Edge modes, boundary dynamics, and entanglement entropy

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May 8th 2017

Outline

1. Motivations

2. Covariant phase space

3. Chern–Simons

4. Gravity

5. Dynamics

6. Perspectives

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  - degrees of freedom attached to subsystems
  - relationships between these subsystems

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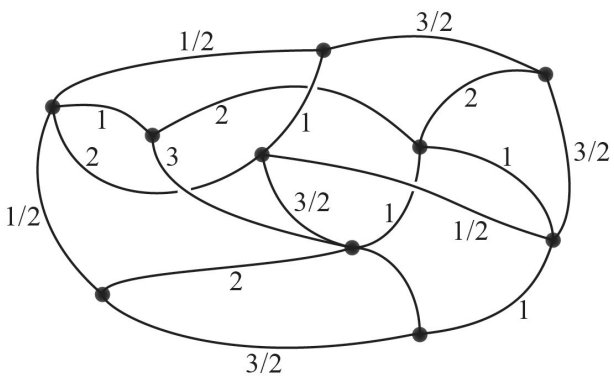
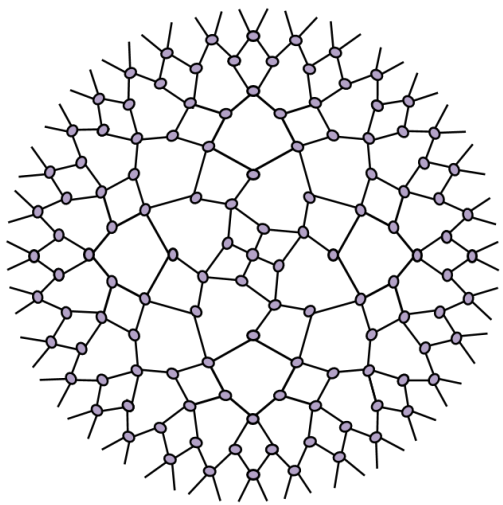


quantum gravity

Ashtekar, Rovelli, Penrose, Smolin



condensed matter

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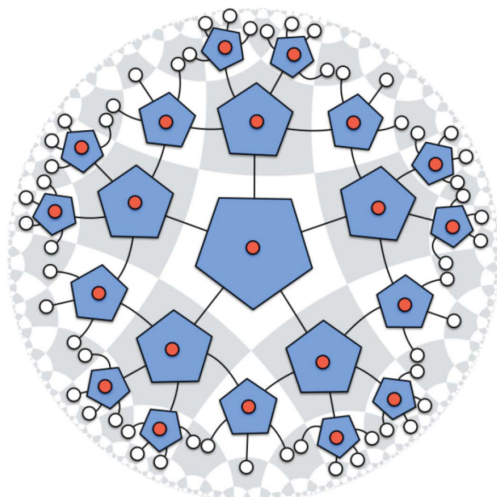

AdS/CFT & quantum info.

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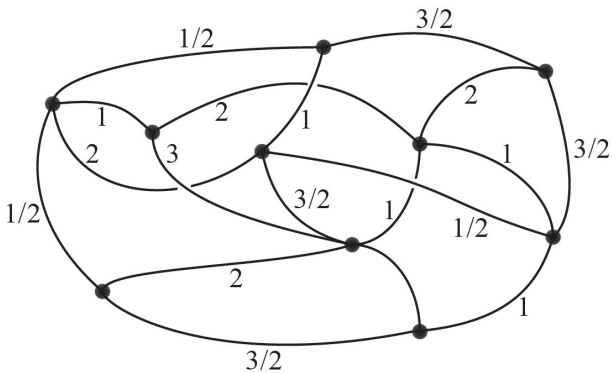

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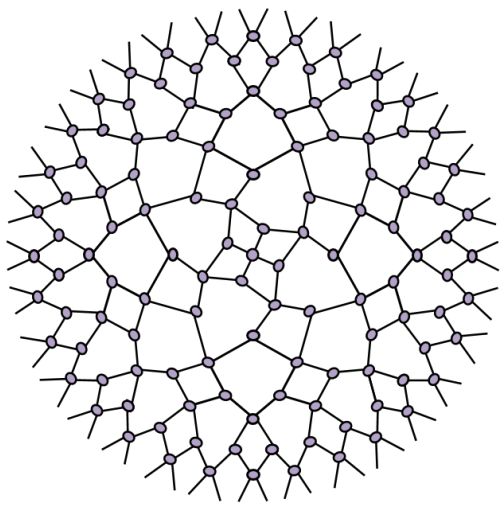

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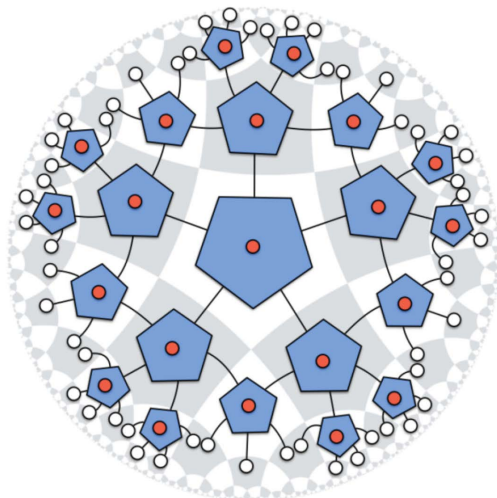

