Supersymmetric GCA_2/BMS_3 blocks

Wout Merbis

Université Libre de Bruxelles, Physique Théorique et Mathématique

May 8, 2018

Based on work in progress with
Ivano Lodato (Fudan U., Shanghai) and Zodinmawia (IIT Kanpur)

Motivation

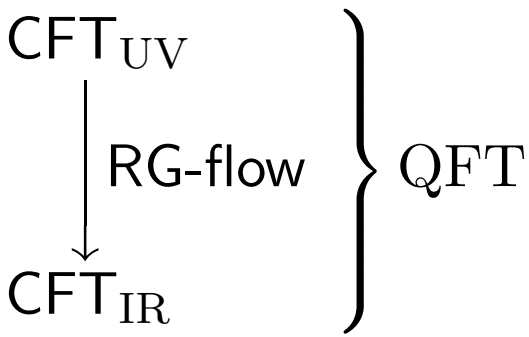
- (Gauge) symmetries are essential guiding principles in constructing Quantum field theories (QFTs)

Motivation

- (Gauge) symmetries are essential guiding principles in constructing Quantum field theories (QFTs)
- We wish to uncover the space of QFTs consistent with given symmetries

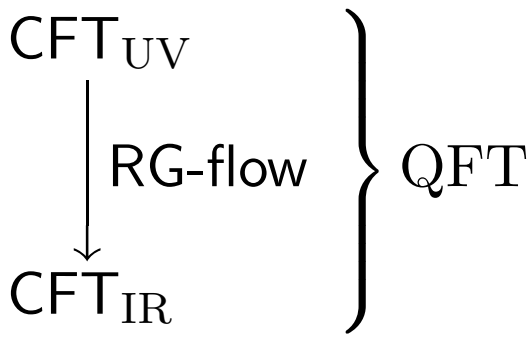
Motivation

- (Gauge) symmetries are essential guiding principles in constructing Quantum field theories (QFTs)
- We wish to uncover the space of QFTs consistent with given symmetries
- A modern day view on (UV complete) QFT:



Motivation

- (Gauge) symmetries are essential guiding principles in constructing Quantum field theories (QFTs)
- We wish to uncover the space of QFTs consistent with given symmetries
- A modern day view on (UV complete) QFT:



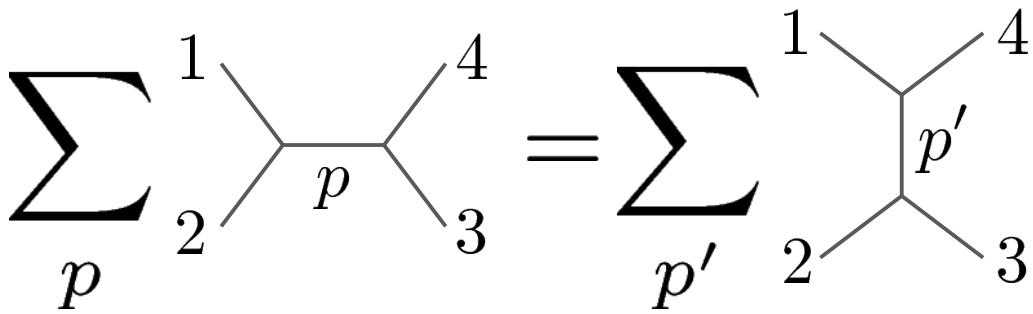
- A systematic and non-perturbative method to obtain consistent CFT's is the conformal bootstrap program [Ferrara, Gatto, and Grillo, '73; Polyakov, '74]

Motivation: the conformal bootstrap

- Use (super)-conformal symmetry and internal consistency to constrain "CFT data" (Operator weights $h_{\mathcal{O}}, \bar{h}_{\mathcal{O}}$ and structure constants c_{mn}^{p})

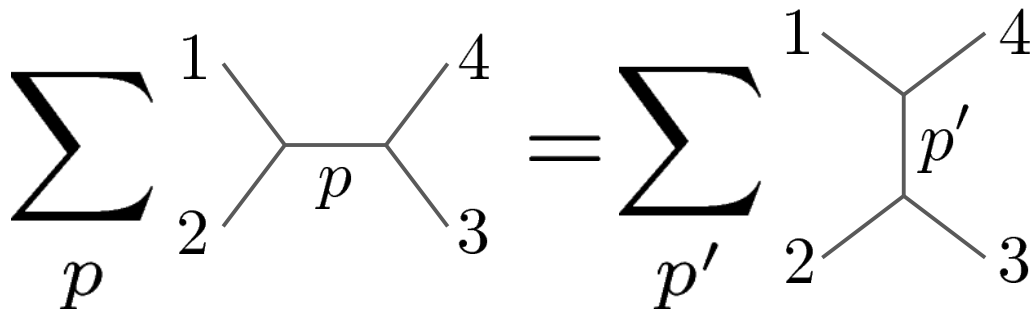
Motivation: the conformal bootstrap

- Use (super)-conformal symmetry and internal consistency to constrain "CFT data" (Operator weights $h_{\mathcal{O}}, \bar{h}_{\mathcal{O}}$ and structure constants c_{mn}^{p})
- Pictorially:



Motivation: the conformal bootstrap

- Use (super)-conformal symmetry and internal consistency to constrain "CFT data" (Operator weights $h_{\mathcal{O}}, \bar{h}_{\mathcal{O}}$ and structure constants c_{mn}^{p})
- Pictorially:

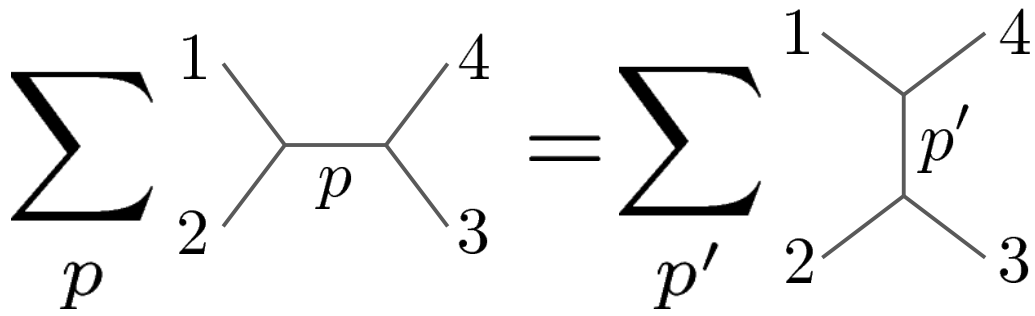


- In terms of conformal blocks $g_{34}^{21}(p|z,\bar{z})$

$$\sum_{p} c_{21}^{p} c_{34}^{p} g_{34}^{21}(p|z,\bar{z}) = \sum_{p'} c_{32}^{p'} c_{41}^{p'} g_{32}^{41}(p|1-z,1-\bar{z})$$

Motivation: the conformal bootstrap

- Use (super)-conformal symmetry and internal consistency to constrain "CFT data" (Operator weights $h_{\mathcal{O}}, \bar{h}_{\mathcal{O}}$ and structure constants c_{mn}^{p})
- Pictorially:



- In terms of conformal blocks $g_{34}^{21}(p|z,\bar{z})$

$$\sum_{p} c_{21}^{p} c_{34}^{p} g_{34}^{21}(p|z,\bar{z}) = \sum_{p'} c_{32}^{p'} c_{41}^{p'} g_{32}^{41}(p|1-z,1-\bar{z})$$

- Knowledge of the (super)-conformal blocks is essential for this program.

