Cosmological constant is a conserved charge

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Recent Trends in String Theory and Related Topics

IPM, May 8$^{\text{th}}$ 2018

Outline

1. History and motivations
2. Λ is a conserved charge
3. Conjugate chemical potential and first law of thermodynamics

History and motivations

100+1 years ago: introduction of Λ

142 Sitzung der physikalisch-mathematischen Klasse vom 8. Februar 1917

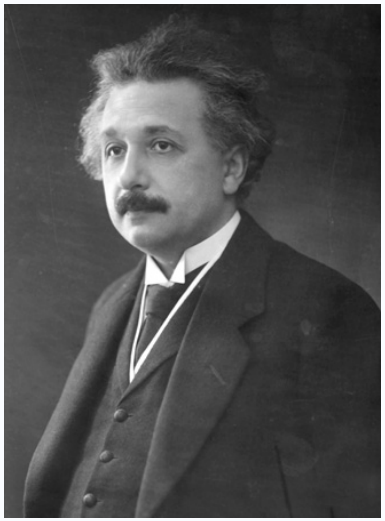
Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta\phi = 4\pi K\rho \qquad (1)$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt.



Konstante $-\lambda$ multiplizierten Fundamentaltensor $g_{\mu\nu}$ hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

$$G_{\mu\nu} - \lambda g_{\mu\nu} = -\varkappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right). \qquad (13a)$$

Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn

Why Einstein introduced Λ?

Einstein static universe

Einstein expected/assumed the universe to be:

- spatially homogenious and isotropic
- static
- with some matter $T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}, \qquad \rho > 0, \quad P \geq 0$

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$$\text{Metric} \qquad ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

Equation of motion: $\begin{cases} \frac{\Lambda}{3} = \frac{4\pi G}{3}(\rho + 3P) \\ \frac{k}{a^2} = \frac{\Lambda}{3} + \frac{8\pi G}{3}\rho \end{cases} \Rightarrow \begin{cases} \Lambda > 0 \\ k > 0 \end{cases}$

- Einstein universe is spatially closed with positive Λ.

Problems with Einstein universe

Instability

- Einstein universe is unstable: $\Lambda \uparrow$ Expansion, $\Lambda \downarrow$ Collapse

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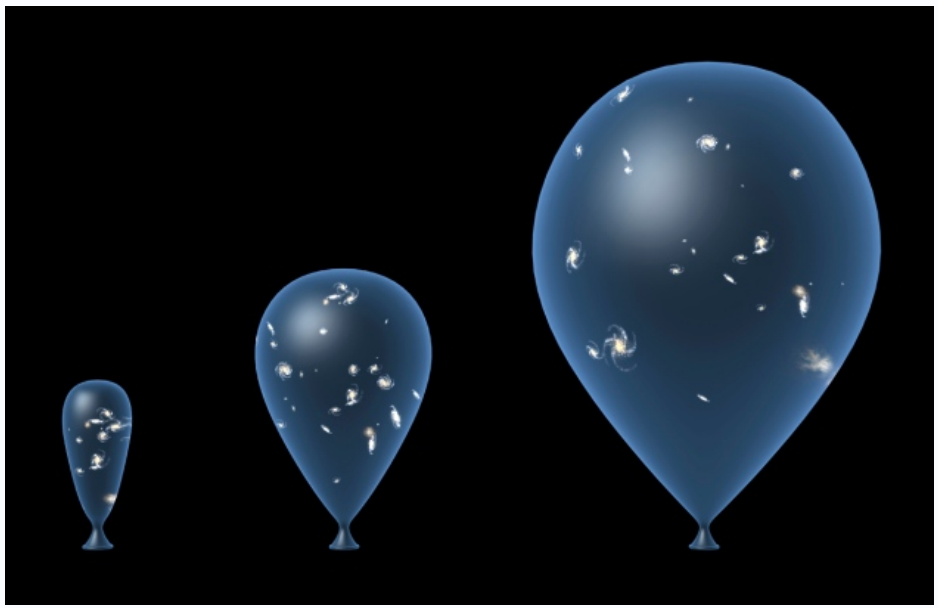
Observation: universe is not static!

