Schwarzian theory from generalized Jackiw-Teitelboim model

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Motivation: SYK

Dilaton gravity: Jackiw-Teitelboim model

Action principle

Example

Reformulation: Particle on a group manifold

Conclusions

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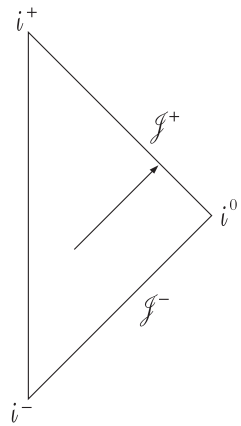
- Asymptotic symmetries, infinite-dimensional symmetries & quantum description of gravity.

$$S_{\mathrm{BH}} = \frac{\mathrm{Area}}{4G_N}$$



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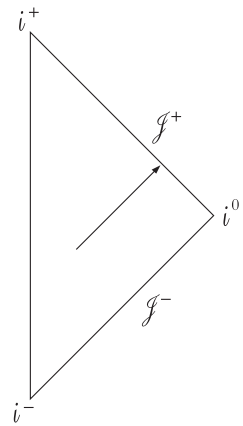

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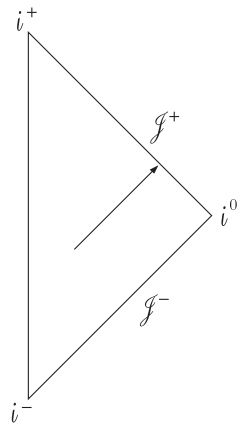

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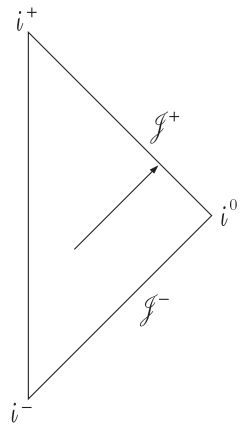

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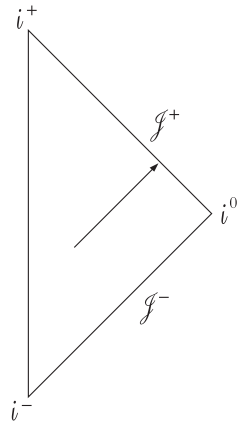

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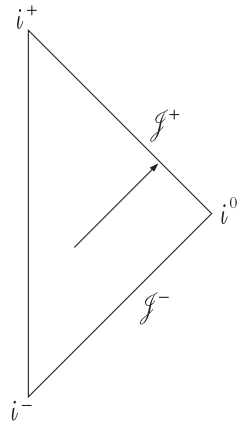

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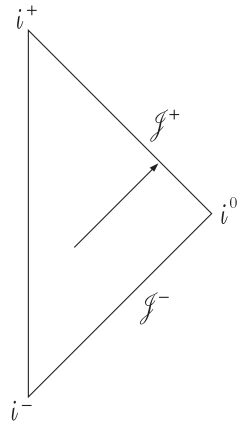

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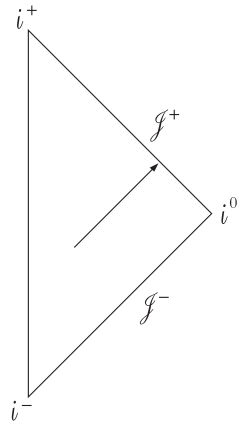

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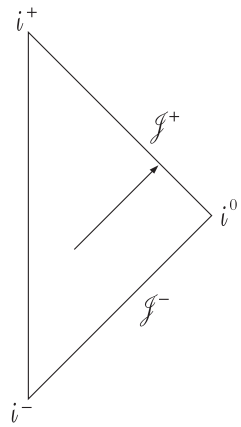

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$$f(\tau) \to \frac{a\, f(\tau) + b}{c\, f(\tau) + d}, \quad ad - cb = 1$$

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$$I[q(\tau)] = \frac{N}{\beta J} \int_0^\beta d\tau \{f; \tau\}, \quad f(\tau) = \cot\left(\frac{1}{2} q(\tau)\right)$$

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- Consistent description: 2D Dilaton gravity models.

Simplest: Jackiw-Teiltelboim model

[Jackiw 1984] [Teiltelboim 1984] :Metric formulation

$$I_{EH}[g_{\mu\nu}] = -\frac{1}{16\pi G_2}\int d^2x\sqrt{g}(R-2\Lambda) \quad \rightarrow \quad \text{No dynamics}$$

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- Results will be presented in terms of a Gauge formulation

Gauge theory formulation: BF theory

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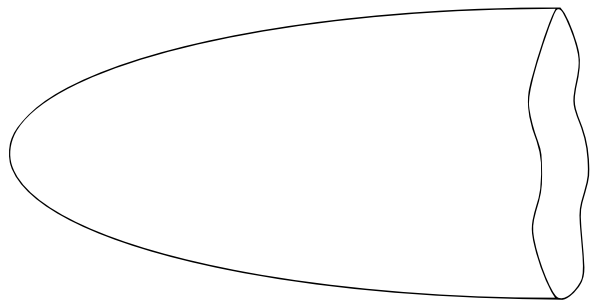

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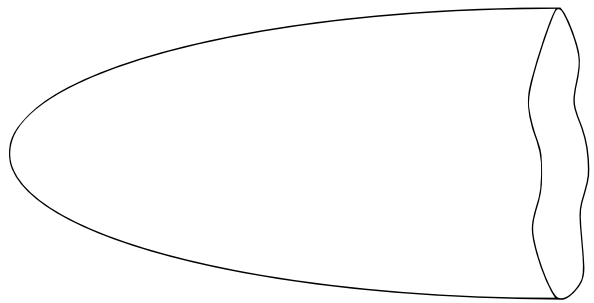

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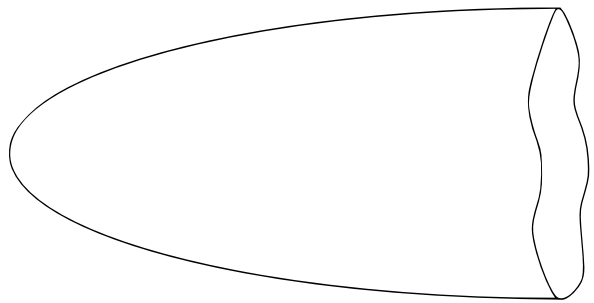

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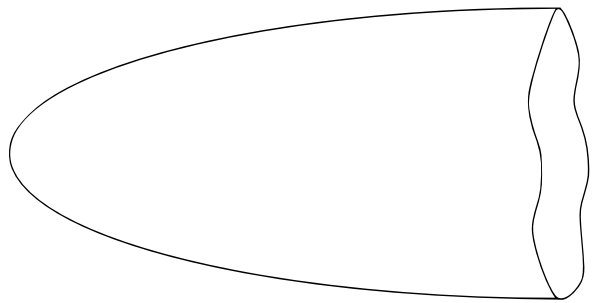

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- J_A generators of a group G.