Motivation: non/ultra-relativistic bootstrap

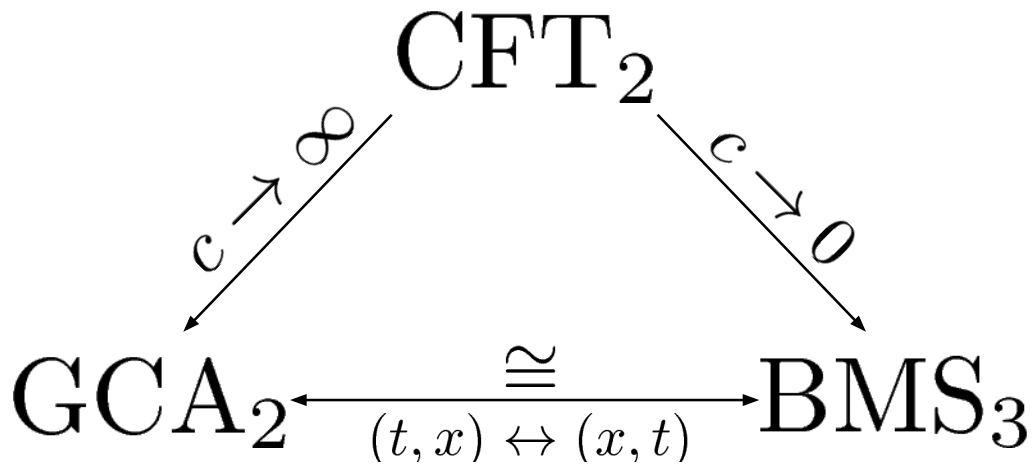
- Much less is known about quantum field theories that are not relativistic.

Motivation: non/ultra-relativistic bootstrap

- Much less is known about quantum field theories that are not relativistic.
- Here we will be interested in non- and ultra-relativistic limits from CFTs (in 2 dimensions) and its supersymmetric extensions

Motivation: non/ultra-relativistic bootstrap

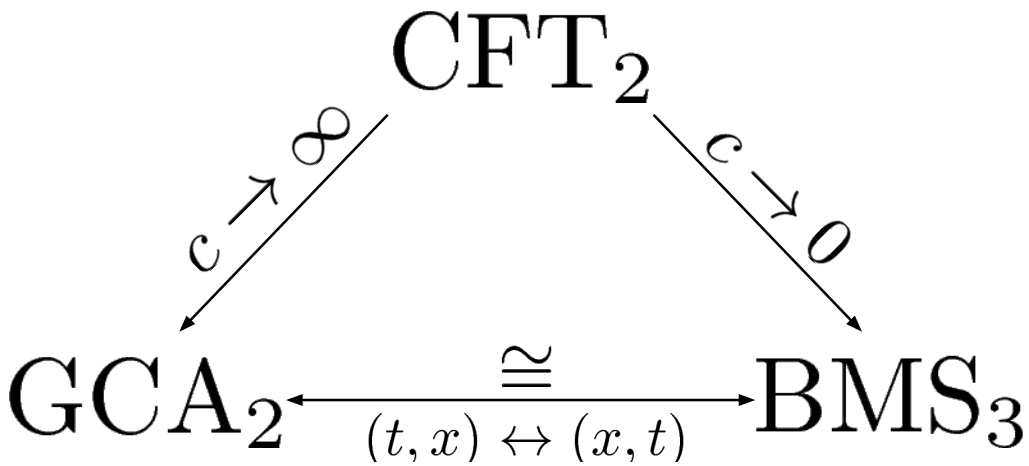
- Much less is known about quantum field theories that are not relativistic.
- Here we will be interested in non- and ultra-relativistic limits from CFTs (in 2 dimensions) and its supersymmetric extensions



[Bagchi, Barnich, Compère, Fareghbal, Grumiller, Gonzalez, Gopakumar, Mandal, Miwa, Naseh, Troessaert, plus many many more]

Motivation: non/ultra-relativistic bootstrap

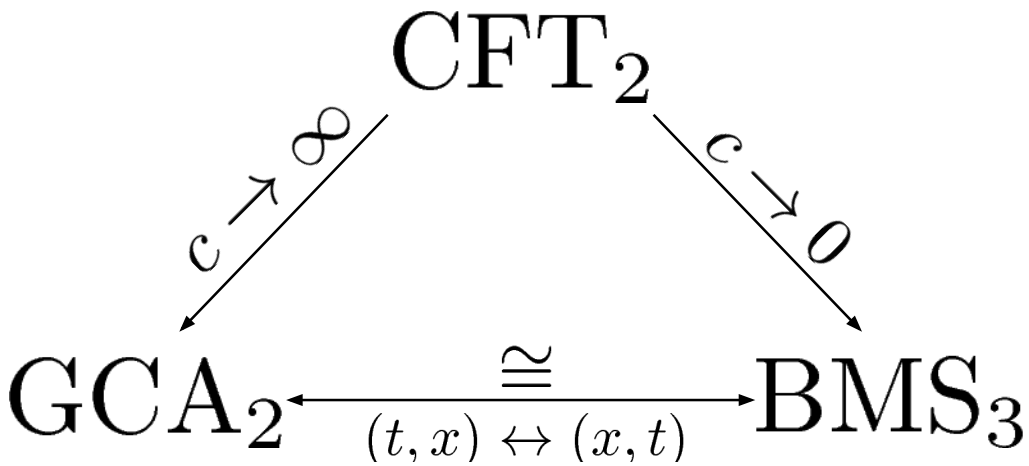
- Much less is known about quantum field theories that are not relativistic.
- Here we will be interested in non- and ultra-relativistic limits from CFTs (in 2 dimensions) and its supersymmetric extensions



- Could the powerful conformal bootstrap technique be modified and applied to theories without relativistic conformal symmetries? [Bagchi, Gary, Zodinmawia, '16 & '17]

Motivation: non/ultra-relativistic bootstrap

- Much less is known about quantum field theories that are not relativistic.
- Here we will be interested in non- and ultra-relativistic limits from CFTs (in 2 dimensions) and its supersymmetric extensions



- Could the powerful conformal bootstrap technique be modified and applied to theories without relativistic conformal symmetries?
- Here we make progress in this direction by deriving the supersymmetric $\mathrm{GCA}_2/\mathrm{BMS}_3$ blocks