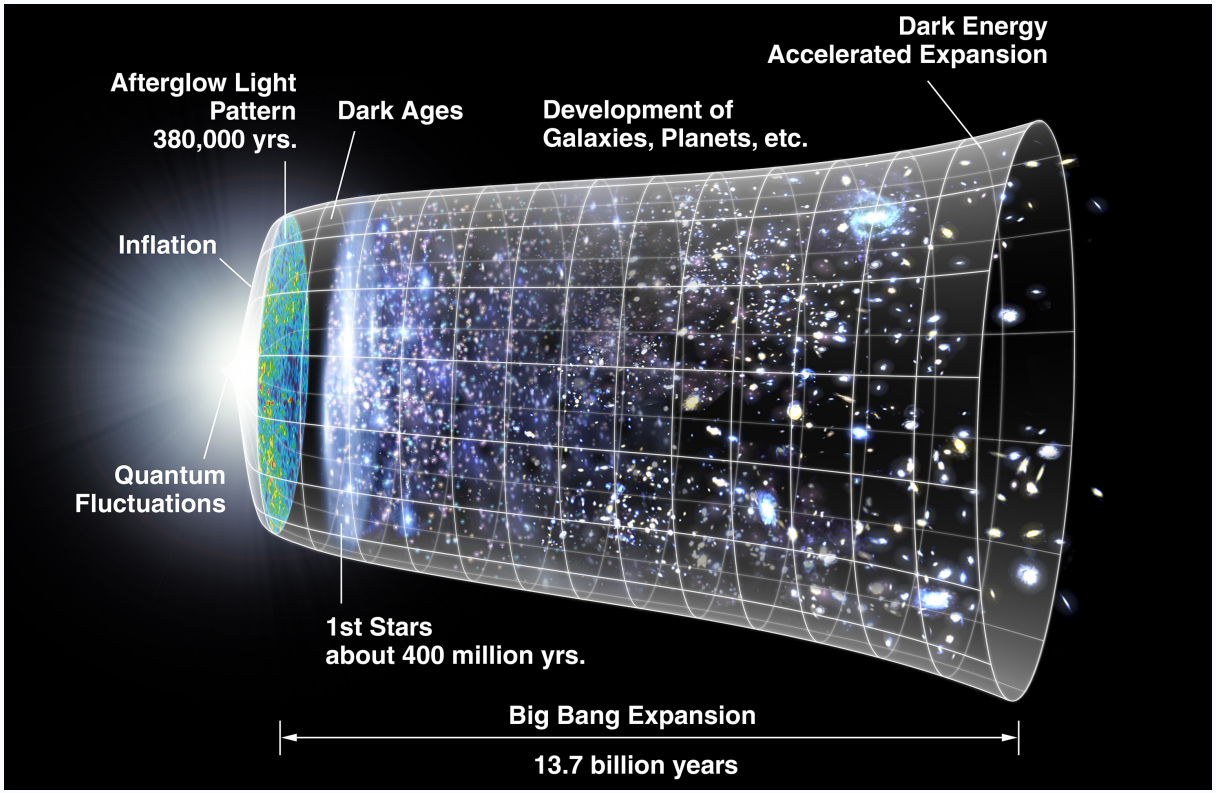
- In 1929 Hubble discovered that the universe is expanding.

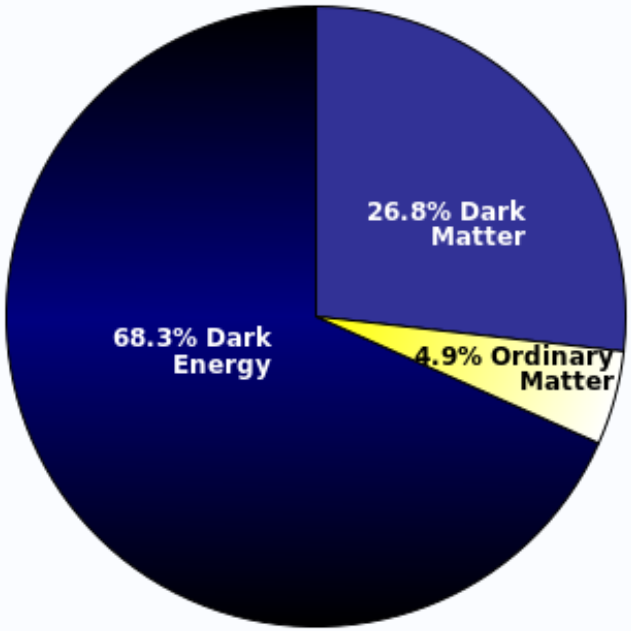


- Λ is not needed anymore. Λ: Biggest Einstein's mistake.

