IPM WORKSHOP
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SUPERSYMMETRIC CORRECTIONS IN 11D SUPERGRAVITY

05/2018
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Supersymmetric corrections

- Motivation:

String theory

$$\downarrow$$

$$\mathcal{S}_{\text{eff}} = \sum_{g=0}^{\infty} g_s^{2g-2} \int d^{10}x \sqrt{g}\, \mathcal{L}_g$$

$$\downarrow$$

$$\mathcal{L}_0 = \mathcal{L}_{\text{sugra}} + \sum_{i=1}^{\infty} (\alpha')^i \mathcal{L}_i$$

M-theory

$$\downarrow$$

11D Supergravity
+ high energy corrective terms:

$$\mathcal{L}_{\text{11D sugra}} + \sum_{i=1}^{\infty} l^i \mathcal{L}_i$$

11D Supergravity

11D Supergravity - Cremmer Julia Scherk

- $$\mathcal{S} = \int dx \Big[\quad -\frac{\sqrt{g}}{4K^2} R(\omega)$$

$$- \frac{i\sqrt{g}}{2} \psi_m \Gamma^{mnr} D_n(\omega) \psi_r$$

$$- \frac{\sqrt{g}}{48} G_{mnrs} \, G^{mnrs}$$

$$+ \frac{\sqrt{g}K}{192} (\bar{\psi}_m \Gamma^{mnpqrs} \psi_n + 12 \bar{\psi}^p \Gamma^{qr} \psi^s) \, G_{pqrs}$$

$$+ \frac{2K}{12^4} \, \epsilon^{p_1 \ldots p_4 q_1 \ldots q_4 mnr} G_{p_1 \ldots p_4} G_{q_1 \ldots q_4} C_{mnr} \Big]$$

11D Supergravity - Cremmer Julia Scherk

- $$\mathcal{S} = \int dx \Big[\quad -\frac{\sqrt{g}}{4K^2} R(\omega)$$ → Scalar curvature R

$$-\frac{i\sqrt{g}}{2}\psi_m \Gamma^{mnr} D_n(\omega)\psi_r$$ → Gravitinos ψ_m

$$-\frac{\sqrt{g}}{48} G_{mnrs}\, G^{mnrs}$$ → Field strength : $G_{\mu\nu\rho\sigma} = \partial_{[\mu} C_{\nu\rho\sigma]}$

$$+\frac{\sqrt{g}K}{192}(\bar{\psi}_m \Gamma^{mnpqrs}\psi_n + 12\bar{\psi}^p \Gamma^{qr}\psi^s)\, G_{pqrs}$$ → Counter term

$$+\frac{2K}{12^4}\,\epsilon^{p_1 \dots p_4 q_1 \dots q_4 mnr} G_{p_1 \dots p_4} G_{q_1 \dots q_4} C_{mnr} \Big]$$ → Chern-Simons

11D Supergravity - Algebra

- Supersymmetric transformations:

$$\delta_Q C_{mnr} = \frac{3}{2}\epsilon\Gamma_{[mn}\psi_{r]}$$

$$\delta_Q \psi_m = \frac{1}{K} D_m \epsilon + \frac{i}{144}\left(\Gamma^{abcd}{}_m - 8\,\Gamma^{bcd}\delta^a_m\right)\epsilon G_{abcd}$$

$$\delta_Q e_m^{\ a} = -iK\bar{\epsilon}\,\Gamma^a\psi_m$$

- Closure of the 11D sugra algebra:

$$[\delta_{Q1}, \delta_{Q2}]\, C_{mnr} = (\delta_\xi + \delta_Q + \delta_\zeta)\, C_{mnr}$$

$$[\delta_{Q1}, \delta_{Q2}]\, \psi_m = (\delta_\xi + \delta_Q + \delta_\Lambda + \text{e.o.m})\, \psi_m$$

On-shell closure

11D Supergravity - Superspace

- From space to superspace: $$(x^m)\ m = 1\dots 11 \quad \longrightarrow \quad z^M = \begin{pmatrix} (x^m) & m = 1\dots 11 \\ (\theta^\mu) & \mu = 1\dots 32 \end{pmatrix}$$