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- Field equations: $\mathcal{F} = d\mathcal{A} + \mathcal{A}^2 = 0, \quad d\mathcal{X} + [\mathcal{A}, \mathcal{X}] = 0.$

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- Casimir function

$$C = -\frac{1}{2}\langle \mathcal{X}, \mathcal{X}\rangle \rightarrow \text{Conserved quantity}: \ \partial_\tau C \overset{\text{eom}}{=} 0.$$

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- Asymptotic symmetries get broken due to finite temperature effects.
- Dynamics at the boundary is controlled by (generalizations) of Schwarzian theory.

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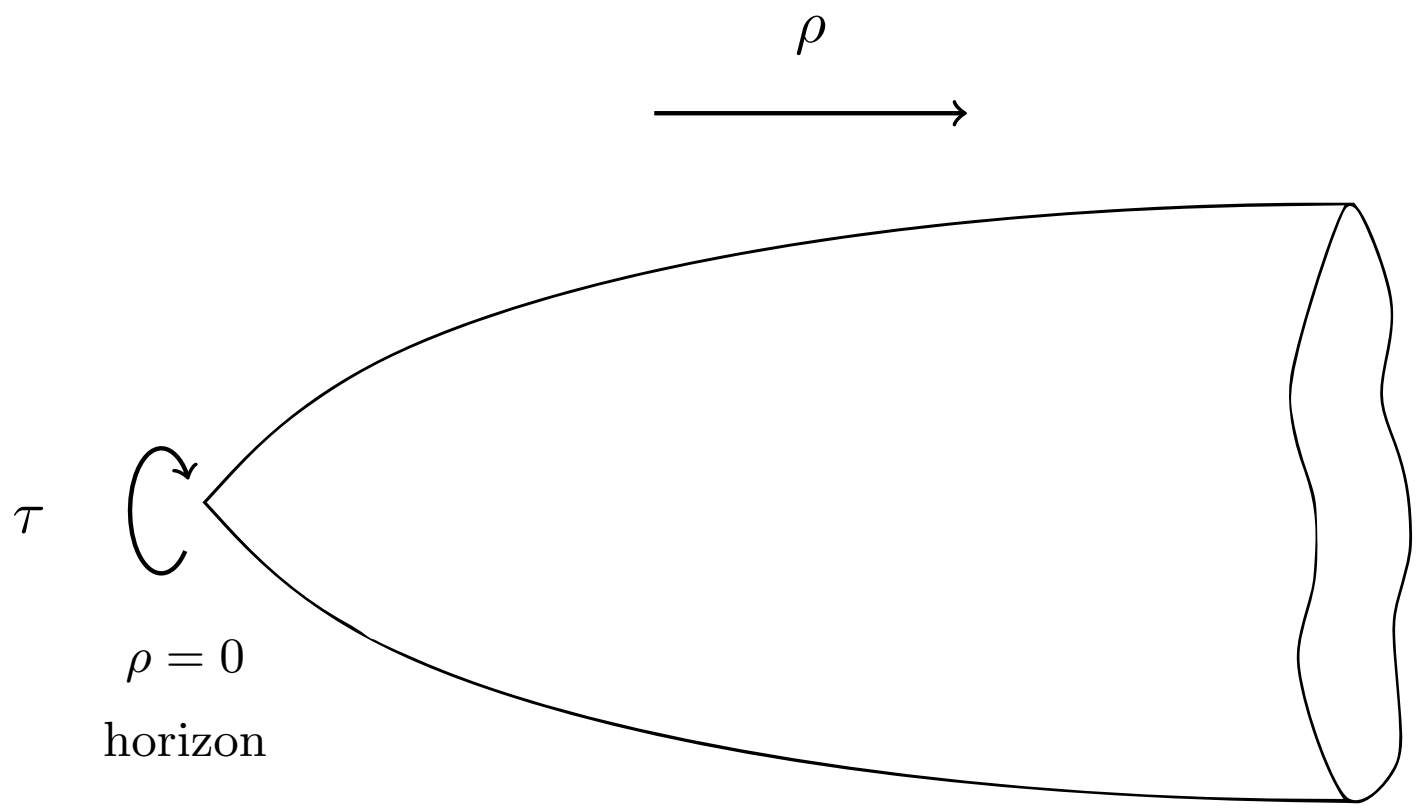

