Soft hair on black hole and cosmological horizons in any dimension

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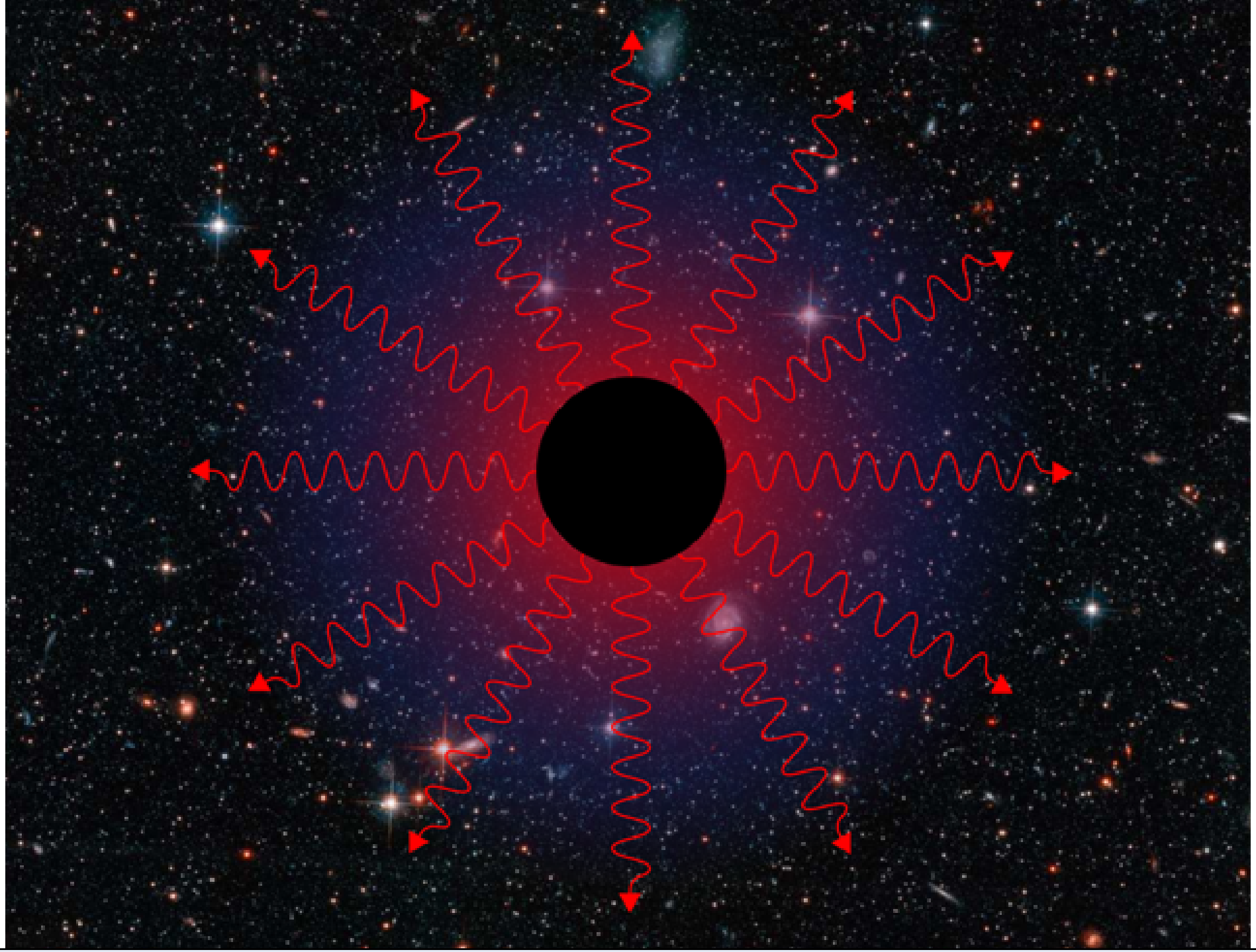

Motivation — horizons

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- Bekenstein–Hawking entropy

$$S_{\mathrm{BH}} = \frac{\mathrm{Area}}{4G} \qquad \sim 10^{89} \quad \text{for Sgr A}^*$$

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  Semi-classical limit of entropy for large area:

$$S_{\text{BH}} = \frac{\text{Area}}{4G} + N_{\text{dof}} \ln(\text{Area}/G) + \mathcal{O}(1)$$

  Up to $\mathcal{O}(1)$ terms everything accessible semi-classically!

