(A)dS$_3$ in the Near Horizon of Asymptotically dS Solutions

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in collaboration with
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Near Horizon Extremal Geometries

NHEG

- are obtained by coordinate transformation(s) and limit procedure on the black holes.
- solve the same equations as their black holes do.

Symmetries of NHEG

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  - for asymptotically AdS/flat spaces, it is AdS_2 or $\mathbb{R}^{1,1}$
  - for asymptotically dS spaces, it can be dS_2, as well.
- The example of the latter case is the near horizon of *Schwarzschild-dS* black hole which is *Nariai* solution. It is a product space: $dS_2 \times S^2$.

Near horizon Extremal Vanishing Horizon Geometries

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- Strong energy condition implies that this 3d part in the case of $\Lambda \leq 0$ is either AdS_3 or 3d flat.
  - Flat case can only occur for $\Lambda = 0$.
- For generic $\Lambda > 0$, the above analysis does not yield a restriction on the sign of 3d curvature.

First example : Kerr-(A)dS black hole

Kerr-(A)dS black hole

- solution to $R_{\mu\nu} = (d-1)\,\lambda\, g_{\mu\nu}$

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- multi-spinning with $\left[\frac{d-1}{2}\right]$ *independent* rotation
- reduces to Myers-Perry black hole in the case of $\boldsymbol{\lambda} = 0$
- reduces to Schwarzschild-(A)dS black hole in the *static* case (once all rotations are turned off)

Kerr-(A)dS black hole

- Metric in $d = 2n + 1 + \alpha$ dimensions*

$$
\begin{aligned}
ds^2 \quad = \quad & -W\,(1-\boldsymbol{\lambda}\, r^2)\, dt^2 + \frac{2\,\boldsymbol{m}}{V F}\Big(W\, dt - \sum_{i=1}^{n} \frac{\boldsymbol{a_i}}{\Xi_i}\, \mu_i^2\, d\varphi_i\Big)^2 \\
& + \frac{V F}{(V - 2\,\boldsymbol{m})}\, dr^2 + \frac{\boldsymbol{\lambda}}{W\,(1-\boldsymbol{\lambda}\, r^2)}\Big(\sum_{i=1}^{n+\alpha} \frac{r^2 + \boldsymbol{a_i}^2}{\Xi_i}\, \mu_i\, d\mu_i\Big)^2 \\
& + \sum_{i=1}^{n} \frac{r^2 + \boldsymbol{a_i}^2}{\Xi_i}\, \mu_i^2\, d\varphi_i^2 + \sum_{i=1}^{n+\alpha} \frac{r^2 + \boldsymbol{a_i}^2}{\Xi_i}\, d\mu_i^{\,2}\,.
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& + \frac{VF}{(V-2\,m)}\, dr^2 + \frac{\lambda}{W\,(1-\lambda\, r^2)}\Big(\sum_{i=1}^{n+\alpha} \frac{r^2 + {a_i}^2}{\Xi_i}\,\mu_i\, d\mu_i\Big)^2 \\
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- $V \equiv r^{\alpha-2}\,(1-\lambda\, r^2)\,\prod_i (r^2 + {a_i}^2)\,, \qquad \Xi_i \equiv 1 + \lambda\, {a_i}^2\,,$

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- $F \equiv \frac{1}{1-\lambda\, r^2}\, \sum \frac{r^2\,\mu_i^2}{r^2 + {a_i}^2}\,, \qquad W \equiv \sum \frac{\mu_i^2}{\Xi_i}\,, \qquad \sum \mu_i^2 = 1\,.$

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Kerr-(A)dS Thermodynamics

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- $M = \frac{m\ \mathcal{A}_{d-2}}{4\,\pi\, G_d\,\prod_j \Xi_j}\sum_i \left(\frac{1}{\Xi_i} + \frac{\alpha-1}{2}\right)$.

