Asymptotic Symmetries in p-form Theories

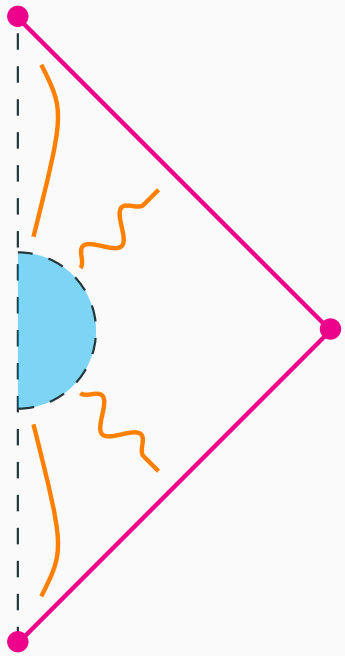
based on arXiv1801.07752 (JHEP '18)

H. Afshar, E. Esmaeili, M.M. Sheikh-Jabbari

May 9, 2018

School of physics, Institute for research in fundamental sciences (IPM)

Prelude



Outline

- p-form theories
- Asymptotic symmetries and motivation
- Action principle
- Surface charges and their algebra
- Exact/coexact forms
- Classification of charges

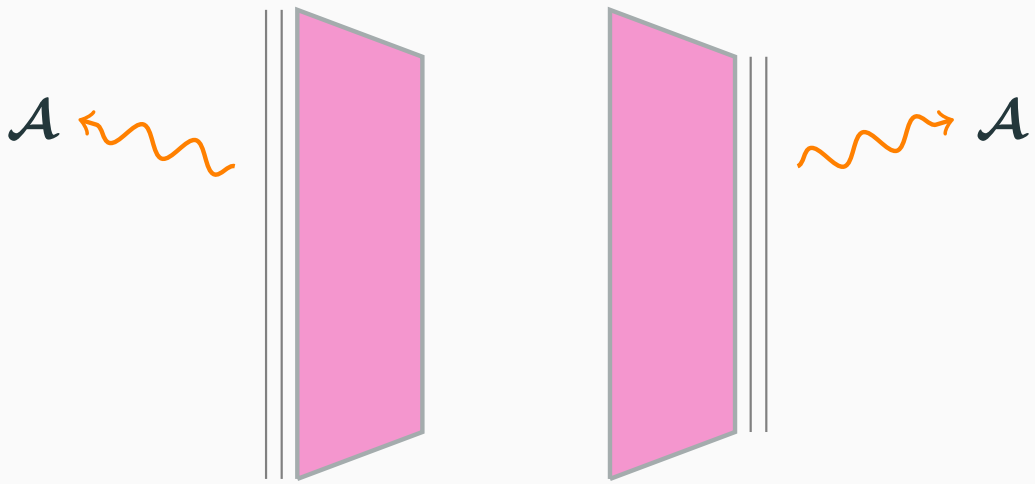
p-FORM THEORIES

- Gauge potential: $\boldsymbol{\mathcal{F}} = \mathrm{d}\boldsymbol{\mathcal{A}}$

$$\boldsymbol{\mathcal{A}} = \frac{1}{(p+1)!}\mathcal{A}_{\nu_0\cdots\nu_p}\mathrm{d}x^{\nu_0}\wedge\cdots\mathrm{d}x^{\nu_p}\,, \quad \boldsymbol{\mathcal{F}} = \frac{1}{(p+2)!}\mathcal{F}_{\nu_0\cdots\nu_{p+1}}\mathrm{d}x^{\nu_0}\wedge\cdots\mathrm{d}x^{\nu_{p+1}}$$