Revival of Λ in 1997: Λ as dark energy

ΛCDM: a successful model for the universe

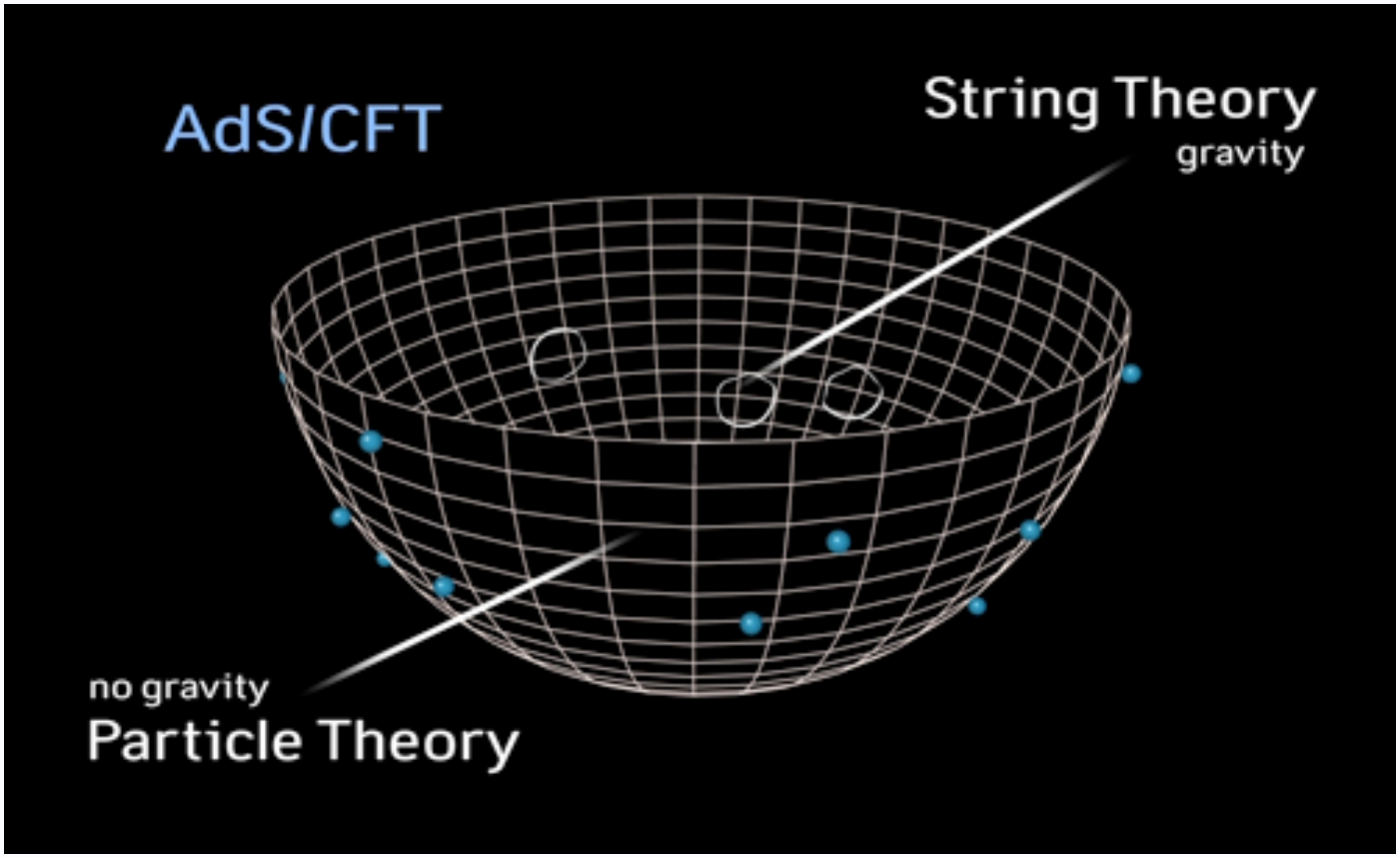




- About 68 percent of the universe is Λ!