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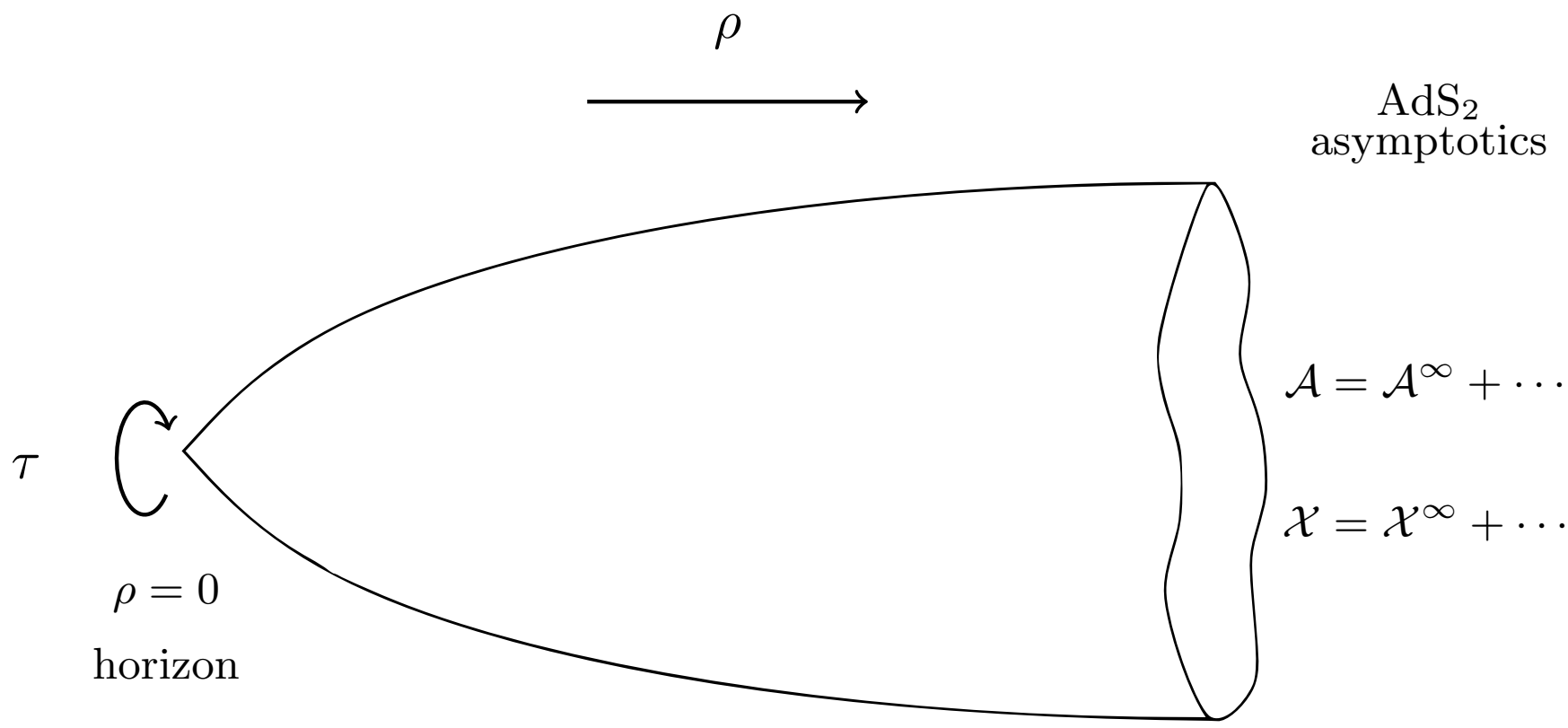

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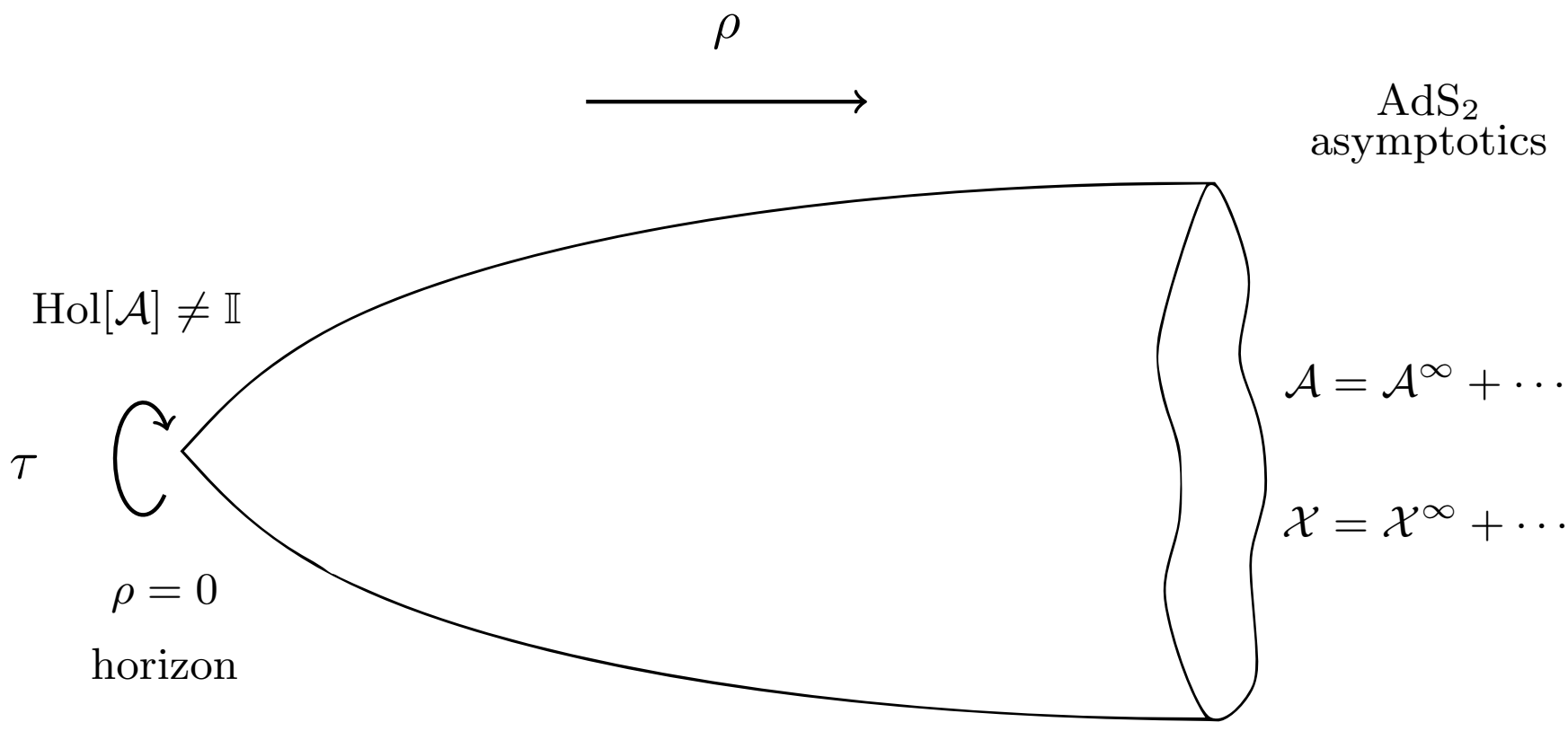