Kerr-(A)dS Thermodynamics

In odd dimensions ; $d = 2n + 1$ $(\alpha = 0)$

- $T = \frac{\kappa_H}{2\pi}$; $\qquad \kappa_H = \frac{(1 - \boldsymbol{\lambda}\, r_h^2)}{4\, \boldsymbol{m}} V'(r_h)$,

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  - $r_h\,, \boldsymbol{a_1} \to 0 \quad \Rightarrow \quad \kappa_H \sim r_h \to 0$

Near horizon geometries in the EVH limit

- EVH limit ($\epsilon \to 0$)

$$r_h = \boldsymbol{\rho_0}\,\epsilon, \qquad \boldsymbol{a_1} = \boldsymbol{a_0}\,\epsilon^2, \quad \boldsymbol{m} = \frac{1}{2}\prod_{i=2}^{n} \boldsymbol{a_i}^2 + \tilde{m}\,\epsilon^2\,,$$

$$\tilde{m} \equiv \frac{\boldsymbol{\rho_0}^2}{2}\left(\frac{\boldsymbol{a_0}^2}{\boldsymbol{\rho_0}^4} - \boldsymbol{\lambda_3}\right)\prod_{i=2}^{n} \boldsymbol{a_i}^2. \qquad \boldsymbol{\lambda_3} \equiv \boldsymbol{\lambda} - \sum_{i=2}^{n} \frac{1}{\boldsymbol{a_i}^2}\,.$$

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- $\Omega^1 \sim \frac{a_0}{{\rho_0}^2}\,, \qquad J_1 \sim a_0\, \epsilon^2\,.$
- Eliminating λ_3 $\Rightarrow$ $\tilde{m}\, \epsilon^2 \sim \left(\frac{1}{2} T S + \Omega^1_H J_1\right).$

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Parameter space

- Black hole parameter space:

$$\{m, a_1, a_2, \cdots, a_n\}$$

- Normal to EVH hyper-surface

$$\{m, a_1\} \quad \text{or} \quad \{\rho_0, a_0\}$$

- Parallel to EVH hyper-surface

$$\{a_2, \cdots, a_n\}$$

EVH as Nariai, cold or ultra-cold limit

$$V(r) - 2\,m = \sum_{p=1}^{n} c_p\, r^{2p}\,, \qquad c_1 = -2\,m\,\lambda_3\,, \qquad c_n = -\lambda\,.$$

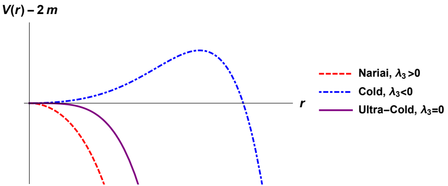

Figure: Roots of Kerr-dS black hole ($\lambda > 0$) in the EVH limit.

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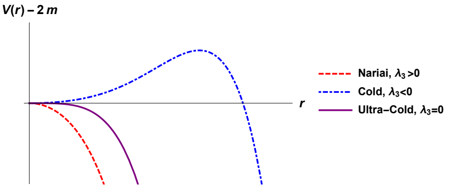

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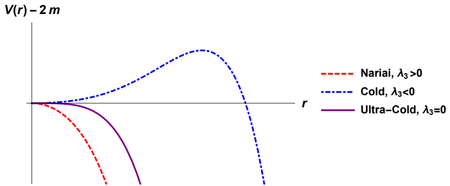

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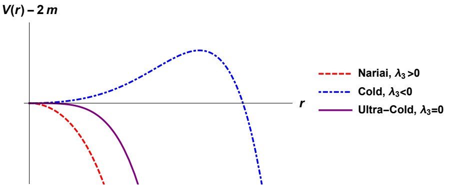

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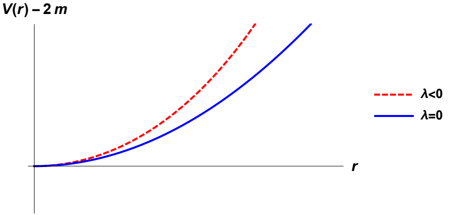

Figure: Root of Kerr black hole for $\boldsymbol{\lambda} \leq 0$ in the EVH limit. There is no horizon for $r > 0$. In these cases, $\boldsymbol{\lambda_3} < 0$.

Near horizon EVH geometry

- Near horizon limit ($\gamma \to 0$)

$$r = \gamma\, \rho + r_h\,, \quad \tau = \frac{v}{\gamma}, \quad \varphi_1 = \frac{\psi}{\gamma}, \quad \varphi_{i\geq 2} = \phi_i - \Omega_H^i\, \tau\,,$$

- In the $\epsilon \ll \gamma$ case,

$$ds^2_{NH} = \mu_1^2 \left(\boldsymbol{\lambda_3}\, \rho^2\, dv^2 + 2\, dv\, d\rho + \rho^2\, d\psi^2 \right)$$

$$+ h_{ij}(r_h)\, d\varphi^i d\varphi^j + k_{ij}(r_h)\, d\mu^i d\mu^j\,,$$

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- If $\boldsymbol{\lambda} > 0 \quad \Rightarrow \boldsymbol{\lambda_3} = \left(\boldsymbol{\lambda} - \sum \frac{1}{\boldsymbol{a_i}^2}\right) \quad \Rightarrow \quad dS_3, R^{2,1}, AdS_3$

