Chiral Effects in QCD Plasma

Navid Abbasi (IPM)

In Collaboration With
D. Allahbakhshi , A. Davody, F. Taghavi

0- Outline

Main goal:

Macroscopic detection of the microscopic anomalies through hydro waves

1. Rev. of relativistic hydrodynamics
2. Chiral hydrodynamics
3. Hydrodynamic excitations
4. Applications
   a. Chiral fluid above the EWPT
   b. QCD fluid
5. Outlook

1- From Short to Large Distances:

Short distances: **microscopic local QFT**

e.g.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

Large distances: **Derivative expansion of QFT**

$$O\left(\frac{E}{\Lambda}\right) \sim O\left(\frac{p}{\Lambda}\right) \sim O\left(\frac{l_{ch}}{\mathrm{L}}\right) \sim O(l_{ch}\,\partial)$$

e.g. **Low energy chiral theory in QCD**

$$U = \exp\left(\frac{i}{f}\phi^a\lambda^a\right) \qquad \mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}(U, \partial U, \partial^2 U, \ldots)$$

2- Derivative Expansion Around Thermodynamic State

In the **long wave-length limit**:

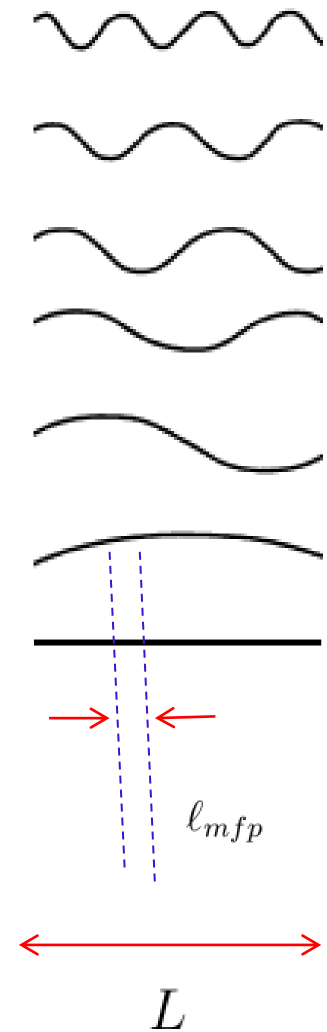
$$\boxed{\frac{\ell_{mfp}}{L} \sim \ell_{mfp}\partial \ll 1}$$

$$T^{\mu\nu}(x) = T^{\mu\nu}_{(o)} + T^{\mu\nu}_{(1)} + \dots$$

$$J^{\mu}(x) = J^{\mu}_{(o)} + J^{\mu}_{(1)} + \dots$$

Ideal Fluid

Viscous Fluid



3- Constitutive Relations

current	component
$T^{\mu\nu}$	10
J^{μ}	4

14 Unknown components

In Terms of

5 Fields and Derivatives

To first order:

$$
\begin{aligned}
T^{\mu\nu} &= (\epsilon+p)u^\mu u^\nu + p\eta^{\mu\nu} - \eta P^{\mu\alpha}P^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha\right) - \left(\zeta - \frac{2}{3}\eta\right)P^{\mu\nu}\partial.u \\
J^\mu &= nu^\mu - \sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu
\end{aligned}
$$

4- Hydrodynamic Equations

Conservation eqs:

$$\partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu$$

$$\partial_\mu J^\mu = 0$$

hydro fields:

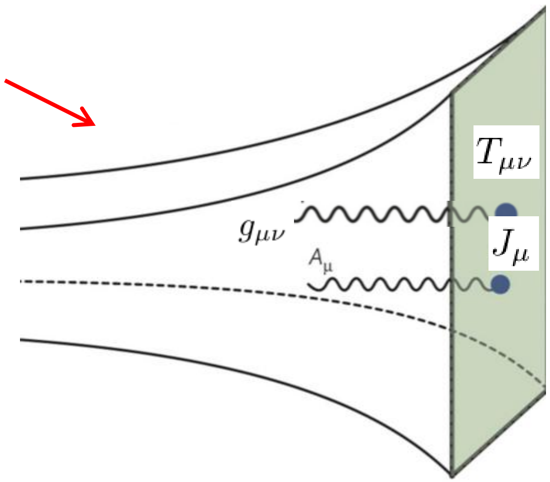
$$T(x),\ \mu(x),\ u^\mu(x)$$

5- Gauge-Gravity Duality

Maldacena, 97

AdS5

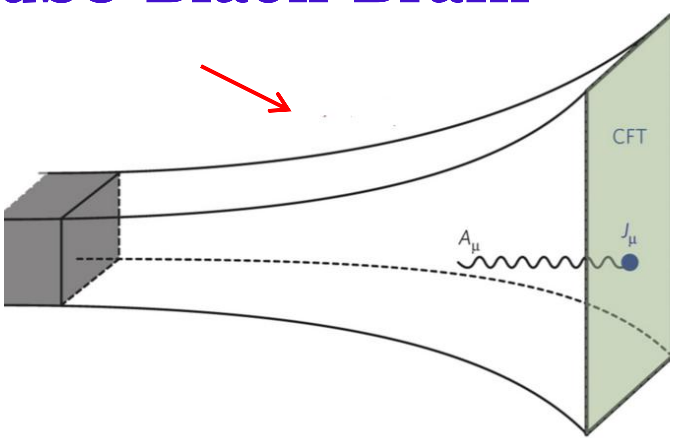
CFT4



$$ds^2 = \frac{dr^2}{r^2} + r^2 \left(dx^\mu dx_\mu \right)$$

AdS5 Black-Brain

Thermal CFT



$$ds^2 = \frac{dr^2}{r^2 f(br)} - r^2 f(br) dt^2 + r^2 d\vec{x}^2$$

$$f(r) = 1 - \frac{1}{r^4}, \qquad b = \frac{1}{\pi T}$$

6- Fluid-Gravity Duality

Hubeny, Minwalla, Rangamani, JHEP,2007

Metric:

$$ds^2 = -2\,u_\mu\,dx^\mu dr - r^2 f(b\,r)\,u_\mu u_\nu\,dx^\mu dx^\nu + r^2\,P_{\mu\nu}\,dx^\mu dx^\nu$$
$$+\,2\,r^2\,b\,F(b\,r)\,\sigma_{\mu\nu}\,dx^\mu dx^\nu + \frac{2}{3}\,r\,u_\mu u_\nu\,\partial_\lambda u^\lambda\,dx^\mu dx^\nu - r\,u^\lambda \partial_\lambda\,(u_\nu u_\mu)\;dx^\mu dx^\nu$$

Constraint Eqs:

$$\partial_\mu T^{\mu\nu} = 0 \quad \longleftrightarrow \quad T^{\mu\nu} = \frac{1}{b^4}\,(4\,u^\mu u^\nu + \eta^{\mu\nu})$$

Energy-Momentum Tensor on the boundary

$$T^\mu_\nu = -2\lim_{r\to\infty} r^4\,(K^\mu_\nu - \delta^\mu_\nu) \quad \Rightarrow \quad T^{\mu\nu} = \frac{1}{b^4}\,(4\,u^\mu u^\nu + \eta^{\mu\nu}) - \frac{2}{b^3}\,\sigma^{\mu\nu}$$

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

7- Generalizations

1. Forced Fluid — MINWALLA, ET. AL JHEP,2009
2. Non-Relativistic Fluid — MINWALLA, ET. AL JHEP,2009
3. **(Chirally)Charged Fluid** — BHATTACHARYYA, ET.AL JHEP 2009

$$S = \frac{1}{16\pi G_5}\int \sqrt{-g_5}\left[R + 12 - F_{AB}F^{AB} - \underbrace{\frac{4\kappa}{3}\epsilon^{LABCD}A_L F_{AB}F_{CD}}_{\text{ANOMALY}}\right]$$

$$T_{\mu\nu} = p(\eta_{\mu\nu} + 4u_\mu u_\nu) - 2\eta\sigma_{\mu\nu} + \ldots$$

$$J_\mu = n\, u_\mu - \mathfrak{D}\, P^\nu_\mu \mathcal{D}_\nu n + \xi\, l_\mu + \ldots$$

$$l^\mu \equiv \epsilon^{\nu\lambda\sigma\mu}u_\nu \partial_\lambda u_\sigma$$

vorticity

8- Macroscopic Manifestation of Quantum Anomalies

Motivated by Fluid/Gravity:

In the presence of anomalies one may add parity odd terms:

$$J^\mu \;=\; nu^\mu - \sigma T P^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right) + \sigma E^\mu \boxed{+\,\xi\omega^\mu + \xi_B B^\mu}$$

$$\partial_\mu j^\mu = C E^\mu B_\mu,$$
$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

CONTRADICTION WITH SECOND LAW!