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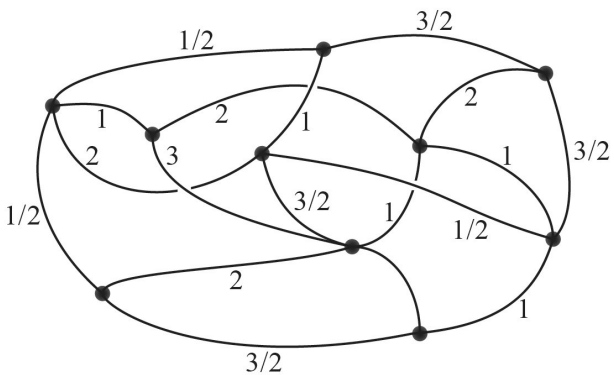

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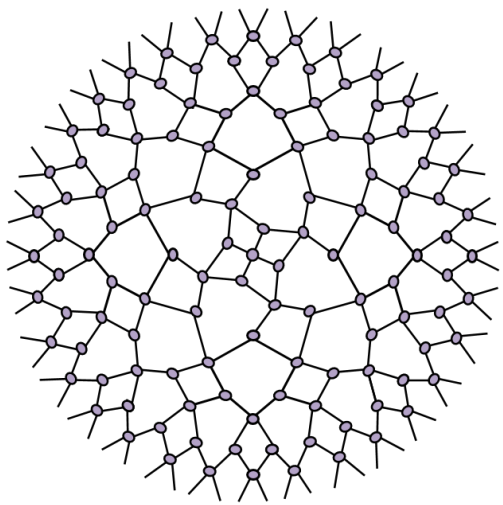

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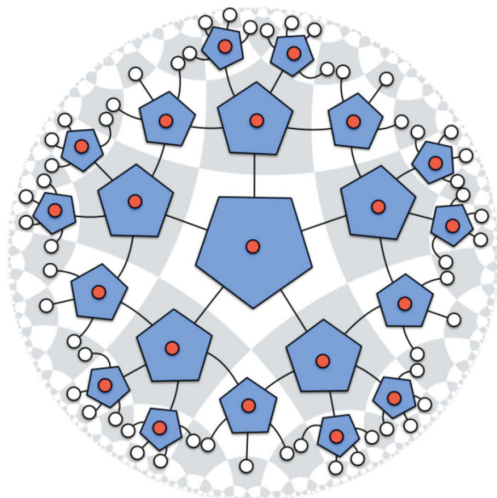

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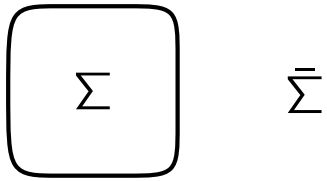

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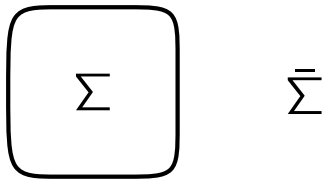

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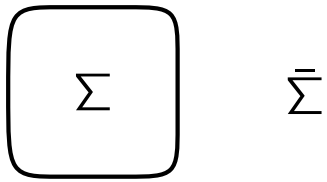

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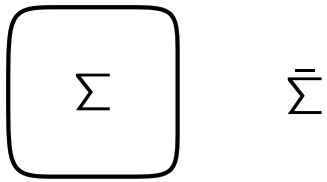

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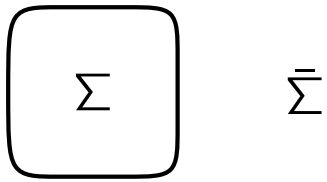

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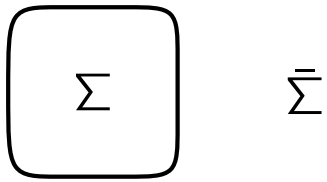

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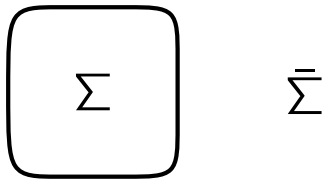

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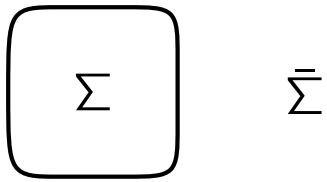

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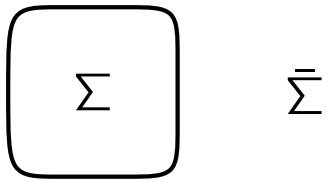

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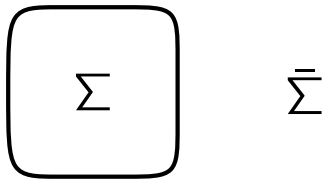

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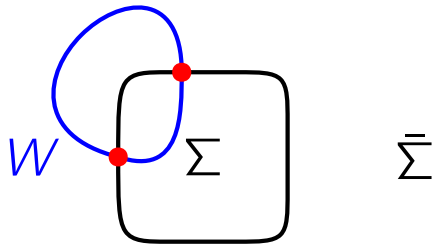