Finite temperature v/s Asymptotic symmetry

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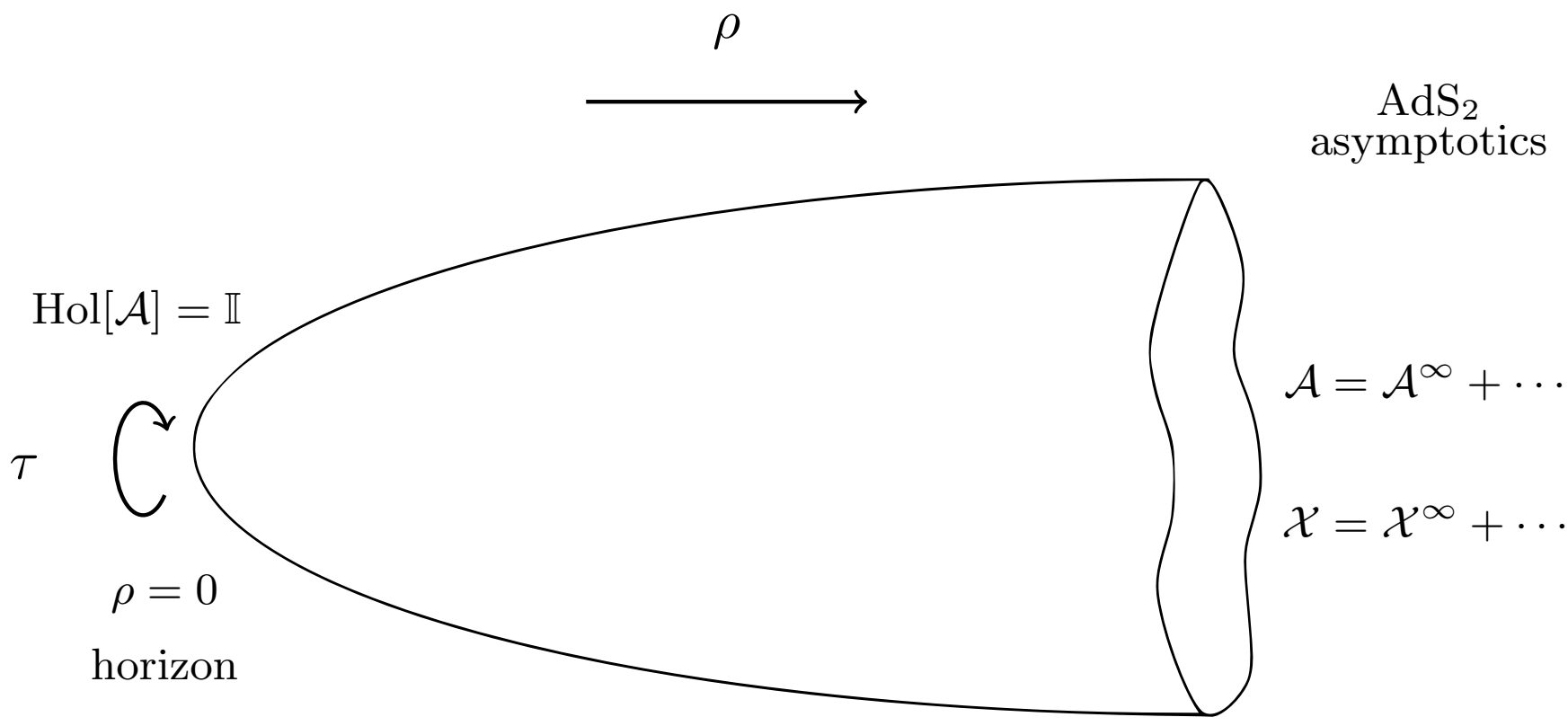

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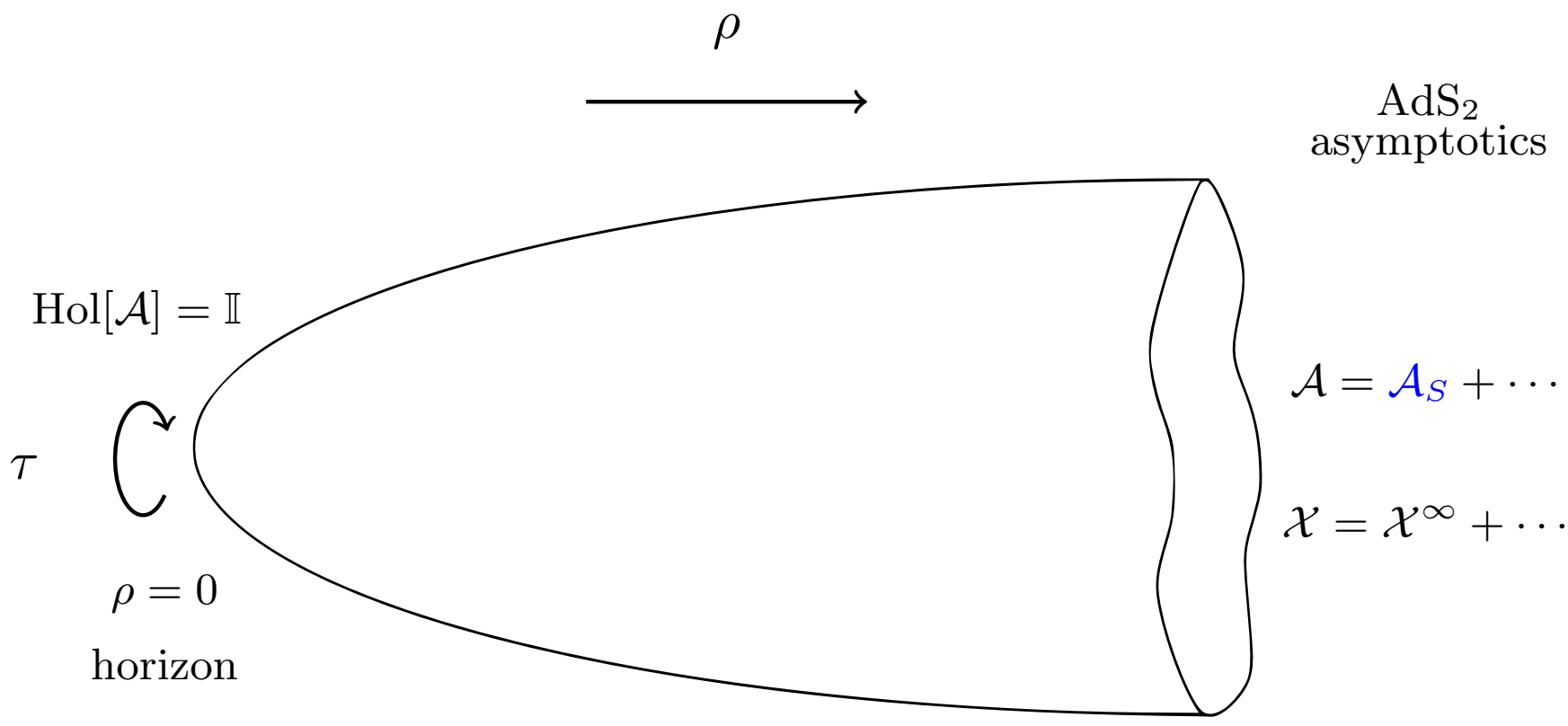