9- Symmetry Considerations

Time Reversal Sym.

Ohm Law: $\boldsymbol{J} = \sigma \boldsymbol{E}$ ➡ **dissipative**

(-) (-)(+)

London 2nd eq.: $\dfrac{\partial \boldsymbol{J}_s}{\partial t} = \sigma_s \boldsymbol{E}$ ➡ **non dissipative**

(+) (+)(+)

Anomalous transport: $\boldsymbol{J} = \xi_B \boldsymbol{B}$ ➡ **Guess What?**

(-) (+)(-)

10- Equality Constraints; "for the First Time"

Constraints on **dissipataive** coefficients:

$$\boxed{\eta \geq 0, \quad \sigma \geq 0}$$

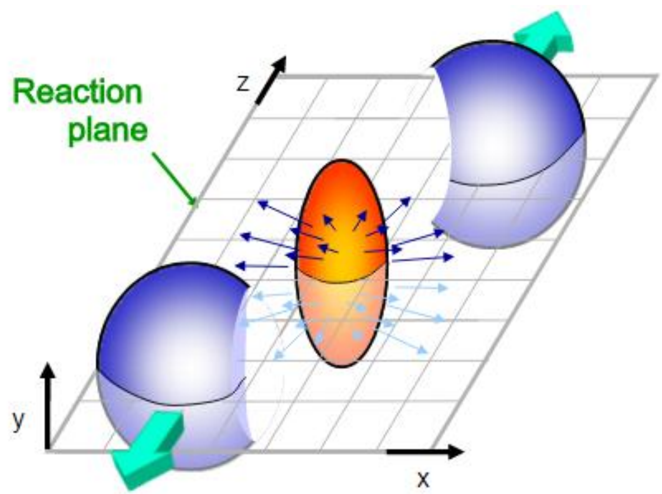
Constarints on **anomalous non-dissipative** transport coefficients:

$$\xi = \mathcal{C}\mu^2\left(1 - \frac{2}{3}\frac{\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right) + \mathcal{D}T^2\left(1 - \frac{2\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right)$$

$$\xi_B = \mathcal{C}\mu\left(1 - \frac{1}{2}\frac{\bar{n}\mu}{\bar{\epsilon}+\bar{p}}\right) - \frac{\mathcal{D}}{2}\frac{\bar{n}T^2}{\bar{\epsilon}+\bar{p}}$$

Son, Surowka(Phys.Rev.Lett. 103 (2009))
Kharzeev, Yee, Phys.Rev. D84 (2011)
Neiman, Oz (JHEP (2011))

11- Quark-Gluon-Plasma Experiment



Steffan Bass: Probing the QGP at RHIC

1- Initial State

2- Hydrodynamic Evolution

3- Particle Spectrum in detectors

12- Axial & Vector Currents in QGP:

Microscopic:

vector current: $J^\mu = \bar{\psi}\gamma^\mu\psi$

axial current: $J_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} J_\lambda$$
$$\partial_\mu J^\mu = 0$$
$$\partial_\mu J_5^\mu = \mathcal{C} E_\mu B^\mu$$

axial anoamly

Macroscopic: **(QCD Plasma)**

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu + p\,\eta^{\mu\nu}$$
$$J^\mu = n u^\mu + \xi\,\omega^\mu + \xi_B B^\mu$$
$$J_5^\mu = n_5 u^\mu + \xi_5\,\omega^\mu + \xi_{B5} B^\mu.$$

13- CMW in QGP:

CME: $$\delta \boldsymbol{j} = \frac{\boldsymbol{B}}{2\pi^2\chi}\delta n_5$$

CSE: $$\delta \boldsymbol{j_5} = \frac{\boldsymbol{B}}{2\pi^2\chi}\delta n$$

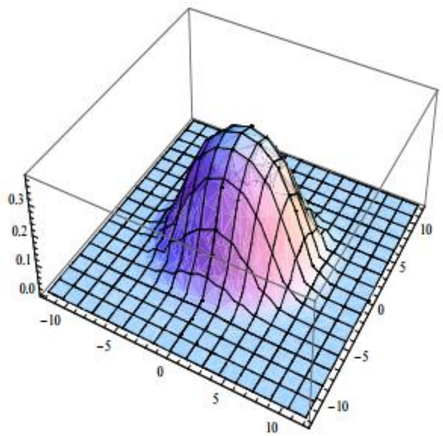
Chiral magnetic wave:
$$(\partial_t^2 - v_{\rm CMW}^2 \delta n(x)_V(x) = 0$$
$$(\partial_t^2 - v_{\rm CM}^2 \delta n_5(x)\,{}_1(x) = 0$$

$$v_{\rm CMW} = \frac{B}{2\pi^2\chi}$$

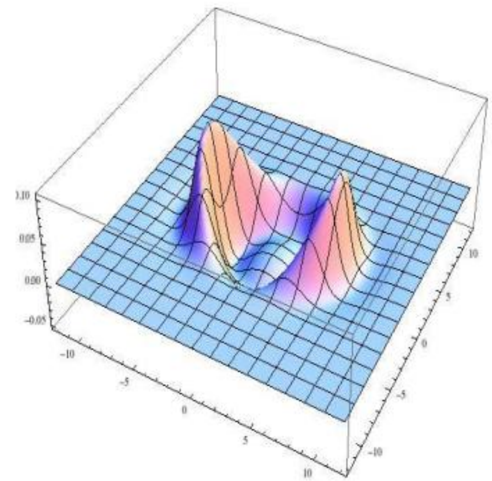
Kharzeev, Yee (2011)

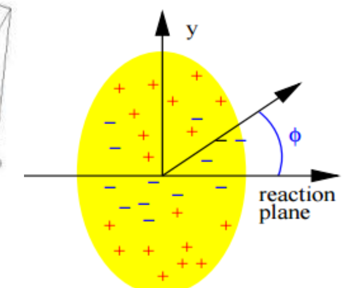
14- From CMW to the charge dependence of v2

Burnier, Kharzeev, Liao, Yee, (2012)



CMW evolution





$$A = \frac{\bar{N}_+ - \bar{N}_-}{\bar{N}_+ + \bar{N}_-}$$

$$\frac{dN_\pm}{d\phi} = \bar{N}_\pm \left[1 + (2v_2 \mp r_e\, A)\cos(2\phi)\right]$$

$$\boxed{v_2^- - v_2^+ = r_e\, A}$$

15- Signature in Experiment

PRL 114, 252302 (2015) PHYSICAL REVIEW LETTERS week ending 26 JUNE 2015

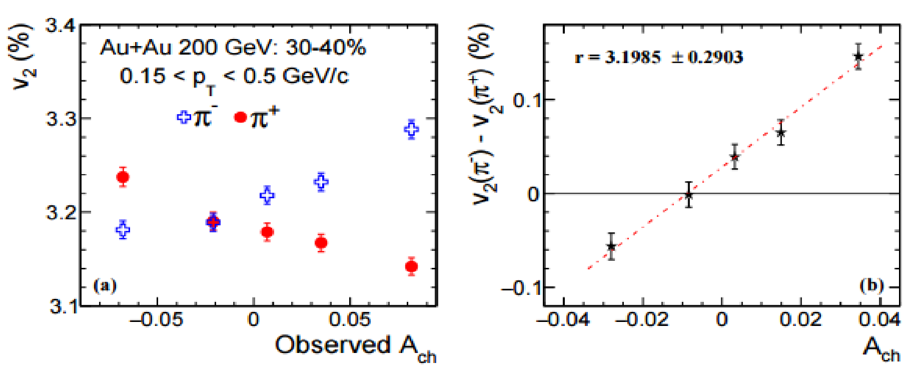
Observation of Charge Asymmetry Dependence of Pion Elliptic Flow and the Possible Chiral Magnetic Wave in Heavy-Ion Collisions