Outlook

1. Motivation
2. BMS_3/GCA_2 correspondence
   - Review BMS_3/GCA_2
   - Supersymmetric extensions
   - Representations
3. GCA_2 blocks
   - Operator product expansion
   - 4-point functions & the GCA_2 blocks
   - Global GCA_2-blocks
4. Supersymmetric GCA_2 blocks
   - $\mathcal{N} = 1$ Super GCA_2-blocks
   - $\mathcal{N} = 2$ "Democratic"-blocks
   - $\mathcal{N} = 2$ "Despotic"-blocks
5. Conclusions & Outlook

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) = \sum_{n\in\mathbb{Z}} e^{2\pi i n(t\pm\epsilon x)} \mathcal{L}_n^{\pm}$$

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} e^{2\pi i n t}(1 \pm 2\pi i n \epsilon x)\mathcal{L}_n^{\pm}$$

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} e^{2\pi i n t}(1 \pm 2\pi i n \epsilon x)\mathcal{L}_n^{\pm}$$

Define the generators

$$L_n = \mathcal{L}_n^+ + \mathcal{L}_n^- , \quad M_n = \epsilon(\mathcal{L}_n^+ - \mathcal{L}_n^-)$$

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} e^{2\pi i n t}(1 \pm 2\pi i n \epsilon x)\mathcal{L}_n^{\pm}$$

Define the generators

$$L_n = \mathcal{L}_n^+ + \mathcal{L}_n^- , \quad M_n = \epsilon(\mathcal{L}_n^+ - \mathcal{L}_n^-)$$

$$Vir_+ \oplus Vir_- \underset{\epsilon\to 0}{\Rightarrow} \begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \tfrac{c_L}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, L_m] &= (n-m)M_{n+m} + \tfrac{c_M}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, M_m] &= 0\,. \end{aligned}$$

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} e^{2\pi i n t}(1 \pm 2\pi i n \epsilon x)\mathcal{L}_n^{\pm}$$

Define the generators

$$L_n = \mathcal{L}_n^{+} + \mathcal{L}_n^{-}\,, \quad M_n = \epsilon(\mathcal{L}_n^{+} - \mathcal{L}_n^{-})$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) = \sum_{n\in\mathbb{Z}} e^{2\pi i n(\epsilon t \pm x)}\mathcal{L}_n^{\pm}$$

$$Vir_+ \oplus Vir_- \underset{\epsilon\to 0}{\Rightarrow} \begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \tfrac{c_L}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, L_m] &= (n-m)M_{n+m} + \tfrac{c_M}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, M_m] &= 0\,. \end{aligned}$$

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} e^{2\pi i n t}(1 \pm 2\pi i n \epsilon x)\mathcal{L}_n^{\pm}$$

Define the generators

$$L_n = \mathcal{L}_n^+ + \mathcal{L}_n^- , \quad M_n = \epsilon(\mathcal{L}_n^+ - \mathcal{L}_n^-)$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} (1 + 2\pi i n \epsilon t) e^{\pm 2\pi i n x}\mathcal{L}_n^{\pm}$$

$$Vir_+ \oplus Vir_- \underset{\epsilon\to 0}{\Rightarrow} \begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \tfrac{c_L}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, L_m] &= (n-m)M_{n+m} + \tfrac{c_M}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, M_m] &= 0\,. \end{aligned}$$

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} e^{2\pi i n t}(1 \pm 2\pi i n \epsilon x)\mathcal{L}^{\pm}_n$$

Define the generators

$$L_n = \mathcal{L}^+_n + \mathcal{L}^-_n\,, \quad M_n = \epsilon(\mathcal{L}^+_n - \mathcal{L}^-_n)$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) = \sum_{n\in\mathbb{Z}} (1 \pm 2\pi i n \epsilon t) e^{2\pi i n x}\mathcal{L}^{\pm}_n$$

$$Vir_+ \oplus Vir_- \underset{\epsilon\to 0}{\Rightarrow} \begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \tfrac{c_L}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, L_m] &= (n-m)M_{n+m} + \tfrac{c_M}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, M_m] &= 0\,. \end{aligned}$$

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} e^{2\pi i n t}(1 \pm 2\pi i n \epsilon x)\mathcal{L}^{\pm}_n$$

Define the generators

$$L_n = \mathcal{L}^+_n + \mathcal{L}^-_n\,, \quad M_n = \epsilon(\mathcal{L}^+_n - \mathcal{L}^-_n)$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) = \sum_{n\in\mathbb{Z}} (1 \pm 2\pi i n \epsilon t) e^{2\pi i n x}\mathcal{L}^{\pm}_{\pm n}$$

Define the generators

$$L_n = \mathcal{L}^+_n - \mathcal{L}^-_{-n}\,, \quad M_n = \epsilon(\mathcal{L}^+_n + \mathcal{L}^-_{-n})$$

$$Vir_+ \oplus Vir_- \underset{\epsilon\to 0}{\Rightarrow} \begin{aligned} [L_n, L_m] &= (n-m)L_{n+m} + \tfrac{c_L}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, L_m] &= (n-m)M_{n+m} + \tfrac{c_M}{12} n(n^2-1)\delta_{n+m,0}\,, \\ [M_n, M_m] &= 0\,. \end{aligned}$$

Starting from a 2D CFT on the cylinder with coordinates $z^{\pm} = e^{2\pi i \omega^{\pm}}$ we can consider two limits

Non-relativistic limit

$$\omega^{\pm} = t \pm \epsilon x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) \sim \sum_{n\in\mathbb{Z}} e^{2\pi i n t}(1 \pm 2\pi i n \epsilon x)\mathcal{L}^{\pm}_n$$

Define the generators

$$L_n = \mathcal{L}^+_n + \mathcal{L}^-_n\,, \quad M_n = \epsilon(\mathcal{L}^+_n - \mathcal{L}^-_n)$$

Ultra-relativistic limit

$$\omega^{\pm} = \epsilon t \pm x, \qquad \epsilon \to 0$$

for the stress tensors this implies:

$$T^{\pm}(t,x) = \sum_{n\in\mathbb{Z}} (1 \pm 2\pi i n \epsilon t) e^{2\pi i n x}\mathcal{L}^{\pm}_{\pm n}$$

Define the generators

$$L_n = \mathcal{L}^+_n - \mathcal{L}^-_{-n}\,, \quad M_n = \epsilon(\mathcal{L}^+_n + \mathcal{L}^-_{-n})$$

non-relativistic

$$c_L = c^+ + c^-\,, \quad c_M = \epsilon(c^+ - c^-)$$

ultra-relativistic

$$c_L = c^+ - c^-\,, \quad c_M = \epsilon(c^+ + c^-)$$

Supersymmetric extensions

The $\mathcal{N} = (1,1)$ superconformal algebra in 2d:

$$
\begin{aligned}
[\mathcal{L}_n^{\pm}, \mathcal{L}_m^{\pm}] &= (n-m)\mathcal{L}_{n+m}^{\pm} + \frac{c^{\pm}}{12} n(n^2-1)\delta_{n+m,0}\,, \\
[\mathcal{L}_n^{\pm}, \mathcal{G}_r^{\pm}] &= (\tfrac{n}{2} - r)\mathcal{G}_{n+r}^{\pm} \\
\{\mathcal{G}_r^{\pm}, \mathcal{G}_s^{\pm}\} &= 2\mathcal{L}_{r+s}^{\pm} + \frac{c^{\pm}}{3}(r^2 - \tfrac{1}{4})\delta_{r+s,0}\,,
\end{aligned}
$$