- Action (coupled to extended sources) Henneaux-Teitelboim '86

$$S_0 = -\int_{\mathcal{M}_d}\left(\tfrac{1}{2}\boldsymbol{\mathcal{F}}\wedge\star\boldsymbol{\mathcal{F}} - (-1)^p\boldsymbol{\mathcal{A}}\wedge\star\boldsymbol{\mathcal{J}}\right)$$



p-FORM THEORIES

- Abelian gauge symmetry $\qquad \boldsymbol{\mathcal{A}}_{p+1} \to \boldsymbol{\mathcal{A}}_{p+1} + \mathrm{d}\boldsymbol{\Lambda}_p$
- Constraints

$$\phi_1^{0i_1\cdots i_p} = \mathcal{F}^{00i_1\cdots i_p}, \qquad [\phi_1^{0i_1\cdots i_p}, H_C] = \partial_k \mathcal{F}^{0ki_1\cdots i_p} \equiv \phi_2^{i_1\cdots i_p}$$

- Number of degrees of freedom

$$\binom{d-2}{p+1}$$

Asymptotic symmetries and motivation

- Asymptotic symmetries are gauge transformations with non-zero charge. ADM '62 Regge-Teitelboim '74 Ashtekar-Hansen '78 Brown-Henneaux '86 etc.
- They are conjectured to be isomorphic to global symmetries of dual theory. Aharony *et al* '99
- In flat space, Weinberg's soft theorems in many gauge theory examples are shown to be Ward identities of asymptotic symmetries.Strominger *et al* '14 Campiglia-Laddha '15 etc.

Motivation for p-forms:

- Algebra

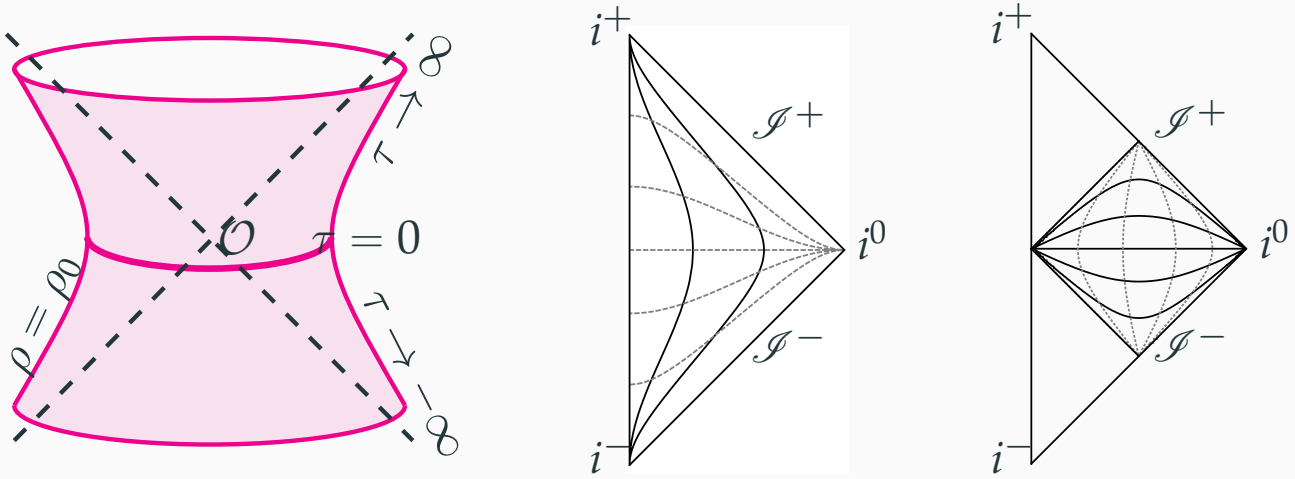
$$\lambda_{\mu_1 \cdots \mu_p}: \qquad \{Q_{\boldsymbol{\lambda}}, Q_{\boldsymbol{\lambda}'}\} = ?$$

- Form fields in super-gravity

We study asymptotic symmetries of p-form theory in $d = 2p + 4$ dimensional Minkowski space at spatial infinity.