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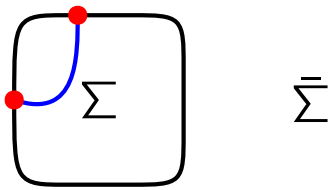

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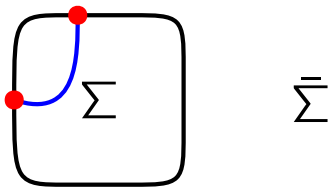

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- We have to **extend the Hilbert space** and introduce boundary degrees of freedom
  - in gauge theory: open ends of Wilson lines, i.e. charges
  - in gravity: coordinates locating the separating surface

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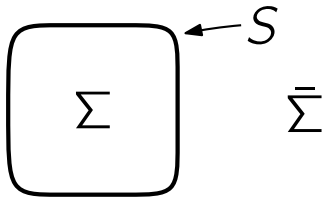

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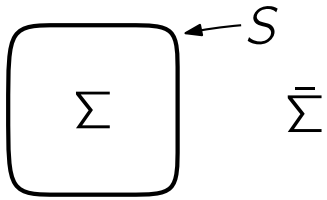

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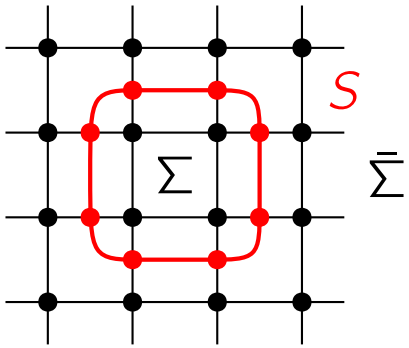

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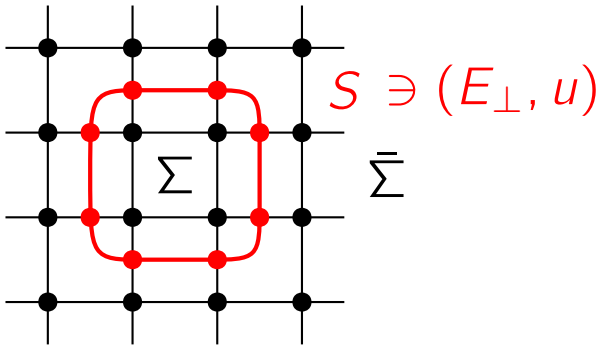

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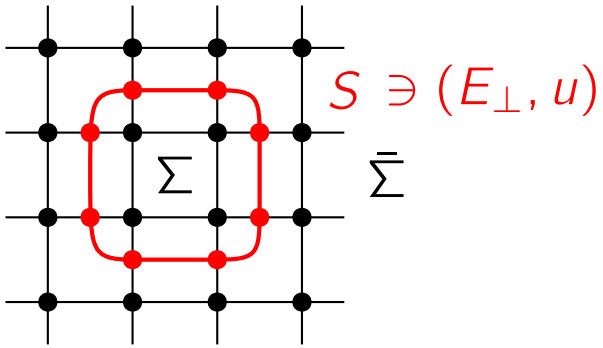

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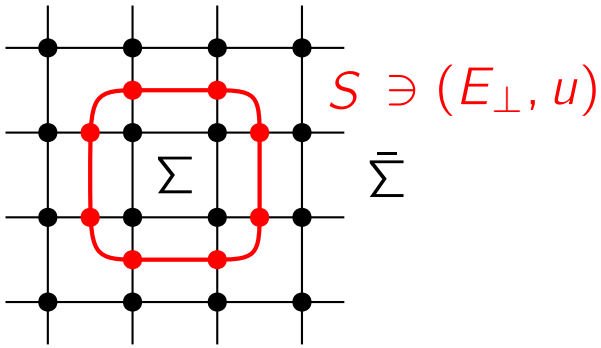

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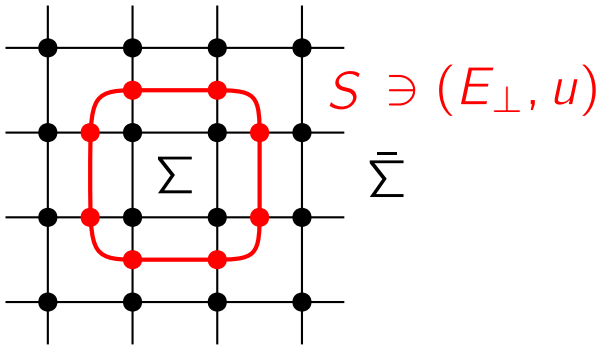

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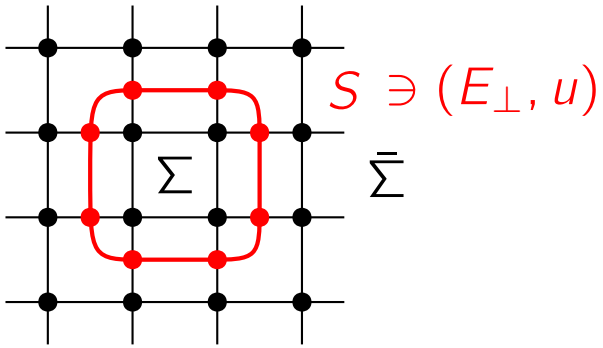

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- Edge modes contribute to the entanglement entropy (Buividovich, Casini, Donnelly, Ghosh, Huerta, Polikarpov, Soni, Trivedi, Wall, . . .)