AdS/CFT correspondence

J. Maldacena 1997



IIb supergravity on $AdS_5 \times S^5 \quad \leftrightarrow \quad$ 3+1 dim $\mathcal{N} = 4$ SYM in large N

- AdS is a solution to $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$ for $\Lambda < 0$

1980's: Λ as pressure

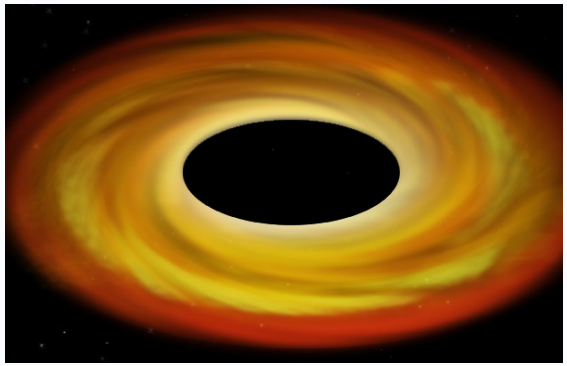
Λ can be considered as a pressure term

$$G_{\mu\nu} = \overbrace{8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}}^{8\pi G \tilde{T}_{\mu\nu}}, \qquad \tilde{T}^{\mu}_{\ \nu} = \begin{pmatrix} -\rho - \frac{\Lambda}{8\pi G} & 0 & 0 & 0 \\ 0 & P - \frac{\Lambda}{8\pi G} & 0 & 0 \\ 0 & 0 & P - \frac{\Lambda}{8\pi G} & 0 \\ 0 & 0 & 0 & P - \frac{\Lambda}{8\pi G} \end{pmatrix}$$

$$\rho \to \rho + \frac{\Lambda}{8\pi G}, \qquad P \to P - \frac{\Lambda}{8\pi G}$$

Generalized first law of thermodynamics

$$T_{\mathrm{H}} \delta S = \delta M - \Omega_{\mathrm{H}} \delta J - \Phi_{\mathrm{H}} \delta Q - V \delta P$$



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T_{H}	Hawking temperature
S	Entropy
M	Mass
Ω_{H}	Horizon angular velocity
J	Angular momentum
Φ_{H}	Horizon electric potential
Q	Electric charge

Some questions about $V\delta P$

1) Λ is a parameter in the Lagrangian. What does $\delta\Lambda$ mean?

cons'd charge ↓ cons'd charge ↓ cons'd charge ↓ cons'd charge ↓

$$T_{\rm H}\delta S = \delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q - V\delta P$$

2) All chemical potentials can be found around horizon. Why V can not be?

on the horizon ↓ on the horizon ↓ on the horizon ↓

$$T_{\rm H}\delta S = \delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q - V\delta P$$

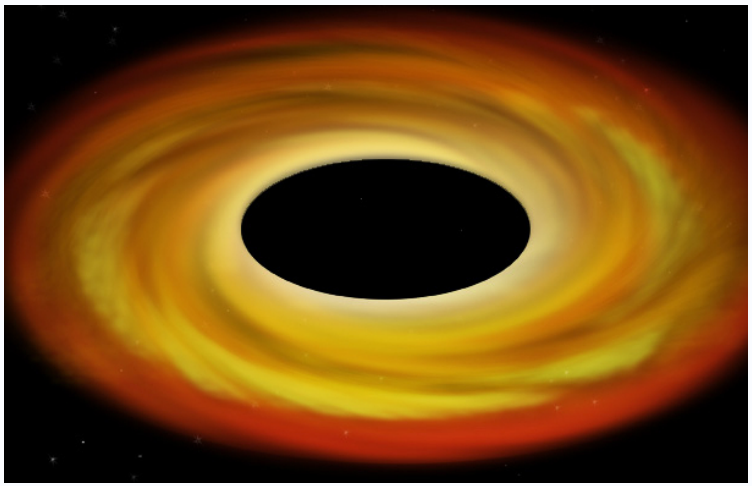
3) Why the order of Intensive/Extensive is reversed in $V\delta P$?

$$T_{\rm H}\delta S = \delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q - V\delta P$$

What is V?