L. Adamczyk,[1] J. K. Adkins,[20] G. Agakishiev,[18] M. M. Aggarwal,[30] Z. Ahammed,[47] I. Alekseev,[16] J. Alford,[19] A. Aparin,[18] D. Arkhipkin,[3] E. C. Aschenauer,[3] G. S. Averichev,[18] A. Banerjee,[47] R. Bellwied,[43] A. Bhasin,[17] A. K. Bhati,[30] P. Bhattarai,[42] J. Bielcik,[10] J. Bielcikova,[11] L. C. Bland,[3] I. G. Bordyuzhin,[16] J. Bouchet,[19] A. V. Brandin,[26] I. Bunzarov,[18]

…

(STAR Collaboration)

We present measurements of π^- and π^+ elliptic flow, v_2, at midrapidity in Au + Au collisions at $\sqrt{s_{NN}} = 200$, 62.4, 39, 27, 19.6, 11.5, and 7.7 GeV, as a function of event-by-event charge asymmetry, A_{ch}, based on data from the STAR experiment at RHIC. We find that π^- (π^+) elliptic flow linearly increases (decreases) with charge asymmetry for most centrality bins at $\sqrt{s_{NN}} = 27$ GeV and higher. At $\sqrt{s_{NN}} = 200$ GeV, the slope of the difference of v_2 between π^- and π^+ as a function of A_{ch} exhibits a centrality dependence, which is qualitatively similar to calculations that incorporate a chiral magnetic wave effect. Similar centrality dependence is also observed at lower energies.



16- Full Hydro Computations

N.A,, D,AllahBakhshi, A.Davody, F.Taghavi (2016)

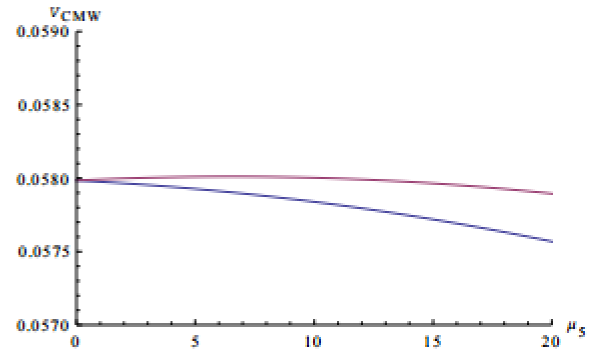
$$
\begin{bmatrix}
-i\alpha_1\omega & ik_j & -i\alpha_2\omega & -i\alpha_3\omega \\
i\alpha_1 v_s^2 k^i & \begin{array}{c} -i\omega\delta^i_j - \epsilon^i{}_{jl}\Omega^l - \frac{\bar{n}}{\bar{w}}\epsilon^i{}_{jl}B^l \\ -i\frac{\xi}{2\bar{w}}\left(\boldsymbol{B}\cdot\boldsymbol{k}\delta^i_j - B_j k^i\right) \end{array} & \begin{array}{c} i\alpha_2 v_s^2 k^i \\ +\left(\frac{\partial\xi}{\partial\mu}\right)\underline{(\boldsymbol{B}\times\boldsymbol{\Omega})^i} \end{array} & \begin{array}{c} i\alpha_3 v_s^2 k^i \\ +\left(\frac{\partial\xi}{\partial\mu_5}\right)\underline{(\boldsymbol{B}\times\boldsymbol{\Omega})^i} \end{array} \\
\begin{array}{c} -i\beta_1\omega + \left(\frac{\partial\xi}{\partial T}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_B}{\partial T}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \end{array} & \begin{array}{c} \frac{\bar{n}}{\bar{w}} ik_j - \frac{2\xi}{\bar{w}} i\omega\Omega_j \\ -\frac{\xi_B}{\bar{w}} i\omega B_j - \frac{\xi_B}{\bar{w}}\underline{(\boldsymbol{B}\times\boldsymbol{\Omega})_j} \end{array} & \begin{array}{c} -i\beta_2\omega + \left(\frac{\partial\xi}{\partial\mu}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_B}{\partial\mu}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \end{array} & \begin{array}{c} -i\beta_3\omega + \left(\frac{\partial\xi}{\partial\mu_5}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_B}{\partial\mu_5}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \end{array} \\
\begin{array}{c} -i\gamma_1\omega + \left(\frac{\partial\xi_5}{\partial T}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_{5B}}{\partial T}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \end{array} & \begin{array}{c} \frac{\bar{n}_5}{\bar{w}} ik_j - \frac{2\xi_5}{\bar{w}} i\omega\Omega_j \\ -\frac{\xi_{5B}}{\bar{w}} i\omega B_j - \frac{\xi_{5B}}{\bar{w}}\underline{(\boldsymbol{B}\times\boldsymbol{\Omega})_j} \end{array} & \begin{array}{c} -i\gamma_2\omega + \left(\frac{\partial\xi_5}{\partial\mu}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_{5B}}{\partial\mu}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \end{array} & \begin{array}{c} -i\gamma_3\omega + \left(\frac{\partial\xi_5}{\partial\mu_5}\right) i\boldsymbol{\Omega}\cdot\boldsymbol{k} \\ +\left(\frac{\partial\xi_{5B}}{\partial\mu_5}\right) i\boldsymbol{B}\cdot\boldsymbol{k} \end{array}
\end{bmatrix}
\begin{bmatrix} \delta T \\ \delta\pi_i \\ \delta\mu \\ \delta\mu_5 \end{bmatrix} = 0
$$