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Soft
theorems

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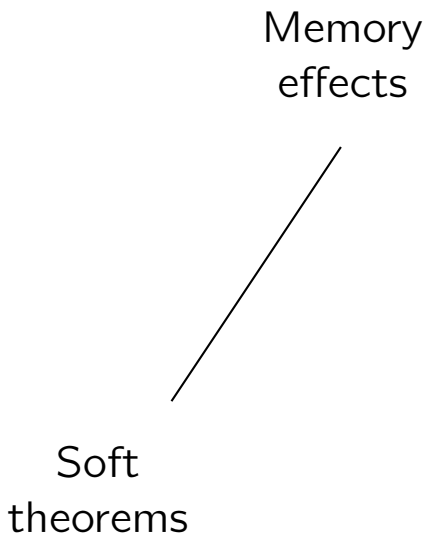

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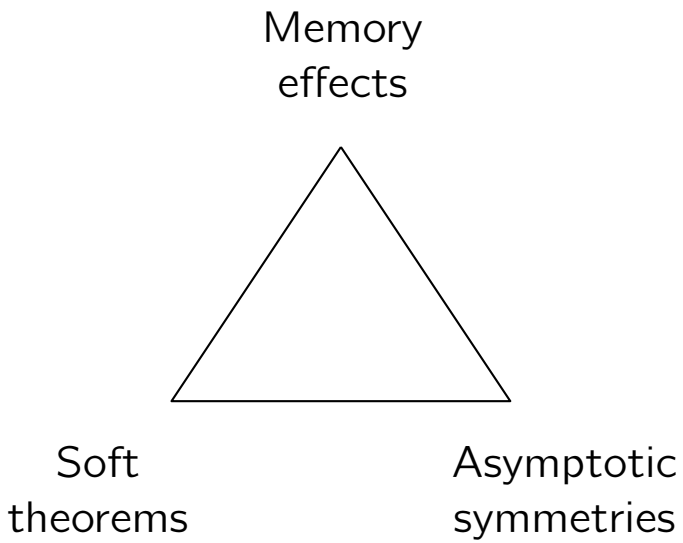

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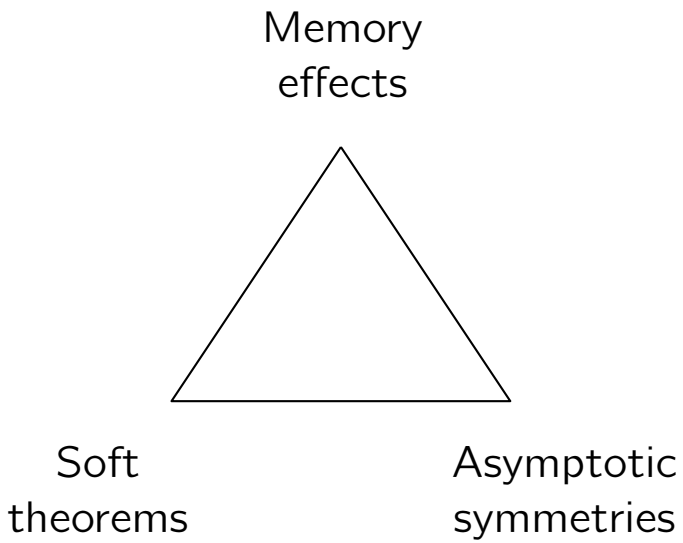

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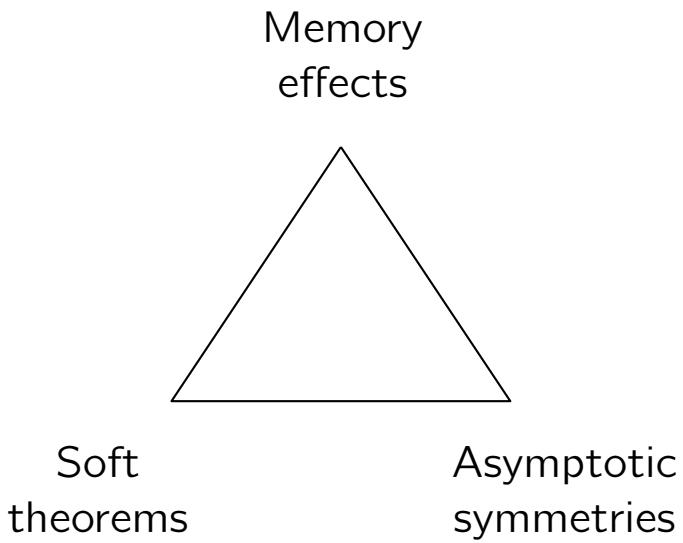