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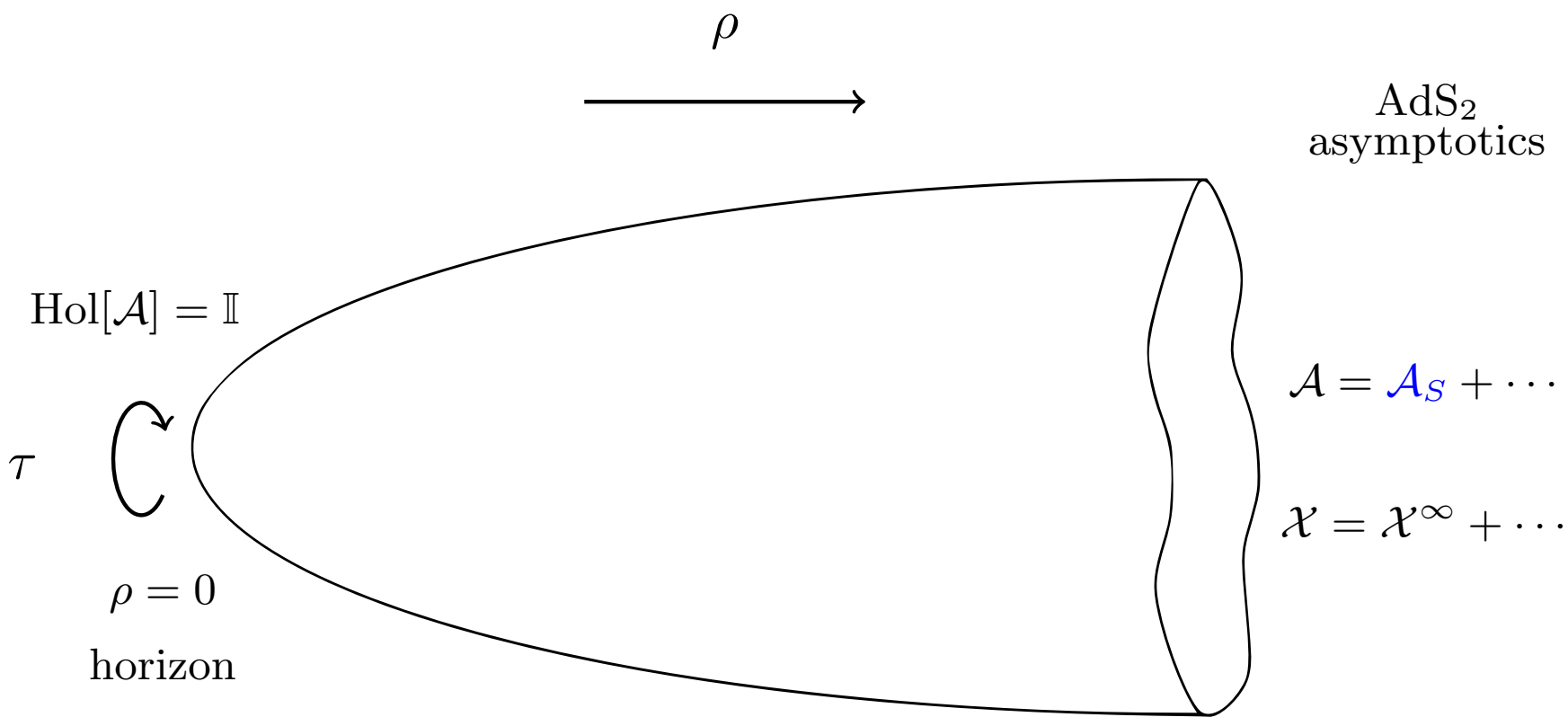
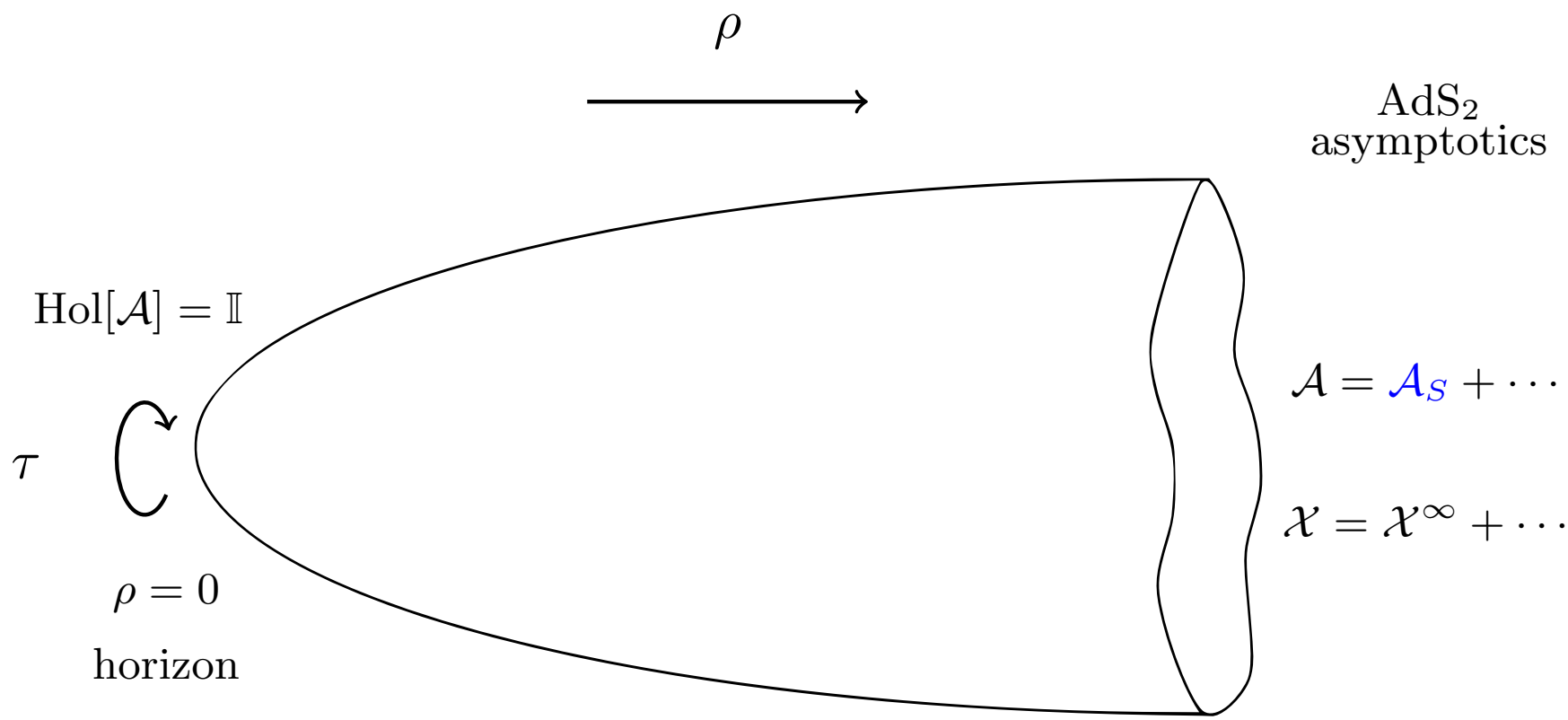


The asymptotic connection $\mathcal{A}^\infty$ becomes $\mathcal{A}_S$
This is consistent with smoothness of the space of solutions.