For both limits, there are two possible ways to contract the fermionic generators

Supersymmetric extensions

The $\mathcal{N} = (1,1)$ superconformal algebra in 2d:

$$
\begin{aligned}
[\mathcal{L}_n^{\pm}, \mathcal{L}_m^{\pm}] &= (n-m)\mathcal{L}_{n+m}^{\pm} + \tfrac{c^{\pm}}{12} n(n^2-1)\delta_{n+m,0}\,, \\
[\mathcal{L}_n^{\pm}, \mathcal{G}_r^{\pm}] &= (\tfrac{n}{2} - r)\mathcal{G}_{n+r}^{\pm} \\
\{\mathcal{G}_r^{\pm}, \mathcal{G}_s^{\pm}\} &= 2\mathcal{L}_{r+s}^{\pm} + \tfrac{c^{\pm}}{3}(r^2 - \tfrac{1}{4})\delta_{r+s,0}\,,
\end{aligned}
$$

For both limits, there are two possible ways to contract the fermionic generators

"Democratic"

NR : $\quad Q_r^{\pm} = \sqrt{\pm\epsilon}\,\mathcal{G}_r^{\pm}\,,$

UR : $\quad Q_r^{\pm} = \sqrt{\epsilon}\,\mathcal{G}_{\pm r}^{\pm}\,,$

Supersymmetric extensions

The $\mathcal{N} = (1,1)$ superconformal algebra in 2d:

$$
\begin{aligned}
[\mathcal{L}_n^{\pm}, \mathcal{L}_m^{\pm}] &= (n-m)\mathcal{L}_{n+m}^{\pm} + \tfrac{c^{\pm}}{12} n(n^2-1)\delta_{n+m,0}\,, \\
[\mathcal{L}_n^{\pm}, \mathcal{G}_r^{\pm}] &= (\tfrac{n}{2} - r)\mathcal{G}_{n+r}^{\pm} \\
\{\mathcal{G}_r^{\pm}, \mathcal{G}_s^{\pm}\} &= 2\mathcal{L}_{r+s}^{\pm} + \tfrac{c^{\pm}}{3}(r^2 - \tfrac{1}{4})\delta_{r+s,0}\,,
\end{aligned}
$$

For both limits, there are two possible ways to contract the fermionic generators

"Democratic"

NR : $\quad Q_r^{\pm} = \sqrt{\pm\epsilon}\,\mathcal{G}_r^{\pm}\,,$

UR : $\quad Q_r^{\pm} = \sqrt{\epsilon}\,\mathcal{G}_{\pm r}^{\pm}\,,$

"Despotic"

NR : $\quad G_r = \mathcal{G}_r^{+} + \mathcal{G}_r^{-}\,,$

$\qquad H_r = \epsilon(\mathcal{G}_r^{+} - \mathcal{G}_r^{-})$

UR : $\quad G_r = \mathcal{G}_r^{+} - i\mathcal{G}_{-r}^{-}$

$\qquad H_r = \epsilon(\mathcal{G}_r^{+} + i\mathcal{G}_{-r}^{-})\,,$

Supersymmetric extensions

"Democratic"

NR : $Q_r^{\pm} = \sqrt{\pm\epsilon}\,\mathcal{G}_r^{\pm}\,,$

UR : $Q_r^{\pm} = \sqrt{\epsilon}\,\mathcal{G}_{\pm r}^{\pm}\,,$

"Despotic"

NR : $G_r^{\pm} = \mathcal{G}_r^{+} + \mathcal{G}_r^{-}\,,$

$H_r = \epsilon(\mathcal{G}_r^{+} - \mathcal{G}_r^{-})$

UR : $G_r = \mathcal{G}_r^{+} - i\mathcal{G}_{-r}^{-}$

$H_r = \epsilon(\mathcal{G}_r^{+} + i\mathcal{G}_{-r}^{-})\,,$

The $\mathcal{N} = 2$ "Democratic" Super-GCA$_2$ (or Super BMS$_3$)-algebra is

$$
\begin{aligned}
[L_n, L_m] &= (n-m)L_{m+n} + \tfrac{c_L}{12}n(n^2-1)\delta_{m+n,0}\,,\\
[L_n, M_m] &= (n-m)M_{m+n} + \tfrac{c_M}{12}n(n^2-1)\delta_{m+n,0}\,,\\
[L_n, Q_r^{\pm}] &= (\tfrac{n}{2} - r)Q_{r+n}^{\pm}\,,\\
\{Q_r^{\pm}, Q_s^{\pm}\} &= M_{r+s} + \tfrac{c_M}{6}(r^2 - \tfrac{1}{4})\delta_{r+s,0}\,,\\
\{Q_r^{\pm}, Q_s^{\mp}\} &= 0 = [M_n, M_m] = [M_n, Q_r^{\pm}]\,.
\end{aligned}
$$

Supersymmetric extensions

"Democratic"

NR : $Q_r^{\pm} = \sqrt{\pm\epsilon}\,\mathcal{G}_r^{\pm}\,,$

UR : $Q_r^{\pm} = \sqrt{\epsilon}\,\mathcal{G}_{\pm r}^{\pm}\,,$

"Despotic"

NR : $G_r^{\pm} = \mathcal{G}_r^{+} + \mathcal{G}_r^{-}\,,$

$H_r = \epsilon(\mathcal{G}_r^{+} - \mathcal{G}_r^{-})$

UR : $G_r = \mathcal{G}_r^{+} - i\mathcal{G}_{-r}^{-}$

$H_r = \epsilon(\mathcal{G}_r^{+} + i\mathcal{G}_{-r}^{-})\,,$

The $\mathcal{N} = 2$ "Despotic" Super-GCA$_2$ (or Super BMS$_3$)-algebra is

$$
\begin{aligned}
[L_n, L_m] &= (n-m)L_{m+n} + \tfrac{c_L}{12}n(n^2-1)\delta_{m+n,0}\,,\\
[L_n, M_m] &= (n-m)M_{m+n} + \tfrac{c_M}{12}n(n^2-1)\delta_{m+n,0}\,,\\
[L_n, G_r] &= (\tfrac{n}{2}-r)G_{r+n}\,, \qquad [L_n, H_r] = (\tfrac{n}{2}-r)H_{r+n}\\
[M_n, G_r] &= (\tfrac{n}{2}-r)H_{r+n}\,, \qquad [M_n, H_r] = 0 = \{H_r, H_s\}\\
\{G_r, G_s\} &= L_{r+s} + \tfrac{c_L}{6}(r^2-\tfrac{1}{4})\delta_{r+s,0}\,,\\
\{G_r, H_s\} &= M_{r+s} + \tfrac{c_M}{6}(r^2-\tfrac{1}{4})\delta_{r+s,0}\,,
\end{aligned}
$$