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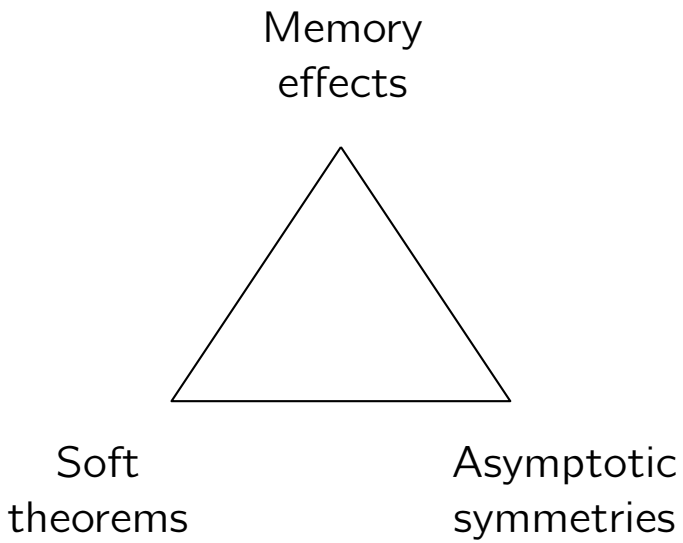

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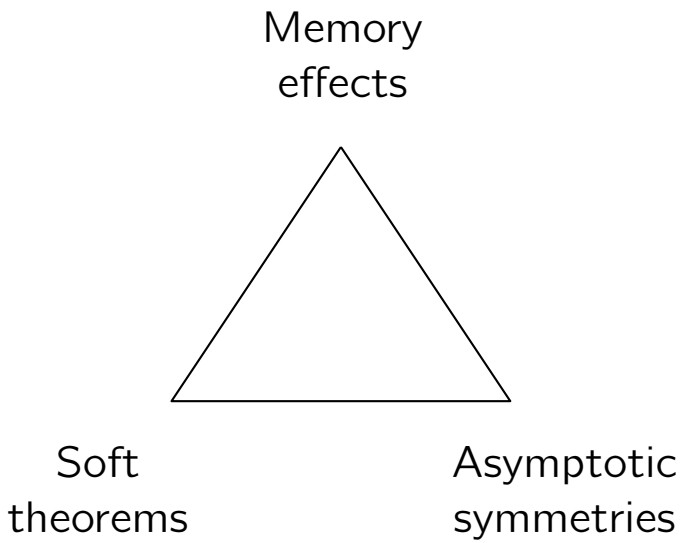

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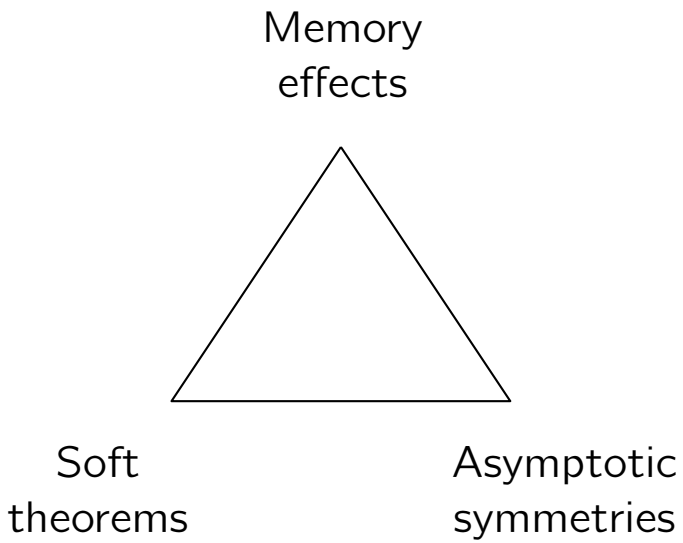

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- Be **general**: finite boundaries with no boundary conditions (recall that S can be arbitrary)

Motivations

Extended phase space constructions

- Metric gravity and Yang–Mills (Donnelly, Freidel, JHEP)
- Chern–Simons theory, BF theory, first order gravity (MG, Nucl. Phys. B + JHEP)
- Higher curvature (Speranza, JHEP)
- String theory (Balasubramanian, Parrikar, PRD)

Covariant phase space

1. Motivations

2. Covariant phase space

3. Chern–Simons

4. Gravity

5. Dynamics

6. Perspectives

Covariant phase space

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- Observables (gauge and symmetries commute) satisfying an algebra $\mathcal{G}_S = \mathrm{U}(1)$ Kač–Moody

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- Insight into factorization and dynamics

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- In addition, diffeomorphisms also move the region $\Sigma \to Y(\Sigma)$