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- If $\boldsymbol{\lambda} \leq 0 \quad \Rightarrow \boldsymbol{\lambda_3} < 0 \quad$ (for all $\boldsymbol{a_i}$'s) $\quad \Rightarrow \quad AdS_3$

Near horizon near-EVH geometry

- In the case of $\epsilon \simeq \gamma$,

$$ds^2_{NH} = \mu_1^2\, ds_3^2 + h_{ij}(r_h)\, d\varphi^i d\varphi^j + k_{ij}(r_h)\, d\mu^i d\mu^j\,,$$

$$ds_3^2 \equiv -f(\rho)\, dv^2 + 2\, d\rho\, dv + \rho^2 (d\psi - \frac{\boldsymbol{a_0}}{\rho^2}\, dv)^2\,,$$

$$f(\rho) \equiv \frac{(\rho^2 - \boldsymbol{\rho_0}^2)(-\boldsymbol{\sigma}\, \rho^2 - \frac{\boldsymbol{a_0}^2\, \boldsymbol{\ell_3}^2}{\boldsymbol{\rho_0}^2})}{\rho^2 \boldsymbol{\ell_3}^2}\,, \qquad \boldsymbol{\lambda_3} \equiv \frac{\boldsymbol{\sigma}}{\boldsymbol{\ell_3}^2}\,.$$

⋆ $dv = dt + \frac{d\rho}{f(\rho)}$, reproduces the standard BTZ form.

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- $\boldsymbol{\sigma} = -1 \quad \Rightarrow \quad \text{BTZ}^\star$

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$$f(\rho) \equiv \frac{(\rho^2 - {\rho_0}^2)(-\sigma\, \rho^2 - \frac{{a_0}^2\, {\ell_3}^2}{{\rho_0}^2})}{\rho^2 {\ell_3}^2}, \qquad \lambda_3 \equiv \frac{\sigma}{{\ell_3}^2}.$$

- $\sigma = -1 \quad \Rightarrow \quad$ BTZ*
- $\sigma = 0 \quad \Rightarrow \quad$ Flat space cosmology

* $dv = dt + \frac{d\rho}{f(\rho)}$, reproduces the standard BTZ form.

Near horizon near-EVH geometry

- In the case of $\epsilon \simeq \gamma$,

$$ds^2_{NH} = \mu_1^2\, ds_3^2 + h_{ij}(r_h)\, d\varphi^i d\varphi^j + k_{ij}(r_h)\, d\mu^i d\mu^j\,,$$

$$ds_3^2 \equiv -f(\rho)\, dv^2 + 2\, d\rho\, dv + \rho^2 (d\psi - \frac{a_0}{\rho^2}\, dv)^2\,,$$

$$f(\rho) \equiv \frac{(\rho^2 - \rho_0{}^2)(-\sigma\, \rho^2 - \frac{a_0{}^2\, \ell_3{}^2}{\rho_0{}^2})}{\rho^2 \ell_3{}^2}\,, \qquad \lambda_3 \equiv \frac{\sigma}{\ell_3{}^2}\,.$$

- $\sigma = -1 \quad \Rightarrow$ BTZ*
- $\sigma = 0 \quad \Rightarrow$ Flat space cosmology
- $\sigma = +1 \quad \Rightarrow$ Kerr-dS$_3$

* $dv = dt + \frac{d\rho}{f(\rho)}$, reproduces the standard BTZ form.

Thermodynamics of the EVH near horizon

- After reduction on $\mathcal{M}_{(d-3)}$ manifold, we get a 3d gravity,

$$G_3 = \frac{2\,\pi\, G_d}{\mathcal{A}_{d-2}} \prod_{i=2}^{n} \frac{\Xi_i}{{a_i}^2}\,.$$

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$$M_3 = \frac{a_0{}^2 \ell_3{}^2 - \sigma \rho_0{}^4}{8 G_3 \ell_3{}^2 \rho_0{}^2}\,\epsilon\,, \qquad S_3 = \frac{\pi \rho_0}{2 G_3}\,\epsilon\,, \qquad J_3 = \frac{a_0}{4 G_3}\,\epsilon\,,$$

$$T_3 = \frac{1}{2\pi \ell_3{}^2} \frac{-\sigma \rho_0{}^4 - a_0{}^2 \ell_3{}^2}{\rho_0{}^3}\,, \qquad \Omega_3 = \frac{a_0}{\rho_0{}^2}\,.$$