Supersymmetric extensions

"Democratic"

$$\text{NR}: \quad Q_r^{\pm} = \sqrt{\pm\epsilon}\,\mathcal{G}_r^{\pm}\,,$$

$$\text{UR}: \quad Q_r^{\pm} = \sqrt{\epsilon}\,\mathcal{G}_{\pm r}^{\pm}\,,$$

"Despotic"

$$\text{NR}: \quad G_r^{\pm} = \mathcal{G}_r^{+} + \mathcal{G}_r^{-}\,,$$
$$H_r = \epsilon(\mathcal{G}_r^{+} - \mathcal{G}_r^{-})$$

$$\text{UR}: \quad G_r = \mathcal{G}_r^{+} - i\mathcal{G}_{-r}^{-}$$
$$H_r = \epsilon(\mathcal{G}_r^{+} + i\mathcal{G}_{-r}^{-})\,,$$

Hermitian conjugate not well-defined for the complex scalings
In the CFT: $(\mathcal{G}_r^{\pm})^\dagger = \mathcal{G}_{-r}^{\pm}$, then for the democratic limits

$$NR: \quad (Q_r^{\pm})^\dagger = \pm Q_{-r}^{\pm} \qquad UR: \quad (Q_r^{\pm})^\dagger = Q_{-r}^{\pm}$$

And for the despotic scalings

$$NR: \quad (G_r)^\dagger = G_{-r} \quad (H_r)^\dagger = H_{-r} \qquad UR: \quad (G_r)^\dagger = \frac{1}{\epsilon} H_{-r}\,, \quad (H_r)^\dagger = \epsilon G_{-r}$$

Supersymmetric extensions

"Democratic"

NR : $Q_r^{\pm} = \sqrt{\pm\epsilon}\,\mathcal{G}_r^{\pm}$,

UR : $Q_r^{\pm} = \sqrt{\epsilon}\,\mathcal{G}_{\pm r}^{\pm}$,

"Despotic"

NR : $G_r^{\pm} = \mathcal{G}_r^{+} + \mathcal{G}_r^{-}$,

$H_r = \epsilon(\mathcal{G}_r^{+} - \mathcal{G}_r^{-})$

UR : $G_r = \mathcal{G}_r^{+} - i\mathcal{G}_{-r}^{-}$

$H_r = \epsilon(\mathcal{G}_r^{+} + i\mathcal{G}_{-r}^{-})$,

Hermitian conjugate not well-defined for the complex scalings
In the CFT: $(\mathcal{G}_r^{\pm})^{\dagger} = \mathcal{G}_{-r}^{\pm}$, then for the democratic limits

$$NR: \quad (Q_r^{\pm})^{\dagger} = \pm Q_{-r}^{\pm} \qquad UR: \quad (Q_r^{\pm})^{\dagger} = Q_{-r}^{\pm}$$

And for the despotic scalings

$$NR: \quad (G_r)^{\dagger} = G_{-r} \quad (H_r)^{\dagger} = H_{-r} \qquad UR: \quad (G_r)^{\dagger} = \frac{1}{\epsilon} H_{-r}\,, \quad (H_r)^{\dagger} = \epsilon G_{-r}$$

The limit of Virasoro highest weight representations

[Campoleoni, Gonzalez, Oblak, Riegler, '16]

Virasoro highest weight reps

$$\mathcal{L}_0^{\pm}|h^{\pm}\rangle = h^{\pm}|h^{\pm}\rangle, \quad \mathcal{L}_n^{\pm}|h^{\pm}\rangle = 0 \quad \text{for} \quad n > 0$$

non-relativistic

$$L_n = \mathcal{L}_n^+ + \mathcal{L}_n^- , \quad M_n = \epsilon(\mathcal{L}_n^+ - \mathcal{L}_n^-)$$

$\Downarrow$

highest weight

$$L_0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle$$
$$M_0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle$$
$$\{M_n, L_n\}|\Delta,\xi\rangle = 0 \quad \text{for} \quad n > 0$$

ultra-relativistic

$$L_n = \mathcal{L}_n^+ - \mathcal{L}_{-n}^- , \quad M_n = \epsilon(\mathcal{L}_n^+ + \mathcal{L}_{-n}^-)$$

$\Downarrow$

induced representations

$$L_0|M,s\rangle = s|M,s\rangle$$
$$M_0|M,s\rangle = M|M,s\rangle$$
$$M_n|M,s\rangle = 0 \quad \text{for} \quad n \neq 0$$

Highest weight representations and modules

In this talk we will restrict to highest weight representations of GCA_2/BMS_3

$$L_n|0\rangle = 0 \qquad M_n|0\rangle = 0 \qquad \text{for : } n \geq -1\,,$$

Highest weight representations and modules

In this talk we will restrict to highest weight representations of GCA_2/BMS_3

$$L_n|0\rangle = 0 \qquad M_n|0\rangle = 0 \qquad \text{for : } n \geq -1\,,$$
$$L_0|\Delta_p, \xi_p\rangle = \Delta_p|\Delta_p, \xi_p\rangle \qquad M_0|\Delta_p, \xi_p\rangle = \xi_p|\Delta_p, \xi_p\rangle$$

Highest weight representations and modules

In this talk we will restrict to highest weight representations of GCA_2/BMS_3

$$L_n|0\rangle = 0 \qquad M_n|0\rangle = 0 \qquad \text{for : } n \geq -1\,,$$
$$L_0|\Delta_p, \xi_p\rangle = \Delta_p|\Delta_p, \xi_p\rangle \qquad M_0|\Delta_p, \xi_p\rangle = \xi_p|\Delta_p, \xi_p\rangle$$
$$L_n|\Delta_p, \xi_p\rangle = 0 \qquad M_n|\Delta_p, \xi_p\rangle = 0 \qquad \text{for : } n > 0$$

Highest weight representations and modules

In this talk we will restrict to highest weight representations of GCA$_2$/BMS$_3$

$$\begin{aligned} L_n|0\rangle &= 0 & M_n|0\rangle &= 0 & \text{for : } n &\geq -1\,, \\ L_0|\Delta_p,\xi_p\rangle &= \Delta_p|\Delta_p,\xi_p\rangle & M_0|\Delta_p,\xi_p\rangle &= \xi_p|\Delta_p,\xi_p\rangle & & \\ L_n|\Delta_p,\xi_p\rangle &= 0 & M_n|\Delta_p,\xi_p\rangle &= 0 & \text{for : } n &> 0 \end{aligned}$$