(A)dS-Schwarzschild black hole $\quad V = \frac{r_{\rm H} \times A_{\rm H}}{3} = \frac{4\pi r_{\rm H}^3}{3}$,

(A)dS-Kerr black hole $\quad V = \frac{4\pi(r_{\rm H}^3 + r_{\rm H} a^2 + \frac{Gma^2}{1+\frac{\Lambda a^2}{3}})}{3(1+\frac{\Lambda a^2}{3})}$,



In general, one can find V from the first law

$\Rightarrow$ V: Thermodynamic volume (not a geometric volume)

Our motivations

Is Λ a conserved charge?

$$\underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}} T_{\text{H}}\delta S = \delta M \underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}} - \Omega_{\text{H}}\delta J \underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}} - \Phi_{\text{H}}\delta Q \underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}} - V\delta P \underset{\text{charge?}}{\overset{\text{cons'd}}{\downarrow}}$$

How to define an alternative to V on the horizon?

$$\underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}} T_{\text{H}}\delta S = \delta M \underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}} - \Omega_{\text{H}}\delta J \underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}} - \Phi_{\text{H}}\delta Q \underset{\text{horizon?}}{\overset{\text{on the}}{\downarrow}} - V\delta P$$

Can we prove the first law?

$$T_{\text{H}}\delta S \overset{?}{=} \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - V\delta P$$

Λ is a conserved charge

Λ from a top form

Introducing a top form $\boldsymbol{F}$ to the theory

Action: $\int d^D x\,(\mathcal{L}_g + \mathcal{L}_M) \quad \rightarrow \quad \int d^D x\,(\mathcal{L}_g + \mathcal{L}_M) \mp \frac{1}{16\pi G}\int d^D x \frac{2}{D!}\,F^2$

$\boldsymbol{F} = d\boldsymbol{A}, \qquad F^2 = F_{\mu_1 \ldots \mu_D} F^{\mu_1 \cdots \mu_D}, \qquad$ gauge symmetry $\begin{cases} \boldsymbol{A} \rightarrow \boldsymbol{A} + d\boldsymbol{\lambda} \\ \boldsymbol{F} \rightarrow \boldsymbol{F} \end{cases}$

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Field equations (equations of motion)

- $E^g_{\mu\nu} \pm \frac{2}{(D-1)!}(F_{\mu\rho_2\ldots\rho_D}F_\nu{}^{\rho_2\ldots\rho_D} - \frac{1}{2D}F^2 g_{\mu\nu}) = 8\pi G T_{\mu\nu},$
- $\nabla_\mu F^{\mu\mu_2\ldots\mu_D} = 0$
- Field equations for the matter field

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- $\boxed{\nabla_\mu F^{\mu\mu_2\ldots\mu_D} = 0} \quad \Rightarrow \quad F_{\mu_1\ldots\mu_D} = c\sqrt{-g}\,\epsilon_{\mu_1\ldots\mu_D}, \quad c = \text{const.} \geq 0$
- Field equations for the matter field

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Field equations (equations of motion)

Replacing on-shell $F_{\mu_1\ldots\mu_D} \Rightarrow$

- $E^g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \qquad \Lambda = \pm c^2$
- Field equations for the matter field

A. Aurilia et al, 1980, M. J. Duff et al, 1980, S. Hawking 1984, M. Henneaux et al, 1984

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In brief

Λ: **parameter of theory $\rightarrow$ parameter of solution**

$$\Lambda = \pm c^2, \qquad F_{\mu_1 \ldots \mu_D} = c\sqrt{-g}\,\epsilon_{\mu_1 \ldots \mu_D}$$

Λ: conserved charge of global part of the gauge symmetry

Noether current and charge for arbitrary $\boldsymbol{\lambda}$

$$J^{\mu}_{\lambda} = \frac{\partial \mathcal{L}}{\partial(\nabla_{\mu} A_{\nu\rho_3\ldots\rho_D})} \delta_{\boldsymbol{\lambda}} A_{\nu\rho_3\ldots\rho_D} = \frac{\mp F^{\mu\nu\rho_3\ldots\rho_D}}{4\pi G(D-2)!} \nabla_{\nu}\lambda_{\rho_3\ldots\rho_D} = \nabla_{\nu} Q^{\mu\nu}$$

$$\text{Surface charge density} \quad \rightarrow \quad \mathrm{Q}^{\mu\nu}_{\lambda} = \frac{\mp F^{\mu\nu\rho_3\ldots\rho_D}}{4\pi G(D-2)!} \lambda_{\rho_3\ldots\rho_D}$$

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$\hat{\boldsymbol{\lambda}}$: global part of the gauge symmetry

Considering the set of closed gauge transformation generators $d\boldsymbol{\lambda} = 0$, global part of the gauge transformation is defined to be $\hat{\boldsymbol{\lambda}} = \frac{\boldsymbol{\lambda}}{\oint_S \boldsymbol{\lambda}}$.