De sitter slicing of flat space

Ashtekar-Romano '92



$$ds^2 = d\rho^2 + \rho^2[-d\tau^2 + \cosh^2\tau\, d\Omega^2]$$

$$(\rho, x^a): \qquad \rho^2 = x^\mu x_\mu = r^2 - t^2\,, \qquad \tanh\tau = \frac{t}{r}\,, \qquad \hat{x} \to \hat{x}$$

Null slicing Bondi *et al.* '62

$$ds^2 = -du^2 - 2dudr + r^2 d\Omega^2$$

De sitter slicing of flat space

- Lorenz invariant division of components

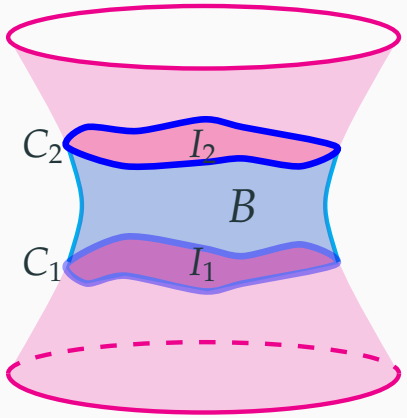
$$\text{electric: } \mathcal{A}_{\rho a_1 \cdots a_p} \qquad \text{magnetic: } \mathcal{A}_{a_0 \cdots a_p} \begin{cases} \mathcal{A}_{\tau B_1 \cdots B_p} \\ \mathcal{A}_{B_0 \cdots B_p} \end{cases}$$

  also for gauge parameter: $\Lambda_{\rho a_2 \cdots a_p}$ and $\Lambda_{a_1 \cdots a_p}$.

- Boundary conditions

$$\begin{aligned} \mathcal{A}^{\rho a_1 \cdots a_p} &= A^{\rho a_1 \cdots a_p}(x)\, \rho^{-d+3} + \mathcal{O}(\rho^{-d+2}), \\ \mathcal{A}^{a_0 \cdots a_p} &= A^{a_0 \cdots a_p}(x)\, \rho^{-d+2} + \mathcal{O}(\rho^{-d+1})\,. \end{aligned}$$

Action principle



- Generic variation of action

$$\delta S_0 = \int_{\mathcal{M}} E(\Phi)\delta\Phi + \int_{I_2} \mathcal{I}_0(\delta\Phi,\Phi) - \int_{I_1} \mathcal{I}_0(\delta\Phi,\Phi) + \int_B \mathcal{B}(\delta\Phi,\Phi)$$

- Suitable boundary term

$$\delta S_b = -\int_B \mathcal{B}(\delta\Phi,\Phi) + \int_{\partial I_2} \mathcal{I}_b(\delta\Phi,\Phi) - \int_{\partial I_1} \mathcal{I}_b(\delta\Phi,\Phi)$$

- p-form theory and special dimensions

$$\mathcal{B}\text{-term} = -\frac{\rho_0^{2p+4-d}}{(p+1)!}\int_B \delta A^{a_0\cdots a_p}\left[(2p+4-d)A_{a_0\cdots a_p} - (p+1)\partial_{a_0}A_{\rho\, a_1\cdots a_p}\right]$$

Action principle

$d = 2p + 4$:

- Scale invariance
- Coulomb $\sim$ Radiation
- Self-dual forms

Well-defined p-form action in Lorenz gauge for $d = 2p + 4$:

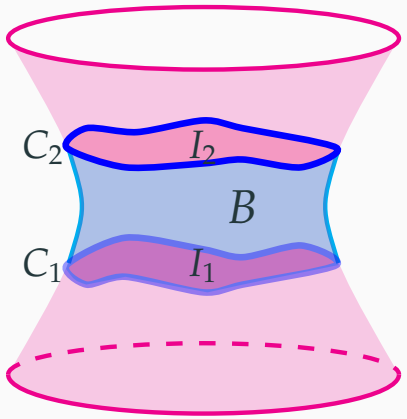
$$S = -\frac{1}{2(p+1)!}\int_{\mathcal{M}} \mathcal{F}_{\mu_0\cdots\mu_{p+1}}\mathcal{F}^{\mu_0\cdots\mu_{p+1}} - \frac{1}{p!}\int_B \mathcal{A}^{\rho a_1\cdots a_p}\mathcal{A}_{\rho a_1\cdots a_p}$$

