Tensor and Matrix Models Romance

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Pablo Díaz

Fields, Gravity and Strings, IBS

IPM, Recent Trends in String Theory and Related Topics
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Definition

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Relation with entanglement

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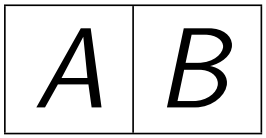

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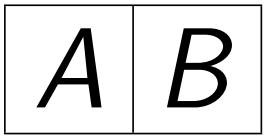

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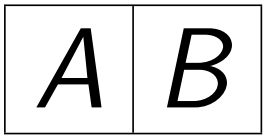

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$$\mathrm{spec}(\rho_A) \sim \mu_A, \ \mathrm{spec}(\rho_B) \sim \mu_B, \ \mathrm{spec}(\rho_{AB}) \sim \mu_{AB}$$ **[Keyl,Werner'01]**

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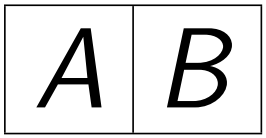

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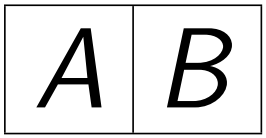

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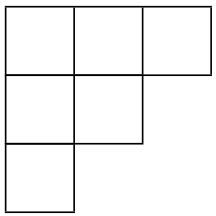

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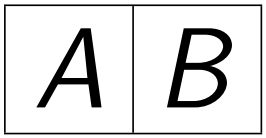

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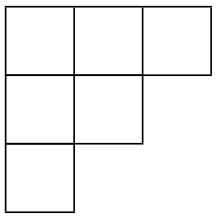
$\mu =$  $\mu \vdash 6$

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Do d-rank tensor models describe d gravity?

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QM system of N fermions with random couplings

$$S = \int dt \, \Big(\frac{i}{2}\sum_j \Psi_j \frac{d}{dt}\Psi_j + j_{i_1 i_2 i_3 i_4}\Psi_{i_1}\Psi_{i_2}\Psi_{i_3}\Psi_{i_4}\Big), \quad i_k = 1, \ldots, N.$$

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- Holography: Conjectured to be dual to 2d BH [Kitaev'15].
- Gauging the theory and using tensor models (instead of random couplings) has been proved to lead to the same large N behaviour [Witten'16].

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- **Holography.** Recently SYK has been linked to tensor models [Witten'16], and matrix models are fundamental tools in holography.
- **Kronecker coefficients vs Littlewood-Richardson numbers.** These numbers organize the spectrum of free TM and MM, respectively. New results in combinatorics and group theory that link those numbers.

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- $n = 4 \longrightarrow 43$ invariants.

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By means of representation theory (and some work):

$$\dim\{\mathcal{O}^{G_d-\mathrm{Inv}}\} = \sum_{\substack{|\mu_1|,\ldots,|\mu_d|=n \\ l(\mu_k)\leq N_k}} g^2_{\mu_1,\ldots,\mu_d}.$$

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whose correlators in the free theory are

$$\langle \mathcal{O}_{\mu_1,\ldots,\mu_d,ij}\overline{\mathcal{O}}_{\nu_1,\ldots,\nu_d,kl}\rangle = n!^2\delta_{ik}\delta_{jl}\prod_{k=1}^{d}\delta_{\mu_k\nu_k}\text{Dim}_{N_k}(\mu_k).$$

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The total number of matrix operators built on n matrices Z and m matrices Y is

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With this counting we can contruct a basis of operators

$$\{\chi_{(\mu;\nu,\lambda)ij}(Z,Y),\quad \mu\vdash n+m,\ \nu\vdash n,\ \lambda\vdash m\quad i,j=1,\ldots,c^{\mu}_{\nu\lambda}\},$$

[de Melo Koch'08].

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$$g_{\mu(r)\nu\lambda} = \mathrm{card}\Big\{ \chi_{(\nu;\rho,\gamma)ii}(Z,Y)\chi_{(\lambda;\rho,\gamma')jj}(Z,Y) \Big|\ \gamma\vdash r,\ \rho\vdash n-r,\ i=1,\dots,c^{\nu}_{\rho\gamma},\ \ j=1,\dots,c^{\lambda}_{\rho\gamma'} \Big\}.$$

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- $\mathcal{O}_{\nu,\lambda;\rho,\gamma;ij} \longrightarrow$ **Fluctuation of depth r about λ.**

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Interpretation: $\mathcal{O}^{+}_{\nu,\lambda;\bar{\rho},\bar{\gamma};kl}$ acting on $|0\rangle$ creates a fluctuation of depth r. $\mathcal{O}_{\nu,\lambda;\rho,\gamma;ij}$ kills a fluctuation of depth r. It seems that there is a tensor operator for each overlap.

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Question:

$$\langle\mathcal{O}_{\mu(r),\nu,\lambda,ij}\overline{\mathcal{O}}_{\mu(r),\nu,\lambda,kl}\rangle \stackrel{?}{=} \sum_{\substack{\rho,\bar{\rho}\vdash n-r\\ \gamma,\bar{\gamma}\vdash r}} \langle\mathcal{O}_{\nu,\lambda;\rho,\gamma;ij}\mathcal{O}^{+}_{\nu,\lambda;\bar{\rho},\bar{\gamma};kl}\rangle$$

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- Is it just a flirt or the beginning of something serious?

Thanks!