Cigar Geometry: Symmetry breaking



$\mathcal{G}_\infty \to \mathcal{G}_S$, Boundary modes live in $\mathcal{G}_\infty/\mathcal{G}_S$

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- f_τ appears as a one-form at the boundary.

Example: Warped–Virasoro boundary conditions

- $\mathrm{SL}(2,\mathbb{R}) \times U(1)$–JT theory: gravity+gauge field.

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- SYK in presence of an internal global symmetry: Bosonic part of $\mathcal{N} = 2$ Super-SYK model [Fu, Gaiotto, Maldacena, Sachdev 2017].

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- We can study higher rank groups by applying condition (a) and (b).

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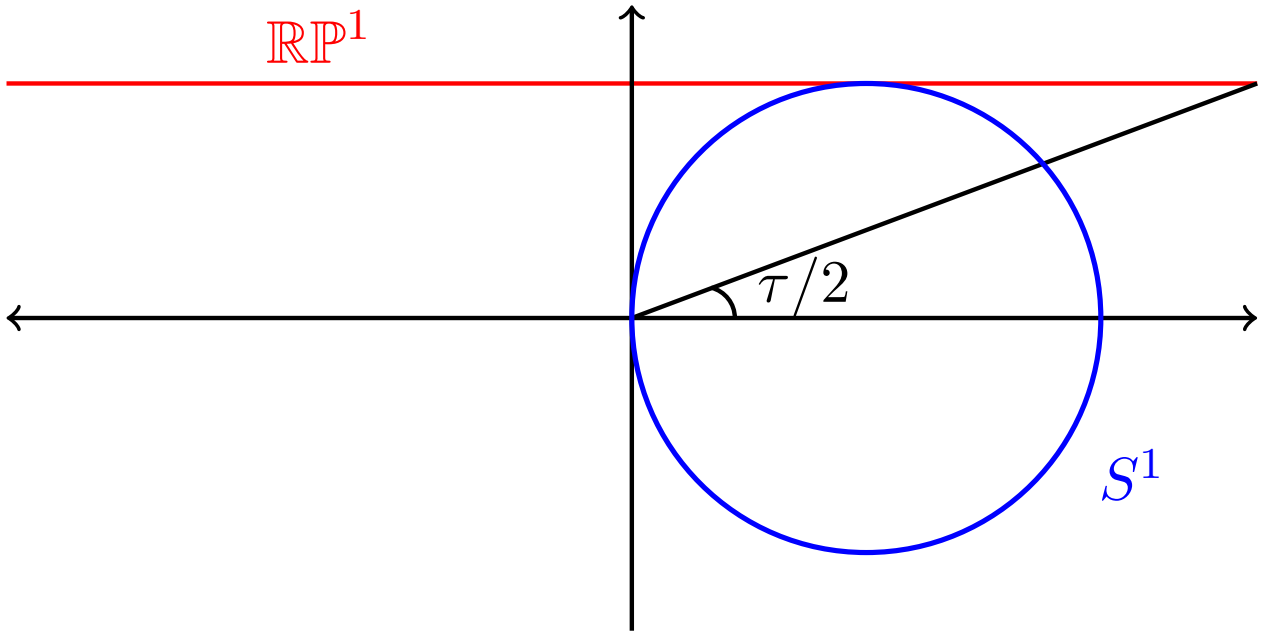

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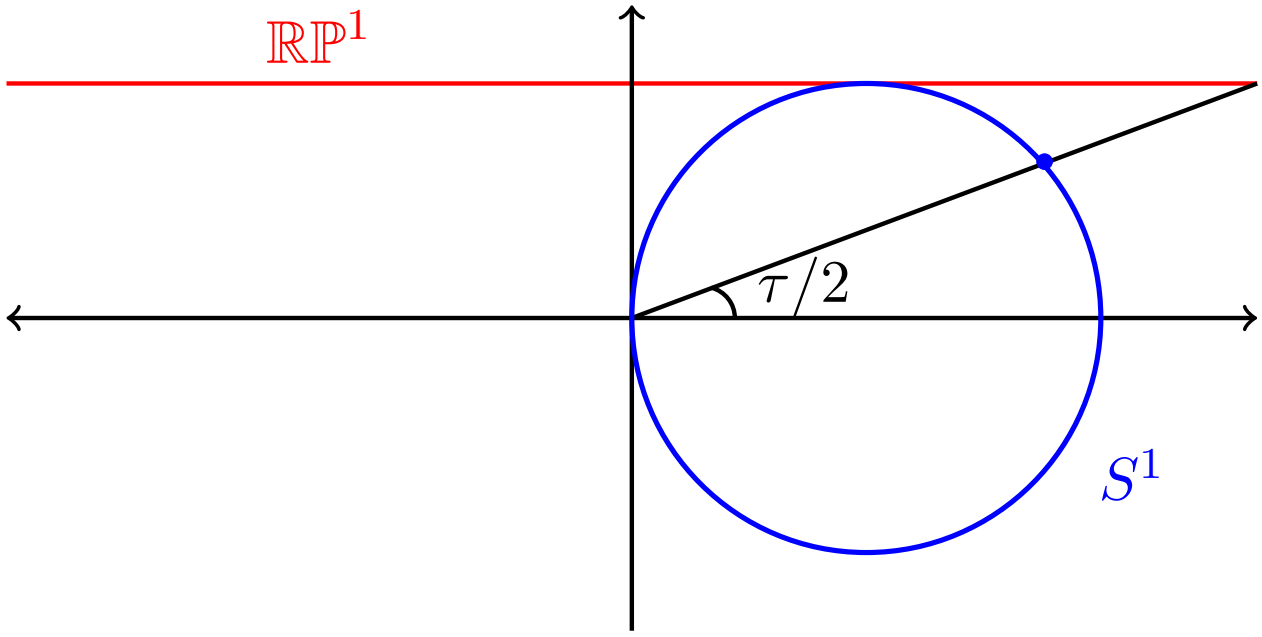