- Superspace fields: $$\left.\begin{array}{c} e^a \longrightarrow E^A \\ T_{ab}{}^c \longrightarrow T_{AB}{}^C \\ G_{abcd} \longrightarrow G_{ABCD} \end{array}\right\} \Longrightarrow \text{New Bianchi identities}$$

- $$E^A{}_\wedge E^B\, H_{AB} = E^a{}_\wedge E^b\, H_{ab} + E^a{}_\wedge E^\alpha\, H_{\alpha b} + E^\beta{}_\wedge E^\alpha\, H_{\alpha\beta}$$

$$H_{AB}(x,\theta) = H_{AB}(x) + \sum_{i=1}^{32} \frac{1}{i!}\, \theta^{\mu_1} \dots \theta^{\mu_i}\, (H_{\mu_1 \dots \mu_2})_{AB}$$

11D Supergravity - Superspace

- Simultaneous resolution of
  - conventionnal constraints ⟵ $T_{\alpha\beta}{}^{a} = -i(\gamma^{a})_{\alpha\beta}$, $G_{\alpha\beta ab} = i(\gamma_{ab})_{\alpha\beta}$
  - Bianchi identitites ⟵ $DT^{A} = E_{B} \wedge R_{A}{}^{B}$, $DG_{ABCD} = 0$

$$\Downarrow$$

11D sugra e.o.m

- $S = \int \left(R \star 1 - \frac{1}{2} G_{4\wedge} \star G_4 - \frac{1}{6} C_{3\wedge} G_{4\wedge} G_4\right)\Big|_{\theta=0}$

$$\begin{pmatrix} E_m^\alpha & = & \psi_m^\alpha \\ T_{mn}{}^{\alpha} & = & D_m \psi_n^\alpha - D_n \psi_m^\alpha \end{pmatrix}$$

- From the flat superspace basis to the non-superspace coordinate basis:

$$L_{m_1 \dots m_D} =$$

$$e_{m_D}{}^{a_D} \cdots e_{m_1}{}^{a_1} L_{a_1 \dots a_D} + D\, e_{m_D}{}^{a_D} \cdots e_{m_2}{}^{a_2} \psi_{m_1}{}^{\alpha_1} L_{\alpha_1 a_2 \dots a_D} + \dots + \psi_{m_D}{}^{\alpha_D} \cdots \psi_{m_1}{}^{\alpha_1} L_{\alpha_1 \dots \alpha_D}$$

Action Principle & Supersymmetric Corrections

Soueres & Tsimpis - Phys. Rev. D **95**, 026013 (2017)

Action Principle - Chern-Simons Term

$$
\begin{aligned}
\mathcal{S}_{\text{Chern-Simons}} \mathcal{S} = \int dx^{11} \Bigg[& -\frac{\sqrt{g}}{4K^2} R(\omega) \\
& - \frac{i\sqrt{g}}{2} \psi_m \Gamma^{mnr} D_n \left(\omega\right) \psi_r \\
& - \frac{\sqrt{g}}{48} G_{mnrs} \, G^{mnrs} \\
& + \frac{\sqrt{g}K}{192} (\bar{\psi}_m \Gamma^{mnpqrs} \psi_n + 12 \bar{\psi}^p \Gamma^{qr} \psi^s) \, G_{pqrs} \\
& + \frac{2K}{12^4} \, \epsilon^{p_1 \ldots p_4 q_1 \ldots q_4 mnr} G_{p_1 \ldots p_4} G_{q_1 \ldots q_4} C_{mnr} \Bigg]
\end{aligned}
$$

Action Principle - Method

$$\mathcal{S}_{\text{Chern-Simons}} = \int dx^{11} \left[\frac{2K}{12^4} \epsilon^{p_1 \dots p_4 q_1 \dots q_4 mnr} G_{p_1 \dots p_4} G_{q_1 \dots q_4} C_{mnr} \right] \sim \int \underbrace{C_3 \wedge G_4 \wedge G_4}$$