Methods to compute charges

Hamiltonian-based Regge-Teitelboim '74

Lagrangian-based Lee-Wald '91 Barnich-Brandt '01

Surface charges from Noether's method



- Gauge invariance of a well-defined action implies a conservation law

$$\delta S \approx \underbrace{\int_{C_2} \mathcal{C}(\mathcal{A}, \delta A)}_{\text{the charge}} - \int_{C_1} \mathcal{C}(\mathcal{A}, \delta A) = 0$$

- For a 'gauge invariant' action, the r.h.s. need not be zero, but still having the same structure as l.h.s. . Charge definition will be modified correspondingly.

p-FORM SURFACE CHARGES

- Conserved charge corresponding to $\mathbf{A} \to \mathbf{A} + \mathrm{d}\boldsymbol{\lambda}$:

$$Q_\lambda[\mathcal{A}] = -\frac{1}{p!}\int_C \left[\lambda_B\, F^{\rho\tau B} + (\mathrm{d}\lambda)^\tau{}_B\, A^{\rho B} - 2p\lambda_\rho\, A^{\rho\tau}\right]$$

- In form notation

$$Q_{\boldsymbol{\lambda}}[\mathcal{A}] = -\int_C \left(\hat{\boldsymbol{\lambda}} \wedge \star\hat{\mathbf{F}}_{\rho\tau} + \hat{\mathbf{A}}_\rho \wedge \star(\widehat{\mathrm{d}\boldsymbol{\lambda}})_\tau - 2\hat{\boldsymbol{\lambda}}_\rho \wedge \star\hat{\mathbf{A}}_{\rho\tau}\right)$$

- Charge algebra

$$\{Q_{\boldsymbol{\lambda}}, Q_{\boldsymbol{\epsilon}}\} = 2\int_C \left(\hat{\boldsymbol{\lambda}}_\rho \wedge \star\partial_\tau\hat{\boldsymbol{\epsilon}}_\rho - \hat{\boldsymbol{\epsilon}}_\rho \wedge \star\partial_\tau\hat{\boldsymbol{\lambda}}_\rho\right)$$

The right-hand-side is a number: the algebra is centrally extended.

Residual G.T.'s , Exact/coexact forms

- Lorentz gauge $\mathrm{d}^\dagger \boldsymbol{\mathcal{A}} = 0, \qquad \text{or} \qquad -\nabla_\alpha \mathcal{A}^{\alpha\mu_1\cdots\mu_p} = 0$
- Residual transformations and temporal gauge

$$\mathrm{d}^\dagger \mathrm{d}\boldsymbol{\Lambda} = 0, \qquad \hat{\boldsymbol{\Lambda}}_\tau = \hat{\boldsymbol{\Lambda}}_{\rho\tau} = 0 \tag{1}$$

- Minkowski form is decomposed into de Sitter forms $\boldsymbol{\lambda}$ and $\boldsymbol{\lambda}_\rho$; and (at leading order) (1) becomes

$$\mathrm{d}^\dagger \mathrm{d}\boldsymbol{\lambda} + 2\mathrm{d}\boldsymbol{\lambda}_\rho = 0\,, \qquad \mathrm{d}^\dagger \mathrm{d}\boldsymbol{\lambda}_\rho = 0 \tag{2}$$