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- Microstate construction for generic black holes?
  Explicit constructions usually exploit (near-)extremality ($T \to 0$), but generic black holes have finite temperature

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Implementing assumptions above yields near horizon metric

$$\mathrm{d}s^2 = -\kappa^2\rho^2\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \Omega_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b + \ldots$$

Donnay, Giribet, González, Pino (2016)

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- Ellipsis denotes higher order terms in ρ and rotation/boost terms

Boundary conditions and near horizon Killing vectors

Boundary conditions on spatial metric, momenta and Lagrange multipliers:

$$g_{\rho\rho} = 1 + \mathcal{O}(\rho^2) \qquad g_{\rho a} = \mathcal{O}(\rho) \qquad g_{ab} = \Omega_{ab} + \mathcal{O}(\rho^2)$$

$$\pi^{\rho a} = \frac{\pi^{\rho a}_{(0)}}{16\pi G} + \mathcal{O}(\rho^2) \qquad \pi^{\rho\rho} = \mathcal{O}(\rho) \qquad \pi^{ab} = \mathcal{O}(\rho)$$

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- This becomes evident once canonical boundary charges are calculated

Canonical realization of near horizon symmetries

Standard procedure yields variation of canonical boundary charges $\delta Q[\xi]$

$$\delta Q[\epsilon^t, \epsilon^a] = \int \mathrm{d}^{D-2}x \left[\epsilon^t \, \delta\mathcal{P} + \epsilon^a \, \delta\mathcal{J}_a\right]$$

with the state-dependent functions

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Transformation laws above assume state-independence of ϵ^t and ϵ^a

Results above compatible with Donnay, Giribet, González, Pino (2016)

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- Expectation 4: twisted warped conformal algebra [Afshar et al (2015)]

Surprising result: none of the above!

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Instead: two $u(1)$ current algebras

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(Why) were the expectations off?

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- Example: start with $u(1)$ current algebras

$$[J_n^{\pm},\, J_m^{\pm}] = \frac{k}{2}\, n\, \delta_{n+m,\,0}$$

and map to Brown–Henneaux variables (Miura-map)

$$L_n^{\pm} = \frac{1}{k} \sum_p J_{n-p}^{\pm} J_p^{\pm} + inJ_n^{\pm}$$

nothing but twisted Sugawara construction! Yields two Virasoros

$$[L_n^{\pm},\, L_m^{\pm}] = (n-m)\, L_{n+m}^{\pm} + \frac{c}{12}\, n^3\, \delta_{n+m,\,0}$$

with $c = \frac{3\ell}{2G}$

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- Works also for higher spins [Prohazka et al (2016), Wutte et al (2017)]

$$[L_n,\, L_m] = (n-m)L_{n+m} + \frac{c_L}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[L_n,\, M_m] = (n-m)M_{n+m} + \frac{c_M}{12}\,(n^3-n)\,\delta_{n+m,\,0}$$

$$[U_n,\, U_m] = (n-m)(2n^2+2m^2-nm-8)L_{n+m} + \frac{192}{c_M}(n-m)\Lambda_{n+m}$$
$$- \frac{96\left(c_L+\frac{44}{5}\right)}{c_M^2}(n-m)\Theta_{n+m} + \frac{c_L}{12}\,n(n^2-1)(n^2-4)\,\delta_{n+m,\,0}$$

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obtained from twisted Sugawara-like construction with four $u(1)$ current algebras

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- Seems like $u(1)$ current algebras are "algebras of everything"
- Universality perhaps easier understood by map to Heisenberg algebras

$$[X_n,\, P_m] = ik\,\delta_{n,\,m} \qquad \text{if } n \neq 0$$

where

$$X_n = J_n^+ - J_{-n}^- \qquad P_0 = J_0^+ + J_0^- \qquad P_n = \frac{i}{n}\left(J_{-n}^+ + J_n^-\right)$$

Note that P_0 commutes with all generators!