Z_{11} not gauge-invariant

$(C \to C + \mathrm{d}\Lambda)$

globally defined

not globally defined

- Let's define K_{11} such that $\mathrm{d}\, K_{11} = \mathrm{d}\, Z_{11}$
- Then $Z_{11} - K_{11}$ is a closed superform $\mathrm{d}(Z_{11} - K_{11}) = 0$
- Then the action $S = \int (Z_{11} - K_{11})|_{\theta=0}$ is superinvariant

Action Principle - Method

- Diffeomorphism in superspace: $\delta_\xi \phi = \mathcal{L}_\xi \phi$ with $\xi_A = \begin{pmatrix} \xi_a \\ \xi_\alpha \end{pmatrix}$

- SuperLie derivative of $Z_{11} - K_{11}$:

$$\mathcal{L}_\xi (Z_{11} - K_{11}) = (d\, i_\xi - i_\xi\, d)(Z_{11} - K_{11})$$
$$= d\, i_\xi (Z_{11} - K_{11})$$

$$\Downarrow$$

$$\delta_\xi S = \int \mathcal{L}_\xi (Z_{11} - K_{11})|$$
$$= \int d(\dots) = 0$$

First Test - Original action

First Test - Original Action

- $S_{\text{Chern-Simons}} = Z_{11} = C_3 {\scriptstyle\wedge} G_4 {\scriptstyle\wedge} G_4$

- Looking for: $\mathrm{d}K_{11} = G_4 {\scriptstyle\wedge} G_4 {\scriptstyle\wedge} G_4$ (superspace equation)

$$\Updownarrow$$

- $D_{[A_1} K_{A_2 \ldots A_{12})} + \frac{11}{2} T_{[A_1 A_2|}{}^{F} K_{F|A_3 \ldots A_{12})} = -\frac{11!}{6(4!)^3} G_{[A_1 \ldots A_4} G_{A_5 \ldots A_8} G_{A_9 \ldots A_{12})}$

  $A_1 \ldots A_{11} = \alpha_1 \ldots \alpha_{11} \rightarrow K_{a_1 \alpha_1 \ldots \alpha_{10}}$

  $A_1 \ldots A_{11} = \alpha_1 \ldots \alpha_{10}\, a_1 \rightarrow K_{a_1 a_2 \alpha_1 \ldots \alpha_9}$

  etc.

- After 13 equations: $S = \int \left(R \star 1 - \frac{1}{2} G_4 {\scriptstyle\wedge} \star G_4 - \frac{1}{6} C_3 {\scriptstyle\wedge} G_4 {\scriptstyle\wedge} G_4\right)\Big|_{\theta=0}$

First Superinvariant

First Superinvariant

- Modified Chern-Simons term: $Z_{11} = l^3 \; C_3{}_\wedge G_4{}_\wedge \mathrm{tr} R^2$

- Looking for: $\mathrm{d}K_{11} = \mathrm{d}Z_{11} = G_4{}_\wedge G_4{}_\wedge R_a{}^b{}_\wedge R_b{}^a$

$$\longrightarrow \; D_{[A_1} \, K_{A_2 \dots A_{12})} + \frac{11}{2} \, T_{[A_1 A_2|}{}^F \, K_{F|A_3 \dots A_{12})} = \frac{11!}{4(4!)^2} \, R_{[A_1 A_2|c_1 c_2} \, R_{|A_3 A_4|}{}^{c_2 c_1} \, G_{|A_5 \dots A_8} \, G_{A_9 \dots A_{12})}$$

- After 13 equations: $\Delta_1 S = l^3 \int \left(C_3{}_\wedge G_4{}_\wedge \mathrm{tr} R^2 + 2 \; G_4{}_\wedge \star \, G_4{}_\wedge \mathrm{tr} R^2 \right)\Big|_{\theta=0}$