Descendants are build by acting with raising operators on the highest weight states $|\Delta_p,\xi_p\rangle$

$$|\Delta_p,\xi_p,\{N\}\rangle \equiv L_{-k_1}\dots L_{-k_n}M_{-l_1}\dots M_{-l_m}|\Delta_p,\xi_p\rangle = L_{-\{k\}}M_{-\{l\}}|\Delta_p,\xi_p\rangle\,.$$

Highest weight representations and modules

In this talk we will restrict to highest weight representations of GCA$_2$/BMS$_3$

$$L_n|0\rangle = 0 \qquad M_n|0\rangle = 0 \qquad \text{for : } n \geq -1\,,$$
$$L_0|\Delta_p,\xi_p\rangle = \Delta_p|\Delta_p,\xi_p\rangle \qquad M_0|\Delta_p,\xi_p\rangle = \xi_p|\Delta_p,\xi_p\rangle$$
$$L_n|\Delta_p,\xi_p\rangle = 0 \qquad M_n|\Delta_p,\xi_p\rangle = 0 \qquad \text{for : } n > 0$$

Descendants are build by acting with raising operators on the highest weight states $|\Delta_p,\xi_p\rangle$

$$|\Delta_p,\xi_p,\{N\}\rangle \equiv L_{-k_1}\dots L_{-k_n}M_{-l_1}\dots M_{-l_m}|\Delta_p,\xi_p\rangle = L_{-\{k\}}M_{-\{l\}}|\Delta_p,\xi_p\rangle\,.$$

Hermitian conjugates

$$(L_n)^\dagger = L_{-n}\,, \qquad (M_n)^\dagger = M_{-n}\,,$$
$$|\Delta_p,\xi_p,\{N\}\rangle^\dagger = \langle\Delta_p,\xi_p,\{N\}| = \langle\Delta_p,\xi_p|M_{l_m}\dots M_{l_1}L_{k_n}\dots L_{k_1}$$

Highest weight representations and modules

Primary states can also be represented by operators $\phi_p(t,x)$ inserted at the origin:

$$|\Delta_p, \xi_p\rangle = \phi_p(0,0)|0\rangle \,.$$

Highest weight representations and modules

Primary states can also be represented by operators $\phi_p(t,x)$ inserted at the origin:

$$|\Delta_p, \xi_p\rangle = \phi_p(0,0)|0\rangle\,.$$

The conjugate state (out-state) is taken at $(t,x) \to (\infty, 0)$

$$\langle\Delta_p, \xi_p| = \lim_{t\to\infty} t^{2\Delta_p}\langle 0|\phi_p(t,0)$$

Highest weight representations and modules

Primary states can also be represented by operators $\phi_p(t,x)$ inserted at the origin:

$$|\Delta_p,\xi_p\rangle = \phi_p(0,0)|0\rangle\,.$$

The conjugate state (out-state) is taken at $(t,x)\to(\infty,0)$

$$\langle\Delta_p,\xi_p| = \lim_{t\to\infty} t^{2\Delta_p}\langle 0|\phi_p(t,0)$$

GCA$_2$ invariance fixes the structure of the two and three point functions as

$$\langle\phi_m(t_m,x_m)\phi_n(t_n,x_n)\rangle = \frac{\delta_{\Delta_m,\Delta_n}\delta_{\xi_m,\xi_n}}{t_{mn}^{\Delta_m+\Delta_n}}e^{-\frac{x_{mn}(\xi_m+\xi_n)}{t_{mn}}}$$

$$\langle\phi_i(t_i,x_i)\phi_j(t_j,x_j)\phi_n(t_n,x_n)\rangle = \frac{c_{ijn}}{t_{ij}^{\Delta_{ijn}}t_{jn}^{\Delta_{jni}}t_{in}^{\Delta_{inj}}}e^{-\frac{x_{ij}\xi_{ijn}}{t_{ij}}-\frac{x_{jn}\xi_{jni}}{t_{jn}}-\frac{x_{in}\xi_{inj}}{t_{in}}}\,,$$

With $\Delta_{ijn}=\Delta_i+\Delta_j-\Delta_n$ and $\xi_{ijn}=\xi_i+\xi_j-\xi_n$.

Highest weight representations and modules

Primary states can also be represented by operators $\phi_p(t,x)$ inserted at the origin:

$$|\Delta_p,\xi_p\rangle = \phi_p(0,0)|0\rangle\,.$$

The conjugate state (out-state) is taken at $(t,x)\to(\infty,0)$

$$\langle\Delta_p,\xi_p| = \lim_{t\to\infty} t^{2\Delta_p}\langle 0|\phi_p(t,0)$$

GCA$_2$ invariance fixes the structure of the two and three point functions as

$$\langle\phi_m(t_m,x_m)\phi_n(t_n,x_n)\rangle = \frac{\delta_{\Delta_m,\Delta_n}\delta_{\xi_m,\xi_n}}{t_{mn}^{\Delta_m+\Delta_n}}e^{-\frac{x_{mn}(\xi_m+\xi_n)}{t_{mn}}}$$

$$\langle\phi_i(t_i,x_i)\phi_j(t_j,x_j)\phi_n(t_n,x_n)\rangle = \frac{c_{ijn}}{t_{ij}^{\Delta_{ijn}}t_{jn}^{\Delta_{jni}}t_{in}^{\Delta_{inj}}}e^{-\frac{x_{ij}\xi_{ijn}}{t_{ij}}-\frac{x_{jn}\xi_{jni}}{t_{jn}}-\frac{x_{in}\xi_{inj}}{t_{in}}}\,,$$

With $\Delta_{ijn}=\Delta_i+\Delta_j-\Delta_n$ and $\xi_{ijn}=\xi_i+\xi_j-\xi_n$.
We can use the global GCA invariance to move 6 coordinates to convenient points

Highest weight representations and modules

Primary states can also be represented by operators $\phi_p(t,x)$ inserted at the origin:

$$|\Delta_p,\xi_p\rangle = \phi_p(0,0)|0\rangle\,.$$

The conjugate state (out-state) is taken at $(t,x)\to(\infty,0)$

$$\langle\Delta_p,\xi_p| = \lim_{t\to\infty} t^{2\Delta_p}\langle 0|\phi_p(t,0)$$

GCA$_2$ invariance fixes the structure of the two and three point functions as

$$\langle\Delta_i,\xi_i|\Delta_n,\xi_n\rangle = \lim_{t_i\to\infty} t_i^{2\Delta_i}\langle\phi_i(t_i,0)\phi_n(0,0)\rangle = \delta_{\Delta_i,\Delta_n}\delta_{\xi_i,\xi_n}$$

$$\langle\Delta_i,\xi_i|\phi_j(1,0)|\Delta_n,\xi_n\rangle = \lim_{t_i\to\infty} t_i^{2\Delta_i}\langle\phi_i(t_i,0)\phi_j(1,0)\phi_n(0,0)\rangle = c_{ijn}$$