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- Redefinitions of transformation parameters required

$$\epsilon^a_{\mathrm{H}} := \epsilon^a \sqrt{\Omega} \qquad\qquad \epsilon^t_{\mathrm{H}} := \epsilon^t + \epsilon^a J^{\mathrm{H}}_a$$

to render canonical boundary charges integrable

$$Q_{\mathrm{H}}[\epsilon^t_{\mathrm{H}}, \epsilon^a_{\mathrm{H}}] = \int \mathrm{d}^{D-2}x \left[\epsilon^t_{\mathrm{H}} \mathcal{P} + \epsilon^a_{\mathrm{H}} \mathcal{J}^{\mathrm{H}}_a\right]$$

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to render canonical boundary charges integrable

$$Q_{\mathrm{H}}[\epsilon^t_{\mathrm{H}}, \epsilon^a_{\mathrm{H}}] = \int \mathrm{d}^{D-2}x \left[\epsilon^t_{\mathrm{H}} \mathcal{P} + \epsilon^a_{\mathrm{H}} \mathcal{J}^{\mathrm{H}}_a\right]$$

Consequences of choices above: entropy, soft hair and Heisenberg

Entropy

- Define near horizon Hamiltonian as normalized time-translation generator

$$H := Q[\epsilon^t_{\rm H} = \kappa,\, \epsilon^a_{\rm H} = 0] = \kappa \int \mathrm{d}^{D-2}x\, \mathcal{P} \equiv \kappa\, \mathcal{P}_0$$

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- From 3D experience: should work beyond Bekenstein–Hawking
- Result for entropy extremely simple; e.g. in 3D equivalent to Cardy

$$S = 2\pi \sqrt{\frac{c}{6} L_0^+} + 2\pi \sqrt{\frac{c}{6} L_0^-}$$

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Notion of "soft hair" introduced by Hawking, Perry and Strominger (2016)

Soft hair = non-trivial zero energy excitations

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Our near horizon boundary conditions naturally lead to soft hair in any D

Near horizon Heisenberg symmetries

Restrict in most of talk to cases of vanishing superrotation field strength

$$F^{\mathrm{H}}_{ab} := \partial_a \mathcal{J}^{\mathrm{H}}_b - \partial_b \mathcal{J}^{\mathrm{H}}_a = 0 \qquad \Rightarrow \qquad \mathcal{J}^{\mathrm{H}}_a = \partial_a \mathcal{Q}$$

- Holds when horizon non-rotating or intrinsically flat
- Examples: Schwarzschild or Kerr-AdS with toroidal horizon

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(come back to general case in the end; Heisenberg persists, so keep calm and carry on)

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Near horizon Heisenberg algebra in any dimension

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Consider as first example Schwarzschild black holes

- Expand in spherical harmonics

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- Could be key in semi-classical description of Schwarzschild black holes

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Consider as second example toroidal Kerr-AdS$_4$

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- Microstate counting, fluff proposal, log corrections to entropy, ...

Kerr

More complicated, but fits perfectly in general near horizon expansion

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- Metric near Kerr black hole event horizon

$$\mathrm{d}s^2 = -\kappa^2\rho^2\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + r_+^2\left[(1+\mathscr{R}\cos^2\theta)\,\mathrm{d}\theta^2 + \frac{(1+\mathscr{R})^2\sin^2\theta}{1+\mathscr{R}\cos^2\theta}\,\mathrm{d}\varphi^2\right]$$
$$+\,\mathcal{O}(\rho^2)\,\mathrm{d}t\,\mathrm{d}\varphi + \mathcal{O}(\rho)\,\mathrm{d}\rho\,\mathrm{d}\theta + \ldots \qquad \text{where } \mathscr{R} := r_-/r_+$$

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- Superrotation field strength does not vanish

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Near horizon symmetries for Kerr

- Transformations of state-dependent functions

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ellipses known operators linear in trafo-parameters η, μ and linear in charge $\mathcal{C}$

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First two equalities yield again Heisenberg algebra!

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Exciting prospect to lift some of 3D results on soft Heisenberg hair to 4D or higher!

Concluding remarks and future research directions

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Soft Heisenberg hair provides novel approach to understand horizons

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Thanks for your attention!