two sectors: **1) scalar** **2) scalar-vector**

17- QCD Fluid Coupled to Magnetic Field

Hydro modes

QGP



	$\boldsymbol{B} \parallel \boldsymbol{k}$	$\boldsymbol{B} \perp \boldsymbol{k}$
Chiral Magnetic Wave	$v_{1,2} = -\frac{\mathcal{A}_1 \pm \sqrt{\mathcal{A}_1^2 - \alpha_1 \mathcal{C}^2 \mathcal{E}}}{\mathcal{E}} B$	$-$
Magnetosonic Wave	$v_{3,4} = \pm c_s$	$v_{3,4} = \pm \frac{c_s^2 k}{\sqrt{c_s^2 k^2 + \Omega_L^2}}$
Chiral Alfvén Wave	$v_{5,6} = -\frac{\xi}{w} B$	$-$

$$v_{CAW}^{\parallel} = -\left\{\mathcal{C}\left(\mu\mu_5 - \frac{n\mu_5}{3w}\left(3\mu^2 + \mu_5^2\right)\right) - \mathcal{D}\,\frac{n\mu_5}{w} T^2\right\} \frac{B}{w}$$

18- Chiral Kinetic Theory: CKT

STEPANOV, YIN, 2012

Chiral particles in kinetic theory!

$$I = \int_{t_i}^{t_f} (\boldsymbol{p}\cdot\dot{\boldsymbol{x}} + \boldsymbol{A}\cdot\dot{\boldsymbol{x}} - \Phi - |\boldsymbol{p}| - \boldsymbol{a}_{\boldsymbol{p}}\cdot\dot{\boldsymbol{p}})dt$$

$$|\hat{\boldsymbol{a}}_p| = \frac{1}{p} \qquad \boldsymbol{b} = \boldsymbol{\nabla}_{\boldsymbol{p}} \times \boldsymbol{a}_{\boldsymbol{p}}$$

Berry Curvature → **Berry Flux**

Chiral Magnetic Effect:

$$\boldsymbol{j} = \int_{\boldsymbol{p}} \sqrt{G} f \dot{\boldsymbol{x}} = \int_{\boldsymbol{p}} f\hat{\boldsymbol{p}} + \boldsymbol{E}\times\int_{\boldsymbol{p}} f\boldsymbol{b} + \underline{\boldsymbol{B}\int_{\boldsymbol{p}} f(\hat{\boldsymbol{p}}\cdot\boldsymbol{b})}$$

$$\boxed{\boldsymbol{j}_{\mathrm{CME}} = \mu\boldsymbol{B}/(2\pi)^2}$$

19- CME Out of Equilibrium

arXiv: 1703.08856 Liao, etall

EoM:

$$\dot{\vec{\mathbf{x}}} = \hat{\mathbf{p}} + \dot{\vec{\mathbf{p}}} \times \mathbf{b}\ , \ \ \dot{\vec{\mathbf{p}}} = q_i\, \dot{\vec{\mathbf{x}}} \times \vec{\mathbf{B}}$$

$$z = z_0 + \frac{p_{z0}}{p}\int_{t_0}^{t} \frac{1}{\sqrt{G}} \cos\alpha dt' + \frac{p_{x0}}{p}\int_{t_0}^{t} \frac{1}{\sqrt{G}} \sin\alpha dt',$$