Highest weight representations and modules

Primary states can also be represented by operators $\phi_p(t,x)$ inserted at the origin:

$$|\Delta_p,\xi_p\rangle = \phi_p(0,0)|0\rangle\,.$$

The conjugate state (out-state) is taken at $(t,x)\to(\infty,0)$

$$\langle\Delta_p,\xi_p| = \lim_{t\to\infty} t^{2\Delta_p}\langle 0|\phi_p(t,0)$$

GCA$_2$ invariance fixes the structure of the two and three point functions as

$$\langle\Delta_i,\xi_i|\Delta_n,\xi_n\rangle = \lim_{t_i\to\infty} t_i^{2\Delta_i}\langle\phi_i(t_i,0)\phi_n(0,0)\rangle = \delta_{\Delta_i,\Delta_n}\delta_{\xi_i,\xi_n}$$

$$\langle\Delta_i,\xi_i|\phi_j(1,0)|\Delta_n,\xi_n\rangle = \lim_{t_i\to\infty} t_i^{2\Delta_i}\langle\phi_i(t_i,0)\phi_j(1,0)\phi_n(0,0)\rangle = c_{ijn}$$

Raising operators act on $\phi_p(t,x)$ as differential operators

$$[L_n,\phi_p(t,x)] = \left[t^{n+1}\partial_t + (n+1)xt^n\partial_x + \xi_p n(n+1)xt^{n-1} + \Delta_p(n+1)t^n\right]\phi_p(t,x)$$

$$[M_n,\phi_p(t,x)] = \left[t^{n+1}\partial_x + \xi_p(n+1)t^n\right]\phi_p(t,x)\,.$$

GCA$_2$ blocks

- Correlators including descendants of primary operators are fixed in terms of those with only primary operators

GCA_2 blocks

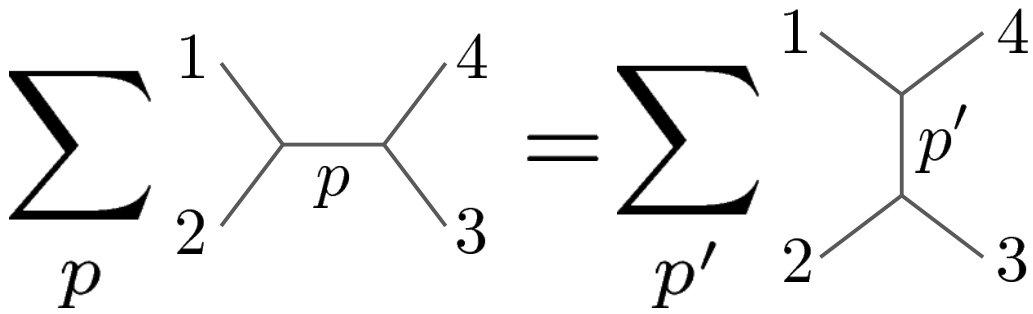
- Correlators including descendants of primary operators are fixed in terms of those with only primary operators
- Product of primary operators are related to a sum over primaries and descendants by the Operator Product Expansion (OPE)

GCA$_2$ blocks

- Correlators including descendants of primary operators are fixed in terms of those with only primary operators
- Product of primary operators are related to a sum over primaries and descendants by the Operator Product Expansion (OPE)
- Therefore the theory is fixed in terms of GCA$_2$ data: $\{\Delta_p, \xi_p, c_{ijk}\}$.

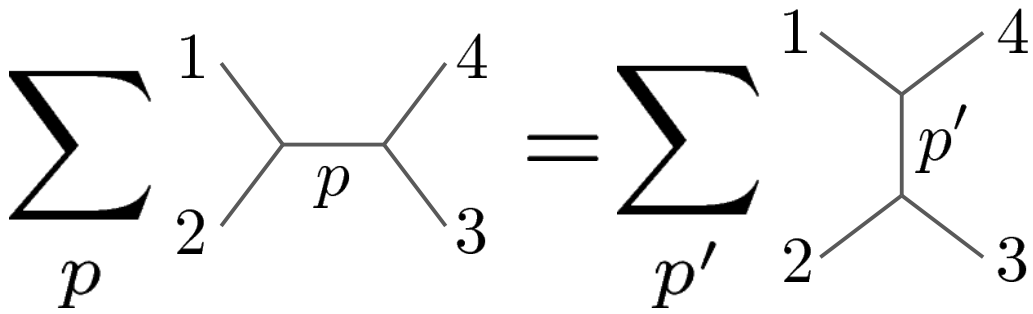
GCA$_2$ blocks

- Correlators including descendants of primary operators are fixed in terms of those with only primary operators
- Product of primary operators are related to a sum over primaries and descendants by the Operator Product Expansion (OPE)
- Therefore the theory is fixed in terms of GCA$_2$ data: $\{\Delta_p, \xi_p, c_{ijk}\}$.
- We can use the OPE to expand the four point function in two different ways



GCA$_2$ blocks

- Correlators including descendants of primary operators are fixed in terms of those with only primary operators
- Product of primary operators are related to a sum over primaries and descendants by the Operator Product Expansion (OPE)
- Therefore the theory is fixed in terms of GCA$_2$ data: $\{\Delta_p, \xi_p, c_{ijk}\}$.
- We can use the OPE to expand the four point function in two different ways



- The functions summed over in these figure are the GCA$_2$ blocks

Operator product expansion

The OPE between to primary fields is

$$\phi_m(t_m,x_m)\phi_n(t_n,x_n) = \sum_{p,\{N\}} \frac{c_{mn}^p}{t_{mn}^{\Delta_{mnp}-N}} e^{-\frac{x_{mn}}{t_{mn}}\xi_{mnp}} \sum_{\alpha=0}^{|I|} \beta_{mn}^{p,\{N\},\alpha} \left(\frac{x_{mn}}{t_{mn}}\right)^{\alpha} \phi_p^{\{N\}}(t_n,x_n)$$

where

$$\phi_p^{\{N\}}(t,x) = L_{-k_1}\dots L_{-k_n} M_{-l_1}\dots M_{-l_m}\phi_p(t,x)$$

Operator product expansion

In terms of states, the OPE reads

$$\phi_m(t,x)|\Delta_n,\xi_n\rangle = \sum_{p,\{N\},\alpha} \frac{C_{mn}^p}{t^{\Delta_{mnp}-N}} \left(\frac{x}{t}\right)^\alpha e^{-\frac{x}{t}\xi_{mnp}} \beta_{mn}^{p,\{N\},\alpha} |\Delta_p,\xi_p,\{N\}\rangle$$

Operator product expansion

In terms of states, the OPE reads

$$\phi_m(t,x)|\Delta_n,\xi_n\rangle = \sum_{p,\{N\},\alpha} \frac{C_{mn}^{p}}{t^{\Delta_{mnp}-N}} \left(\frac{x}{t}\right)^{\alpha} e^{-\frac{x}{t}\xi_{mnp}} \beta_{mn}^{p,\{N\},\alpha} |\Delta_p,\xi_p,\{N\}\rangle$$