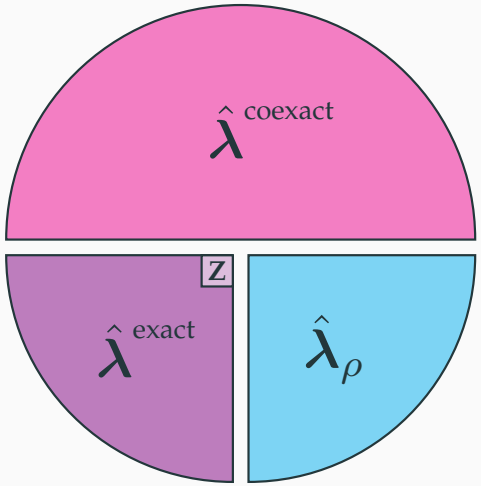
- The residual gauge parameters on sphere are decomposed into exact and coexact forms by Hodge decomposition (no harmonic forms in our case)

$$\hat{\boldsymbol{\lambda}} = \hat{\boldsymbol{\lambda}}^{\text{exact}} + \hat{\boldsymbol{\lambda}}^{\text{coexact}} + \hat{\boldsymbol{\lambda}}^{\text{harmonic}} = \mathrm{d}\boldsymbol{\epsilon} + \mathrm{d}^\dagger \boldsymbol{\psi} + \boldsymbol{\chi}.$$

- Since exact/coexact forms are orthogonal on sphere (w.r.t. usual inner product), equation (2) is separable into exact and coexact parts
- What about $\boldsymbol{\lambda}_\rho$? Exact parameters $\hat{\boldsymbol{\lambda}} = \mathrm{d}\hat{\boldsymbol{\epsilon}}$ are related to $\hat{\boldsymbol{\lambda}}_\rho$ by $\hat{\boldsymbol{\epsilon}}'' \cong -\frac{2\hat{\boldsymbol{\lambda}}_\rho}{(1-y^2)^2}$

Classification of charges

Conserved charges are classified into the following



- **Coexact.** $\hat{\lambda} = \mathrm{d}^\dagger \hat{\psi}$ is
  a coexact form. These are direct generalization of conserved charges in electromagnetism, and form an Abelian algebra.
- **Exact.** $\hat{\lambda} = \mathrm{d}\hat{\epsilon}$ is
  an exact form and in general time-dependent. These are new features of p-form theories.
- **Zero-mode.** $\hat{\epsilon}$ is constant in time, with no restriction on its angular dependence. These are brane charges, which generalize electric charge of electromagnetism.

Algebra of charges

$$\{Q_\epsilon^{\text{coexact}}, Q_\lambda^{\text{coexact}}\} = -\delta_\varepsilon Q_\lambda^{(\text{coexact})} = 0\,,$$

$$\{Q_\epsilon^{\text{exact}}, Q_\lambda^{\text{exact}}\} = -\delta_\varepsilon Q_\lambda^{(\text{exact})} = \text{Heisenberg algebra on } S^{2p+2}\,,$$

$$\{Q_\epsilon^{\text{coexact}}, Q_\lambda^{\text{exact}}\} = 0\,.$$

The zero mode charges $Q_\epsilon^{\text{zero-mode}}$ commute with all other charges.

$$\{Q_{\boldsymbol{\lambda}}, Q_{\boldsymbol{\epsilon}}\} = 2\int_C \left(\hat{\boldsymbol{\lambda}}_\rho \wedge \star\partial_\tau \hat{\boldsymbol{\epsilon}}_\rho - \hat{\boldsymbol{\epsilon}}_\rho \wedge \star\partial_\tau \hat{\boldsymbol{\lambda}}_\rho\right).$$

Summary

- p-form theories

  Generalized electromagnetism, coupled to extended objects
- Asymptotic symmetries

  Computed on flat background in special dimensions, motivated by possible non-trivial algebra
- Action principle

  Needs modification in de Sitter slicing, already for electromagnetism
- Surface charges and their algebra:

  Noether's method was applied; central extension
- Exact/coexact forms

  Orthogonal division of forms using Hodge decomposition
- Classification of charges

  Three classes among which, exact charges are non-commuting

Next steps

- General dimensions ($d \neq 2p+4$), other gauge fixings, *AdS* background.
- Self-dual forms and supergravity.
- Boundary gauge fields and their action.
- p-form memory and soft p-photon theorem(*in progress*)

Thank you

and

many thanks to Vahid Hosseinzadeh and Ali Seraj