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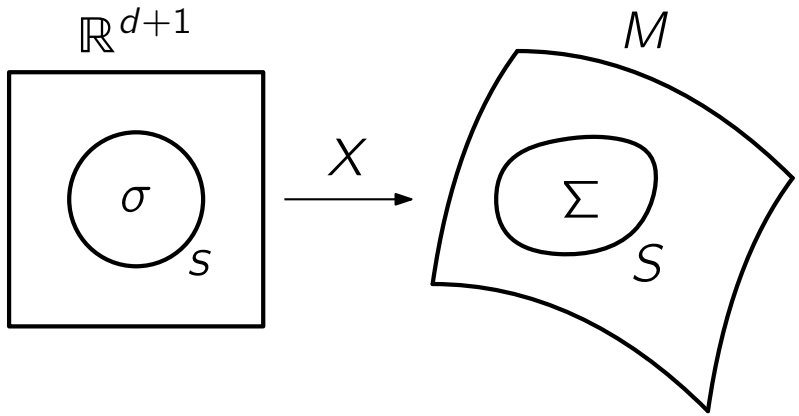

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- In addition, diffeomorphisms also move the region $\Sigma \to Y(\Sigma)$
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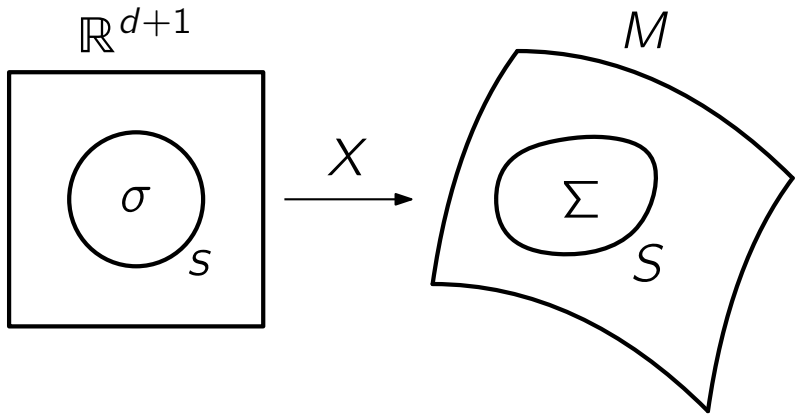
Gravity



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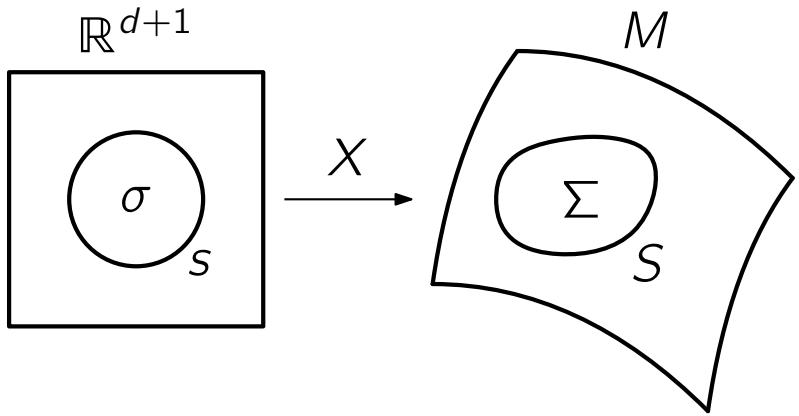

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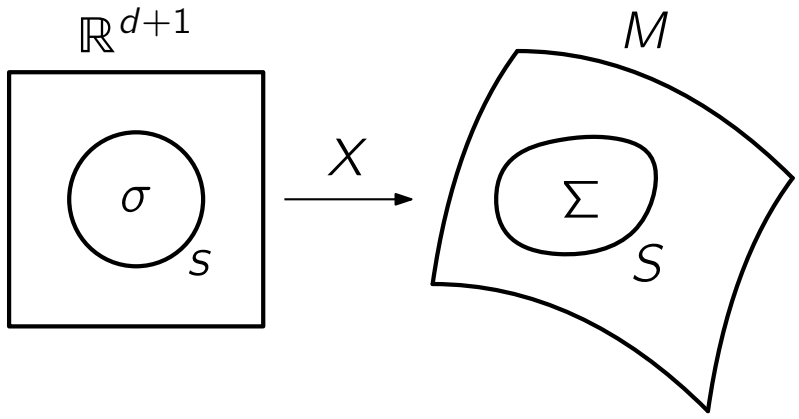
- This quantity is now **invariant under diffeomorphisms** $Y : M \to M$, $X \mapsto Y^*X = Y^{-1} \circ X$



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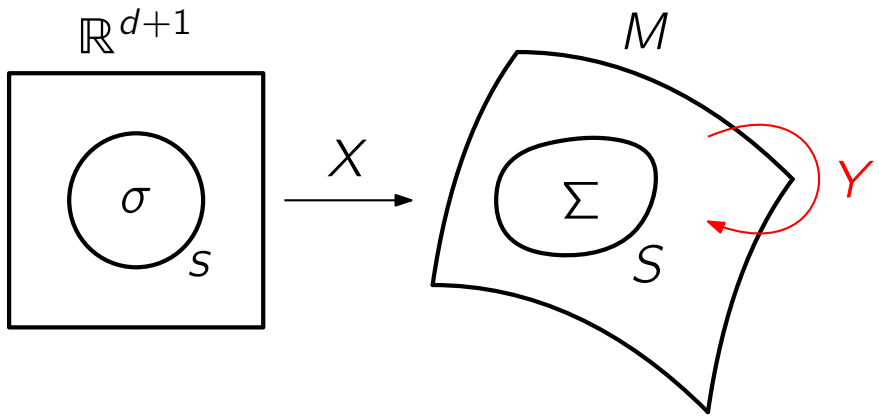