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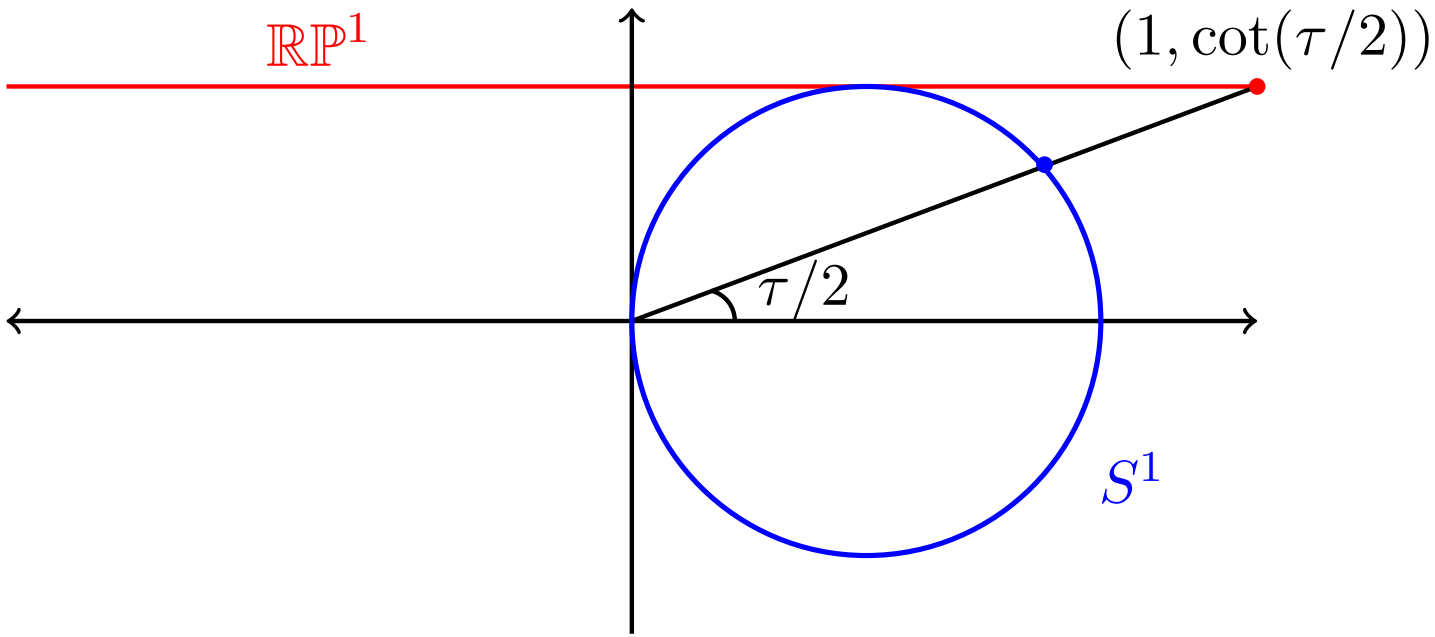
Projective map: $\tau \sim \tau + 2\pi$



$\mathrm{SL}(N=3,\mathbb{R})$-JT theory

$$\mathcal{L} = \mathcal{L}(\hat{s}_1, \hat{s}_2; f), \quad (\hat{s}_1, \hat{s}_2): \text{Central projection}$$

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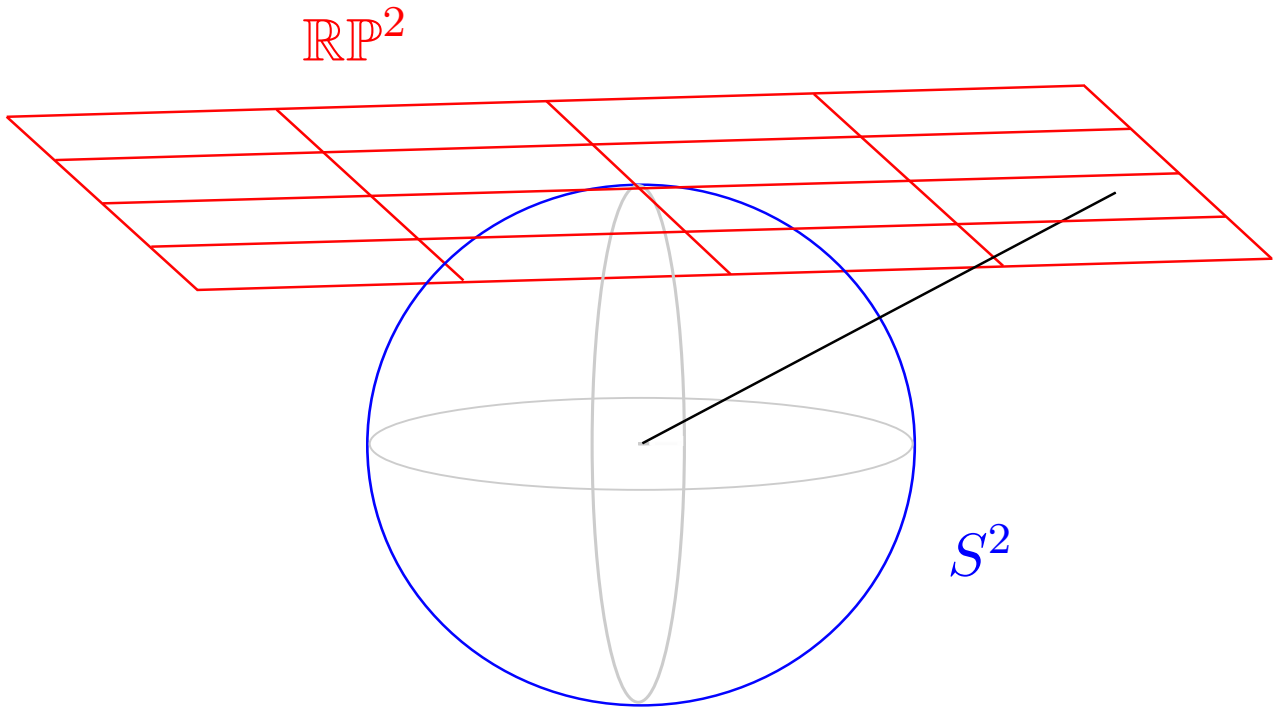