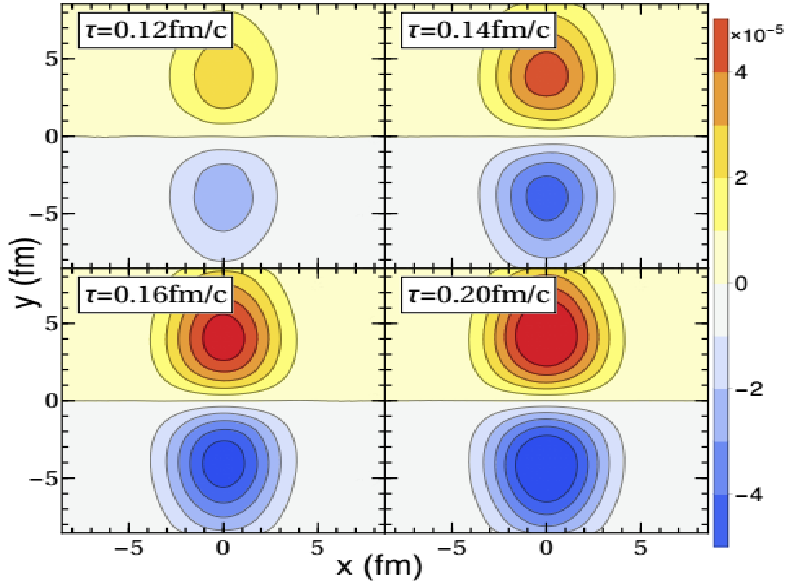
$$x = x_0 - \frac{p_{z0}}{p}\int_{t_0}^{t} \frac{1}{\sqrt{G}} \sin\alpha dt' + \frac{p_{x0}}{p}\int_{t_0}^{t} \frac{1}{\sqrt{G}} \cos\alpha dt',$$

$$y = y_0 + \frac{p_{y0}}{p}\int_{t_0}^{t} \frac{1}{\sqrt{G}} dt' + \frac{\chi}{2p}\int_{t_0}^{t} \frac{q_i B(t)}{p\sqrt{G}} dt'.$$

Solving Boltzmann Eq:

$$f_i(\vec{\mathbf{x}}, \vec{\mathbf{p}}, t) = f_{i\,0}\Big(\vec{\mathbf{x}}_0(\vec{\mathbf{x}}, \vec{\mathbf{p}}, t), \vec{\mathbf{p}}_0(\vec{\mathbf{x}}, \vec{\mathbf{p}}, t)\Big)$$

Initial Condition For Hydro



20- Mixed CMVW in Scalar Sector

At $\mu_5 = 0$ **:**

Kharzeev-Yee

$$v_{CMVW} = \pm\left(v_{CVW} + v_{CMW}\right)$$

$$v_{CMW} = \frac{\mathcal{C}\,B}{\chi}\frac{\left(1 - \frac{n\mu}{w}\right)}{\mathcal{K}},$$

$$v_{CVW} = \frac{\mathcal{C}\,\mu\,\Omega}{\chi}\frac{\left(1 - \frac{n\mu}{w} - \frac{\mathcal{D}}{\mathcal{C}}\frac{nT^2}{\mu w}\right)}{\mathcal{K}}$$

Jiang-Huang-Liao

$$\omega_{1,2} = \pm(v_{CMW} + v_{CVW})k$$

$$= \pm\frac{Bk}{2\pi^2\chi}\frac{1 - \frac{\mu n}{w}}{\sqrt{1 - \frac{\mu n}{w} - \frac{n}{\chi w}\left(\frac{n}{c_s^2} - \chi\mu\right)}} \pm \frac{\Omega\mu k}{2\pi^2\chi}\frac{1 - \frac{\mu n}{w} - \frac{\pi^2 nT^2}{3w}}{\sqrt{1 - \frac{\mu n}{w} - \frac{n}{\chi w}\left(\frac{n}{c_s^2} - \chi\mu\right)}}$$

Gravitational Anomaly: New observation

Thank You
For Your Attention