Gauge transformations

- Diffeomorphism $Y : M \to M$
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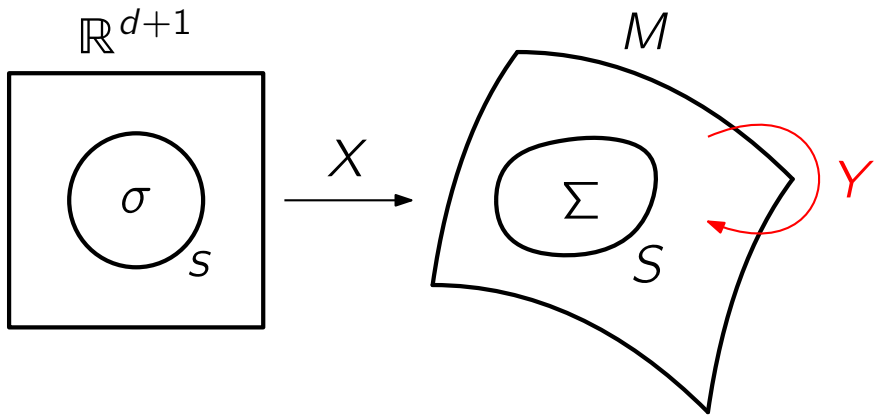

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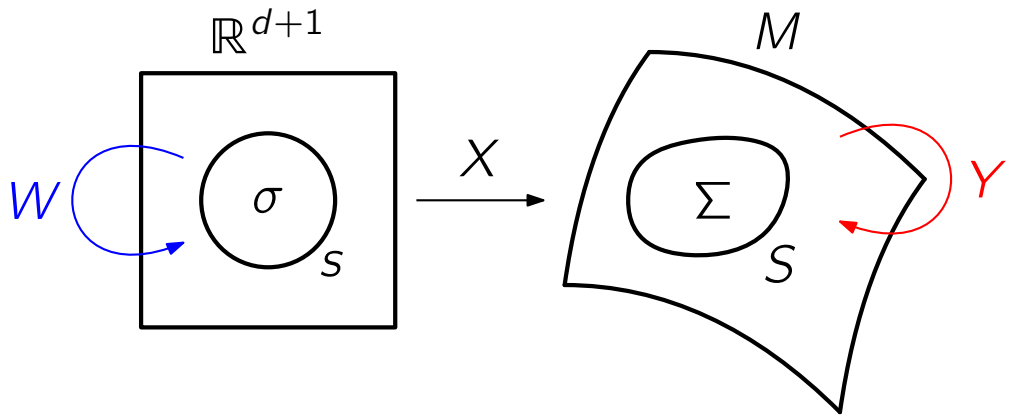

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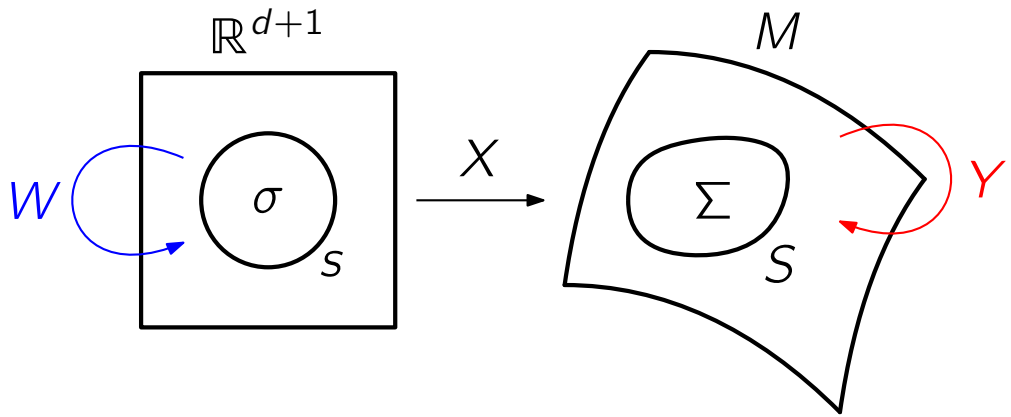

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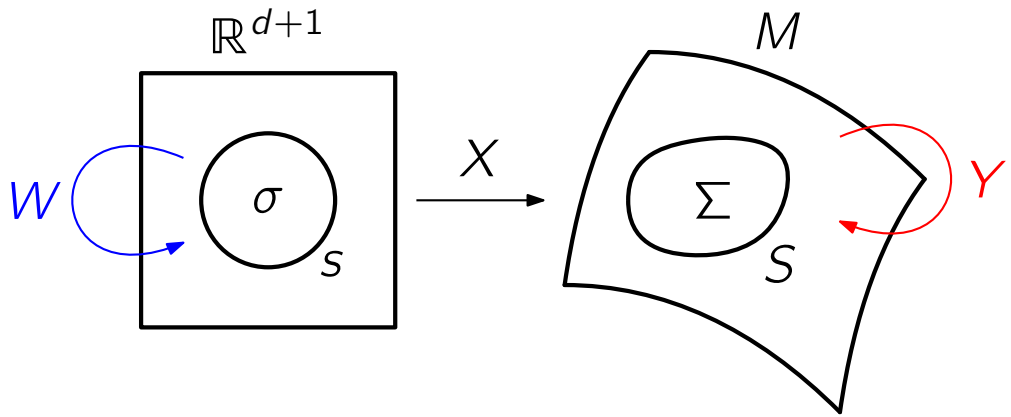