Second Superinvariant

Second Superinvariant

- Modified Chern-Simons term: $Z_{11} = l^6\, C_3{\scriptstyle\wedge} G_4{\scriptstyle\wedge} \overbrace{\left(\mathrm{tr}R^4 - \frac{1}{4}(\mathrm{tr}R^2)^2\right)}^{X^{(8)}}$

- Looking for: $\mathrm{d}K_{11} = \mathrm{d}Z_{11} = G_4{\scriptstyle\wedge} G_4{\scriptstyle\wedge} X^{(8)}$

- Main obstacle: $X^{(8)}_{\alpha_1\ldots\alpha_8} \propto \gamma^f_{(\alpha_1\alpha_2|}G_{f|\alpha_3\ldots\alpha_8)}$ $\longrightarrow$ Probably the hardest part
  The rest should follow « easily »

  (τ-exact)

Second Superinvariant

- $X^{(8)}_{\alpha_1\ldots\alpha_8} = \gamma^{f}_{(\alpha_1\alpha_2|} G_{f|\alpha_3\ldots\alpha_8)}$?

$\downarrow$

12 terms $\sim \gamma\gamma\gamma\gamma GGGG$ with symmetrized fermionic indices...

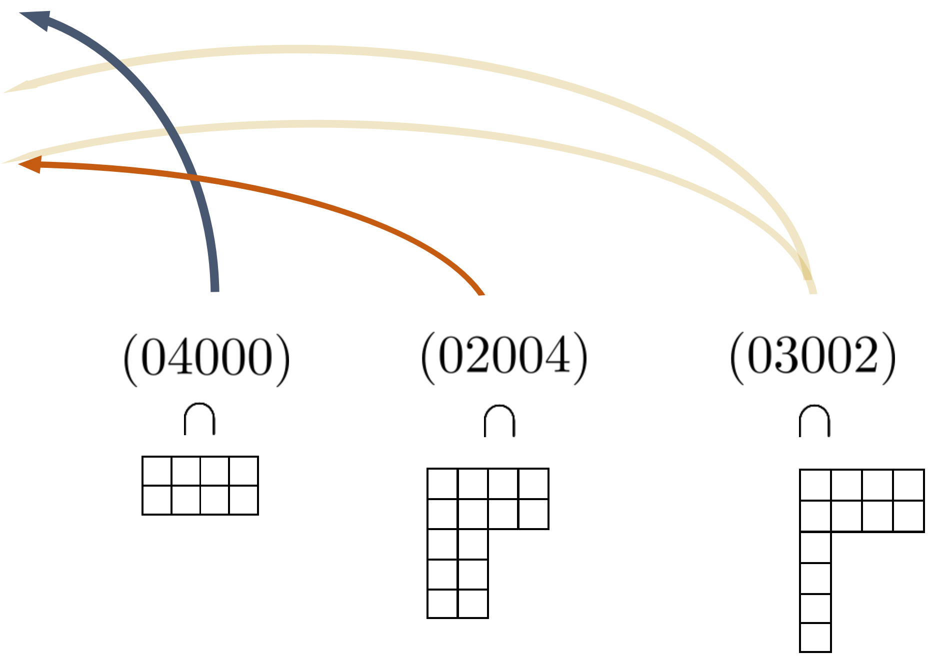
$$(\gamma^{u_0\ldots u_3}{}_{y_0 y_1})(\gamma^{v_0\ldots v_3 y_0}{}_{z_0})(\gamma^{w_0\ldots w_3 y_1}{}_{z_1})(\gamma_{x_0\ldots x_3}{}^{z_0 z_1})\ G_{u_0\ldots u_3} G_{v_0\ldots v_3} G_{w_0\ldots w_3} G_{x_0\ldots x_3} + \ldots$$

- Huge fierzing:

$$X^{(8)}_{\alpha_1\ldots\alpha_8} = \gamma^{(2)}\gamma^{(2)}\gamma^{(2)}\gamma^{(2)}\ (GGGG + \ldots) \quad (1)$$
$$+ \gamma^{(2)}\gamma^{(2)}\gamma^{(2)}\gamma^{(5)}\ (GGGG + \ldots) \quad (2)$$
$$+ \gamma^{(2)}\gamma^{(2)}\gamma^{(5)}\gamma^{(5)}\ (GGGG + \ldots) \quad (3)$$