C: conserved charge of $\hat{\boldsymbol{\lambda}}$

$$\begin{cases} \boldsymbol{\lambda} \rightarrow \hat{\boldsymbol{\lambda}} \\ \boldsymbol{F} \rightarrow c\sqrt{-g}\, \epsilon_{\mu_1\ldots\mu_D} \end{cases} \quad \Rightarrow \quad C = \oint_S \mathbf{Q}_{\hat{\lambda}} = \frac{\pm c}{4\pi G} \oint_S \hat{\boldsymbol{\lambda}} = \frac{\pm c}{4\pi G}.$$

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$$\boxed{C = \frac{\pm\sqrt{|\Lambda|}}{4\pi G}, \quad \text{(or)} \quad \Lambda = \pm(4\pi G)^2 C^2}$$

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In brief

Λ:

parameter of theory $\rightarrow$ parameter of solution $\rightarrow$ conserved charge

$$\Lambda = \pm(4\pi G)^2 C^2 \qquad C : \text{conserved charge of } \hat{\boldsymbol{\lambda}}$$

Conjugate chemical potential of Λ and generalized first law of thermodynamics

Conjugate chemical potential

Inspiration: electric potential on the horizon

Maxwell field:

$$\mathcal{L} = -\frac{1}{4}F^2, \qquad F = dA, \qquad A \to A + d\lambda, \qquad \Phi_{\text{H}} \equiv \zeta_{\text{H}} \cdot A\Big|_{\text{H}}$$

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Θ_{H}: conjugate chemical potential for C

$$\boxed{\Theta_{\text{H}} \equiv \oint_{\text{H}} \zeta_{\text{H}} \cdot \boldsymbol{A}}$$

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Generalized first law of thermodynamics

$$T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - \Theta_{\text{H}}\delta C$$

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Generalized first law of thermodynamics

$$T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - \Theta_{\text{H}}\delta C$$

$$\begin{cases} \delta P = -\sqrt{|\Lambda|}\delta C \\ V \equiv -\dfrac{\Theta_{\text{H}}}{\sqrt{|\Lambda|}} \end{cases} \quad \Rightarrow \quad T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - V\delta P$$

Proof of the generalized first law

- Based on "K.H and M. M. Sheikh-Jabbari, Phys. Rev. D **93**, no. 4, 044074 (2016)".

Basics:

- Focusing on backgrounds with symplectic symmetry generators
- Diffs + gauge transformations $\eta = \{\xi^\mu, \lambda, \boldsymbol{\lambda}\}$

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Symplectic symmetry generators of charges

entropy: $\eta_S = \dfrac{1}{T_{\rm H}}\{\zeta^\mu_{\rm H}, -\Phi_{\rm H} 1, -\Theta_{\rm H}\hat{\boldsymbol{\lambda}}\}$, angular momentum: $\eta_J = \{-\partial_\varphi, 0, 0\}$

mass: $\eta_M = \{\partial_t, 0, 0\}$, electric charge: $\eta_Q = \{0, 1, 0\}$, $C:\ \eta_C = \{0, 0, \hat{\boldsymbol{\lambda}}\}$

Linearity of conserved charges w.r.t generators:

$$T_{\rm H}\eta_S = \eta_M - \Omega_{\rm H}\eta_J - \Phi_{\rm H}\eta_Q - \Theta_{\rm H}\eta_C$$

$$\text{L.o.c.c.w.r.t.g} \quad \Rightarrow \quad T_{\rm H}\delta S = \delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q - \Theta_{\rm H}\delta C$$