Thermodynamics of the EVH near horizon

- They are related to black hole quantities via,

$$T = \epsilon\, T_3\ , \quad \Omega_3 = \Omega_H^1\ , \quad S = S_3\ , \quad J_1 = \epsilon\, J_3\ ,$$

$$M_3 = \frac{\mathcal{A}_{d-2}}{8\pi\, G_d\, \prod_{i=2}^{n} \Xi_i}\, \tilde{m}\, \epsilon\,.$$

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$$\text{BH}: \qquad T\, \delta_\perp S = \delta_\perp M - \sum_i \Omega^i_H\, \delta_\perp J_i \ ,$$

$$\Downarrow$$

$$\text{NH}: \qquad T_3\, \delta_\perp S_3 = \delta_\perp M_3 - \Omega_3\, \delta_\perp J_3 \ .$$

Second example : Cosmological Soliton

Cosmological Soliton metric

The solution to $R_{\mu\nu} = \frac{d-1}{\ell^2}\, g_{\mu\nu}$ in $d = 2n+1$ dimensions*

$$ds^2 = -\,g(r)\, dt^2 + \frac{dr^2}{g(r)f(r)} + \frac{r^2}{2n}\sum_{i=1}^{n-1}\left(d\theta_i^2 + \sin^2\theta_i\, d\phi_i^2\right)$$

$$+\left(\frac{r}{n}\right)^2 f(r)\Big(d\psi + \sum_{i=1}^{n-1}\cos\theta_i\, d\phi_i\Big)^2,$$

- $g(r) = 1 - \frac{r^2}{\ell^2}$, $\quad f(r) = 1 - \left(\frac{a^2}{r^2}\right)^n$,
- θ_i and ϕ_i parametrize 2-spheres, so $\theta_i \in [0,\pi]$ and $\psi, \phi_i \in [0, 2\pi]$.

*Clarkson & Mann, Phys. Rev. Lett. 96 (2005).

Static CTC-free patch

In the case of $a < \ell$,

r	0	a	ℓ	∞
$g(r)$	$+$		$+$	$-$
f(r)	$-$		$+$	$+$
$\|\|\partial_r\|\|^2$	$-$		$+$	$-$
$\|\|\partial_t\|\|^2$	$-$		$-$	$+$
$\|\|\partial_\psi\|\|^2$	$-$		$+$	$+$

Static CTC-free region is $a \leq r \leq \ell$.

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- The first law of thermodynamics holds $\delta M = T\, \delta S$.

Extremal Vanishing Horizon limit

In the $a = \ell \left(1 - \frac{b^2}{2}\epsilon^2\right)$, as $\epsilon \to 0$, limit, horizon metric is

$$ds_H^2 \sim \frac{\ell^2\, b^2}{n}\,\epsilon^2 \left(d\psi + \sum_{i=1}^{n-1} \cos(\theta_i)\, d\phi_i\right)^2 + \frac{\ell^2}{2n} \sum_{i=1}^{n-1} d\Sigma_i^2 + \mathcal{O}\left(\epsilon^2\right) .$$

- $T = \tilde{T}\,\epsilon + \mathcal{O}(\epsilon^2)\,,$
- $S = \tilde{S}\,\epsilon + \mathcal{O}(\epsilon^2)\,,$
- $M = M^{(0)} + M^{(2)}\,\epsilon^2 + \mathcal{O}(\epsilon^4)\,.$

Near horizon geometry

Near horizon limit is obtained by

$$r = a\,(1 + \frac{n}{2\,\ell^2}\,\gamma^2\rho^2)\,, \quad t = \frac{\tau}{\gamma}\,, \quad \psi = \frac{\Psi}{\gamma}\,, \quad \gamma \to 0\,.$$

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$$ds^2_{NH} = \left(\frac{\rho^2}{\ell_3{}^2}\,d\tau^2 - \ell_3{}^2\,\frac{d\rho^2}{\rho^2} + \rho^2\,d\Psi^2\right) + \frac{\ell^2}{2n}\sum_{i=1}^{n-1} d\Sigma_i^2\,,$$

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where ${\ell_3}^2 = \frac{\ell^2}{n}\,, \quad f(\rho) = \frac{\rho^2}{{\ell_3}^2} - b^2\,.$

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- We studied the thermodynamics of near horizon EVH geometries and its relation to black hole thermodynamics in the EVH limit.