$ds^2 = \frac{dr^2}{f(r)} - f(r)dt^2 + r^2\left(d\phi - \frac{8GJ_0}{2r^2}dt\right)^2,$$
$$f(r) = \frac{r^2}{\ell^2} - 8GM_0 + \frac{(8GJ_0)^2}{4r^2},$$

- The ADM mass and angular momentum of this solution is

$$\mathcal{M} = M_0\left(1 - \frac{1}{2\ell^2 m^2}\right), \quad \mathcal{J} = J_0\left(1 - \frac{1}{2\ell^2 m^2}\right).$$

- Note that at the critical point, $m^2 = \frac{1}{2\ell^2}$ where the model develops a log gravity, the rate of growth vanishes.

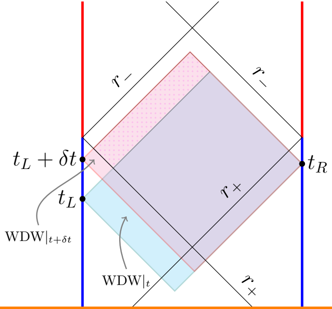
Complexity growth in NMG: Stationary (BTZ)

- bulk term

$$\begin{aligned}\delta I_{\mathcal{M}} &= I_{\mathcal{M}}\left[\mathrm{WDW}|_{t+\delta t}\right] - I_{\mathcal{M}}\left[\mathrm{WDW}|_{t}\right] \\ &= \int_{r_-}^{r_+}\int_{t}^{t+\delta t}\int_0^{2\pi} \mathcal{L}_{\mathrm{NMG}} \quad \text{(at late time)}\end{aligned}$$

- boundary term

$$\delta I_{\partial\mathcal{M}} = \int_t^{t+\delta t}\int_0^{2\pi} (\text{Boundary Term})\Big|_{r\to 0}^{r_h}$$



Complexity growth

$\dot{\mathcal{C}} = 2\sqrt{\mathcal{M}^2 - \frac{\mathcal{J}^2}{\ell^2}} < 2\mathcal{M}$. It respects to Lloyd bound.

Intuitively, the conservation of angular momentum set a barrier to rapid complexification.

Outline

Sum up

- Using "complexity=action" proposal we study complexity growth of $f(R)$ and critical gravity.
- We show action that the action growth for neutral black hole saturate the complexity growth bound.
- We also study effect of shock wave on black hole in critical gravity.
- The presence of massive spin-2 slows down the rate of growth.

Outlook

- Higher derivative terms in critical gravity may slow down complexity growth. What does it mean in dual CFT?
- Finding the boundary terms for null boundary and intersections.
- Using them one may study complexity Not complexity growth.
- What about Gauss-Bonnet and Lovelock or any theory with Riemann? They suffers from singularity at $r = 0$ where $Riem^2$ blows up!!!
- What is the role of quantum gravity near the singularity!
- Promotion CV duality for higher curvature theory. We suggest

$$C_V = -\frac{1}{\ell}\int_{\mathcal{B}} \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\alpha\beta}} \epsilon_{\mu\nu}\epsilon_{\alpha\beta} \tag{1}$$

$$C_V = -\frac{1}{\ell}\int_{\mathcal{B}} \left[\frac{\partial \mathcal{L}}{\partial R_{\mu\nu\alpha\beta}} \left(a\, n_\mu n_\beta h_{\nu\alpha} + \frac{d+5}{d(d+1)}\, h_{\mu\beta} h_{\nu\alpha}\right) + c\right] \tag{2}$$

- Lots to explore!

Thank you!