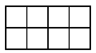
Second Superinvariant

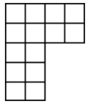
$$
\begin{aligned}
X^{(8)}_{\alpha_1\dots\alpha_8} &= \gamma^{(2)}\gamma^{(2)}\gamma^{(2)}\gamma^{(2)}\,(GGGG+\dots) \\
&+\gamma^{(2)}\gamma^{(2)}\gamma^{(2)}\gamma^{(5)}\,(GGGG+\dots) \\
&+\gamma^{(2)}\gamma^{(2)}\gamma^{(5)}\gamma^{(5)}\,(GGGG+\dots)
\end{aligned}
$$

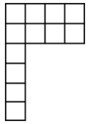


- $(00001)^{\otimes_s 8}$ contains 48 irreps, only 3 non τ-exact:

- Those 3 representations must vanish:

$\prod (1) = 0$ 

$\prod (3) = 0$ 

$\prod ((2)+(3)) \overset{?}{=} 0$ 

Miraculous cancellation depending on the factor 1/4:

$$X^{(8)} = \mathrm{tr}R^4 - \frac{1}{4}(\mathrm{tr}R^2)^2$$

Computational Tools: Mathematica Package Superspace GAMMA

Computational Tools: Mathematica Package Superspace GAMMA

- Basic Young diagrams manipulations:

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IrrYT[{1, 1, 1}, {2, 1}, 1]
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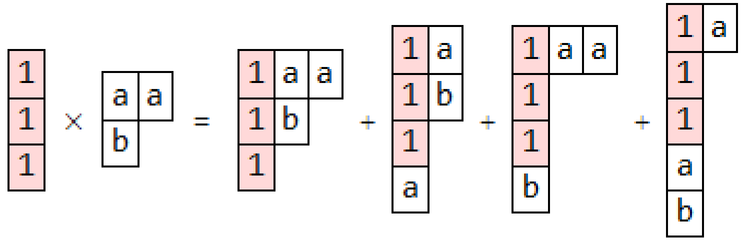

- Tensorial representation of Young diagrams:

$$T^{\boxplus}_{a_1 a_2; b_1 b_2} \longrightarrow \frac{4}{3}\left(\frac{1}{16}\left(T_{a1,a2,a3,a4} - T_{a1,a2,a4,a3}\right) + \frac{1}{16}\left(-T_{a1,a3,a4,a2} + T_{a1,a4,a3,a2}\right) + \frac{1}{16}\left(-T_{a2,a1,a3,a4} + \text{etc.}\right.\right.$$

Computational Tools: Mathematica Package Superspace GAMMA

- Highly non-trivial Fierzing: $(00001)^{\otimes_S 6} = 1(00012) \oplus \dots$

$$(\gamma^{a_1\dots a_4 e_1})_{(\alpha_1\alpha_2}(\gamma^{[b_1\dots b_3|}{}_{e_1e_2e_3})_{\alpha_3\alpha_4}(\gamma^{|b_4\dots b_6]e_2e_3})_{\alpha_5\alpha_6)} \propto 15\,(\gamma^{[a_1a_2|})_{(\alpha_1\alpha_2}(\gamma^{|a_3a_4]})_{\alpha_3\alpha_4}(\gamma^{b_1\dots b_6})_{\alpha_5\alpha_6)}$$

- High number of anti-symmetrized indices: $D_{[A_1}K_{A_2\dots A_{12})} + \frac{11}{2}\,T_{[A_1A_2|}{}^F\,K_{F|A_3\dots A_{12})} = \dots$

- Superspace fields (bosonic & fermionic indices): $D_{(\alpha_1}K_{\alpha_2\dots\alpha_5)[a_1\dots a_6]} + \dots$

THANK YOU