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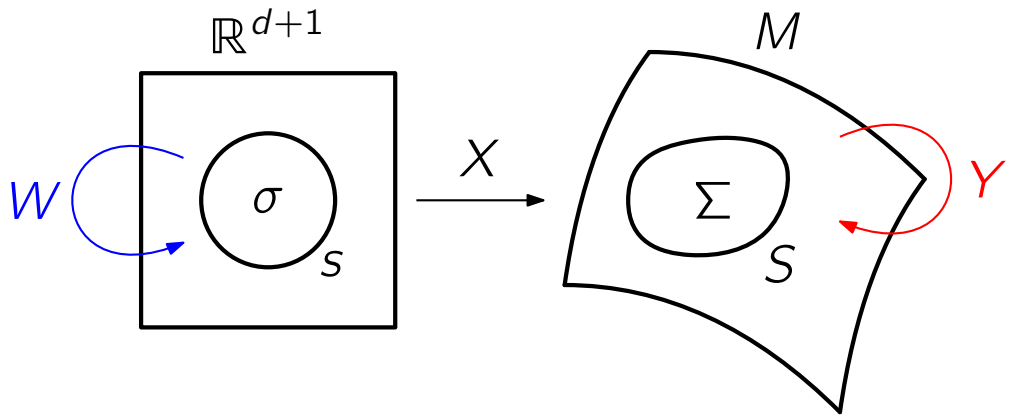

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- E.g. flat Euclidean: Kač–Moody algebra of Poincaré, i.e. ISU(2) + central extension

Dynamics

1. Motivations

2. Covariant phase space

3. Chern–Simons

4. Gravity

5. Dynamics

6. Perspectives

Dynamics

Mixed variational principle (MG, Jai-akson)

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- We recover the extended symplectic potential

$$\theta_\mathrm{e} = \theta + \mathrm{d}(\delta a J - a\delta A) \simeq \theta + \mathrm{d}(\delta a \mathrm{D}a - a\delta A)$$

Factorization and gluing of partition functions

- For two regions M and $\bar{M}$ we can write

$$\mathcal{Z} = \int \mathcal{D}[A, \bar{A}, J, \bar{J}, a, \bar{a}]\ \delta(J + \bar{J})\ \delta(a + \bar{a})$$
$$\exp \mathrm{i} \left(\int_M A \wedge F + \int_{\bar{M}} \bar{A} \wedge \bar{F} + \int_{\partial M} aF + \bar{a}\bar{F} + J \wedge \mathrm{D}a + \bar{J} \wedge \bar{\mathrm{D}}\bar{a} + \frac{1}{2} * J \wedge J - \frac{1}{2} * \bar{J} \wedge \bar{J} \right)$$
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- The left/right entanglement entropy is (Furukawa, Kim, Das, Datta, Wen, Matsuura, Ryu)

$$S_{\mathrm{EE}} = \frac{\mathrm{Ar}}{2\pi} \frac{\pi^2 \sqrt{\lambda}}{12} - \frac{1}{2} \ln \mathrm{k} + \mathcal{O}\left(\frac{1}{\ell}\right)$$

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$$\delta S \approx -\int_{\partial M} \mathrm{Tr}\Big(\delta A \wedge \big(\star F - uJu^{-1}\big) + \delta J \wedge u^* A + \delta u u^{-1}\mathrm{d}\big(uJu^{-1}\big) - \mathrm{d}\big(uJu^{-1}\delta u u^{-1}\big)\Big)$$

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- The boundary dynamics accounts for the Kabat contact term in Maxwell S_{EE} (I'm fairly sure) (Donnelly, Wall, Agarwal, Karabali, Nair, Pretko, Soni, Trevidi, . . .)

Perspectives

1. Motivations

2. Covariant phase space

3. Chern–Simons

4. Gravity

5. Dynamics

6. Perspectives

Perspectives

What we learned

- Gauge-invariance in the presence of a boundary requires **new degrees of freedom**
- Systematic way of **extending the phase space** and introducing boundary degrees of freedom
- Disentangles gauge transformations and boundary symmetries
- Exhibits a large boundary symmetry group for physical diffeomorphisms
- As far as edge modes are concerned, Chern–Simons is the same as any other gauge theory
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- Precise relation between boundary dynamics and entanglement entropy contribution
- Representations of the boundary symmetry groups
- Surface translations
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- Link with current hot topics (BMS particles, soft modes, asymptotic symmetries, . . .)
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Thanks!