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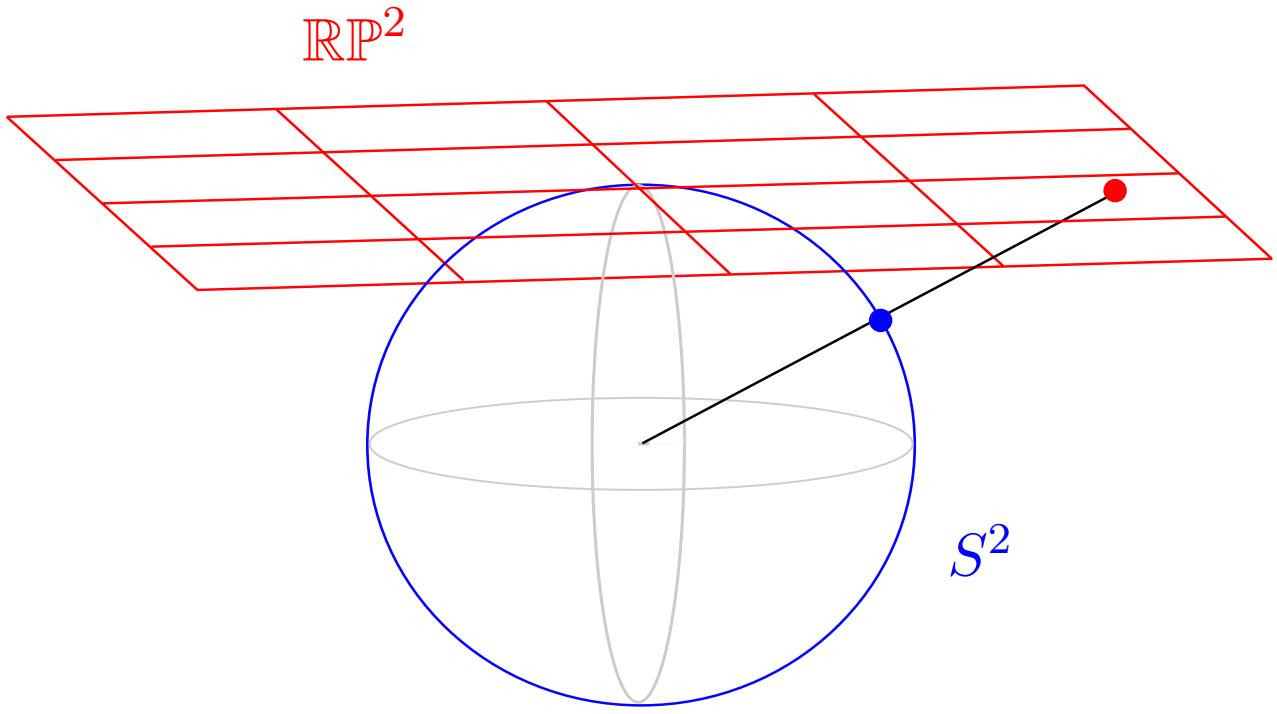

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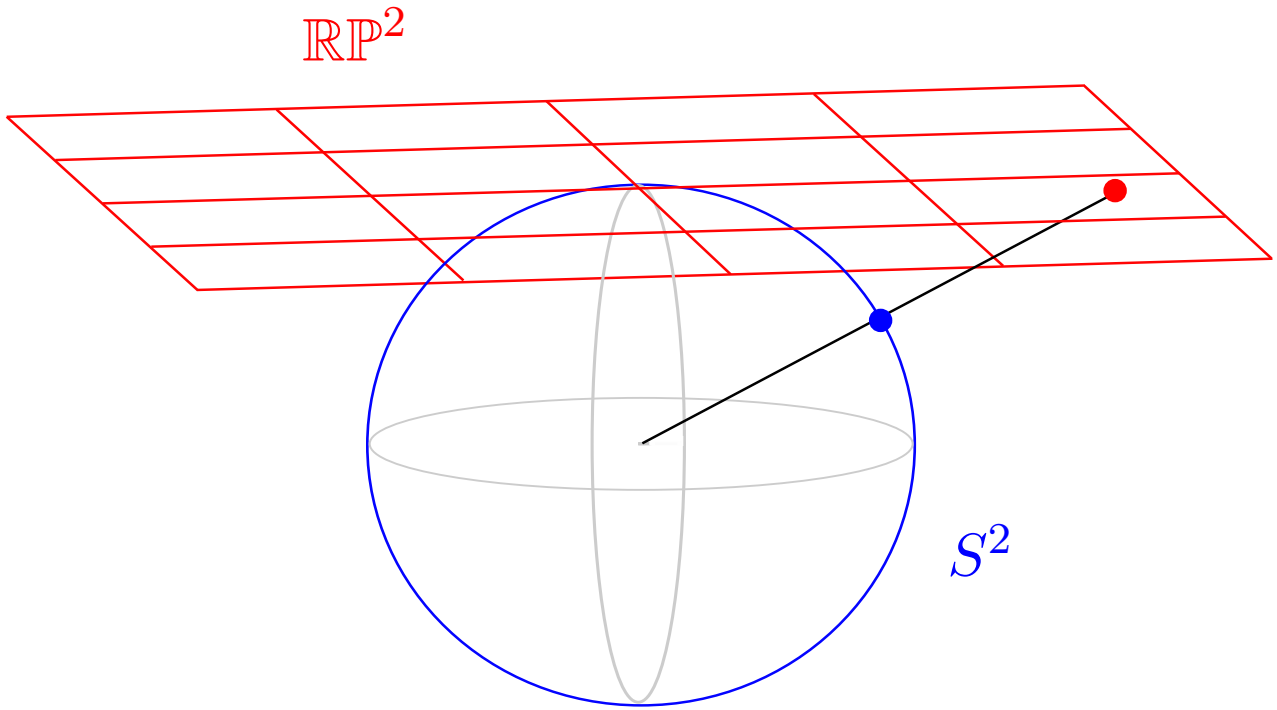

$(\hat{s}_1, \hat{s}_2)$ are the modes responsibles of the $\mathcal{W}_3$ symmetry breaking.

Conclusions

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- Boundary action is controlled by the Casimir function.
- The corresponding action match with effective description for (extensions of) SYK model.
- Extended SL(N) Schwarzian are found. Higher spin SYK?

Thank you!