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In brief

- Θ_{H} **is defined independent of the first law, and on the horizon**

$$\Theta_{\mathrm{H}} \equiv \oint_{\mathrm{H}} \zeta_{\mathrm{H}} \cdot \boldsymbol{A}$$

- **The generalized first law is proved**

$$T_{\mathrm{H}} \delta S = \delta M - \Omega_{\mathrm{H}} \delta J - \Phi_{\mathrm{H}} \delta Q - \Theta_{\mathrm{H}} \delta C$$

Example: Kerr-AdS black hole

$$ds^2 = -\Delta_\theta\Big(\frac{1-\frac{\Lambda r^2}{3}}{\Xi} - \Delta_\theta f\Big)dt^2 - 2\Delta_\theta f a\sin^2\theta\, dt d\varphi + \Big(\frac{r^2+a^2}{\Xi} + f a^2\sin^2\theta\Big)\sin^2\theta\, d\varphi^2$$

$$+ \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2, \qquad \boldsymbol{A} = -\frac{\sqrt{|\Lambda|}\big(r^3 + 3ra^2\cos^2\theta + \frac{Gma^2}{\Xi}\big)\sin\theta}{3\Xi}dt\wedge d\theta\wedge d\varphi$$

$$\rho^2 \equiv r^2 + a^2\cos^2\theta\,, \quad \Delta_r \equiv (r^2+a^2)\Big(1-\frac{\Lambda r^2}{3}\Big) - 2Gmr,$$

$$\Delta_\theta \equiv 1 + \frac{\Lambda a^2}{3}\cos^2\theta\,, \qquad \Xi \equiv 1 + \frac{\Lambda a^2}{3}\,, \quad f \equiv \frac{2Gmr}{\rho^2\Xi^2}$$

$$\kappa_{\rm H} = \frac{r_{\rm H}\big(1-\frac{\Lambda a^2}{3} - \Lambda r_{\rm H}^2 - \frac{a^2}{r_{\rm H}^2}\big)}{2(r_{\rm H}^2+a^2)}, \quad \Omega_{\rm H} = \frac{a\big(1-\frac{\Lambda r_{\rm H}^2}{3}\big)}{r_{\rm H}^2+a^2}, \quad \Theta_{\rm H} = -\frac{\sqrt{|\Lambda|}\ 4\pi\big(r_{\rm H}^3 + r_{\rm H}a^2 + \frac{Gma^2}{\Xi}\big)}{3\Xi}$$

$$M = \frac{m}{\Xi^2}, \qquad J = \frac{ma}{\Xi^2}, \qquad S = \frac{\pi(r_{\rm H}^2+a^2)}{G\Xi}, \qquad C = \frac{\pm\sqrt{|\Lambda|}}{4\pi G}$$

$$T_{\rm H}\delta S = \delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q - \Theta_{\rm H}\delta C$$

Conclusion

We showed that Λ is a conserved charge

$$\underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}}\ \underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}}\ \underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}}\ \underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}}\ \underset{\text{charge}}{\overset{\text{cons'd}}{\downarrow}}$$

$$T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - \Theta_{\text{H}}\delta C, \qquad \Lambda = \pm(4\pi G)^2 C^2$$

We defined Θ_{H} on the horizon, and independent of first law

$$\underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}}\ \underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}}\ \underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}}\ \underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}}$$

$$T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - \Theta_{\text{H}}\delta C, \qquad \Theta_{\text{H}} = \oint_{\text{H}} \zeta_{\text{H}} \cdot \boldsymbol{A}$$

We proved the generalized first law

$$T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - \Theta_{\text{H}}\delta C$$

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$$T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - V\delta P, \qquad -(8\pi G)P = \Lambda = \pm(4\pi G)^2 C^2$$

We defined Θ_{H} on the horizon, and independent of first law

$$\underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}}\qquad \underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}}\ \underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}}\ \underset{\text{horizon}}{\overset{\text{on the}}{\downarrow}}$$

$$T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - V\,\delta P, \qquad -\sqrt{\Lambda}V \equiv \Theta_{\text{H}} = \oint_{\text{H}} \zeta_{\text{H}} \cdot \boldsymbol{A}$$

We proved the generalized first law

$$T_{\text{H}}\delta S = \delta M - \Omega_{\text{H}}\delta J - \Phi_{\text{H}}\delta Q - V\ \delta P$$

THANK YOU