The OPE coefficients $\beta_{mn}^{p,\{N\},\alpha}$ are computed by comparing coefficients of x on both sides of

$$\sum_{\alpha=0}^{|I|} \beta_{mn}^{p,\{N\},\alpha} x^{\alpha} = \sum_{\{N'\}} e^{x\,\xi_{mnp}}\, \mathfrak{M}^{-1}_{\{N\},\{N'\}} \frac{\langle \Delta_p,\xi_p,\{N'\}|\phi_m(t,x)|\Delta_n,\xi_n\rangle}{\langle \Delta_p,\xi_p|\phi_m(1,0)|\Delta_n,\xi_n\rangle}\Big|_{t=1},$$

Operator product expansion

In terms of states, the OPE reads

$$\phi_m(t,x)|\Delta_n,\xi_n\rangle = \sum_{p,\{N\},\alpha} \frac{C_{mn}^p}{t^{\Delta_{mnp}-N}} \left(\frac{x}{t}\right)^\alpha e^{-\frac{x}{t}\xi_{mnp}} \beta_{mn}^{p,\{N\},\alpha} |\Delta_p,\xi_p,\{N\}\rangle$$

The OPE coefficients $\beta_{mn}^{p,\{N\},\alpha}$ are computed by comparing coefficients of x on both sides of

$$\sum_{\alpha=0}^{|I|} \beta_{mn}^{p,\{N\},\alpha} x^\alpha = \sum_{\{N'\}} e^{x\,\xi_{mnp}}\, \mathfrak{M}^{-1}_{\{N\},\{N'\}} \frac{\langle \Delta_p,\xi_p,\{N'\}|\phi_m(t,x)|\Delta_n,\xi_n\rangle}{\langle \Delta_p,\xi_p|\phi_m(1,0)|\Delta_n,\xi_n\rangle}\Big|_{t=1},$$

Here $\mathfrak{M}^{-1}_{\{N\},\{N'\}}$ is the inverse GCA$_2$ Kac matrix

$$\langle \Delta_p,\xi_p,\{N\}|\Delta_p,\xi_p,\{N'\}\rangle = \delta_{|N|,|N'|}\mathfrak{M}_{\{N\},\{N'\}}$$

4-point functions & the GCA$_2$ blocks

A generic four point functions (at equal external weights) is constraint to be

$$\langle \phi(t_i, x_i)\phi(t_j, x_j)\phi(t_m, x_m)\phi(t_n, x_n)\rangle = \frac{1}{(t_{ij}\, t_{mn})^{2\Delta}} e^{-\frac{2x_{ij}\xi}{t_{ij}} - \frac{2x_{mn}\xi}{t_{mn}}} F_{\mathrm{GCA}}(T, X)\,,$$

where T and X are the global GCA$_2$ invariant cross-ratios

$$T = \frac{t_{ij}\, t_{mn}}{t_{im}\, t_{jn}} \qquad \frac{X}{T} = \frac{x_{ij}}{t_{ij}} + \frac{x_{mn}}{t_{mn}} - \frac{x_{im}}{t_{im}} - \frac{x_{jn}}{t_{jn}}\,.$$

4-point functions & the GCA$_2$ blocks

A generic four point functions (at equal external weights) is constraint to be

$$\langle \phi(t_i,x_i)\phi(t_j,x_j)\phi(t_m,x_m)\phi(t_n,x_n)\rangle = \frac{1}{(t_{ij}t_{mn})^{2\Delta}} e^{-\frac{2x_{ij}\xi}{t_{ij}}-\frac{2x_{mn}\xi}{t_{mn}}} F_{\mathrm{GCA}}(T,X)\,,$$

where T and X are the global GCA$_2$ invariant cross-ratios

$$T = \frac{t_{ij}t_{mn}}{t_{im}t_{jn}} \qquad \frac{X}{T} = \frac{x_{ij}}{t_{ij}} + \frac{x_{mn}}{t_{mn}} - \frac{x_{im}}{t_{im}} - \frac{x_{jn}}{t_{jn}}\,.$$

we can use the global GCA$_2$ invariance to move the coordinates to

$$\{(t_i,x_i),(t_j,x_j),(t_m,x_m),(t_n,x_n)\} = \{(\infty,0),(1,0),(t,x),(0,0)\}$$

4-point functions & the GCA$_2$ blocks

A generic four point functions (at equal external weights) is constraint to be

$$\langle \phi(t_i,x_i)\phi(t_j,x_j)\phi(t_m,x_m)\phi(t_n,x_n)\rangle = \frac{1}{(t_{ij}t_{mn})^{2\Delta}} e^{-\frac{2x_{ij}\xi}{t_{ij}}-\frac{2x_{mn}\xi}{t_{mn}}} F_{\mathrm{GCA}}(T,X)\,,$$

where T and X are the global GCA$_2$ invariant cross-ratios

$$T = \frac{t_{ij}t_{mn}}{t_{im}t_{jn}} \qquad \frac{X}{T} = \frac{x_{ij}}{t_{ij}} + \frac{x_{mn}}{t_{mn}} - \frac{x_{im}}{t_{im}} - \frac{x_{jn}}{t_{jn}}\,.$$

we can use the global GCA$_2$ invariance to move the coordinates to

$$\{(t_i,x_i),(t_j,x_j),(t_m,x_m),(t_n,x_n)\} = \{(\infty,0),(1,0),(t,x),(0,0)\}$$

then $T \to t$, $X \to x$ and the four point function becomes

$$\langle \Delta_i,\xi_i|\phi_j(1,0)\phi_m(t,x)|\Delta_n,\xi_n\rangle = \frac{e^{-2\frac{x}{t}\xi}}{t^{2\Delta}} F_{\mathrm{GCA}}(t,x)$$

4-point functions & the GCA$_2$ blocks

A generic four point functions (at equal external weights) is constraint to be

$$\langle \phi(t_i,x_i)\phi(t_j,x_j)\phi(t_m,x_m)\phi(t_n,x_n)\rangle = \frac{1}{(t_{ij}t_{mn})^{2\Delta}} e^{-\frac{2x_{ij}\xi}{t_{ij}}-\frac{2x_{mn}\xi}{t_{mn}}} F_{\mathrm{GCA}}(T,X)\,,$$

where T and X are the global GCA$_2$ invariant cross-ratios

$$T = \frac{t_{ij}t_{mn}}{t_{im}t_{jn}} \qquad \frac{X}{T} = \frac{x_{ij}}{t_{ij}} + \frac{x_{mn}}{t_{mn}} - \frac{x_{im}}{t_{im}} - \frac{x_{jn}}{t_{jn}}\,.$$

we can use the global GCA$_2$ invariance to move the coordinates to

$$\{(t_i,x_i),(t_j,x_j),(t_m,x_m),(t_n,x_n)\} = \{(\infty,0),(1,0),(t,x),(0,0)\}$$

then $T \to t$, $X \to x$ and the four point function becomes

$$\langle \Delta_i,\xi_i|\phi_j(1,0)\phi_m(t,x)|\Delta_n,\xi_n\rangle = \sum_p c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t,x)$$

4-point functions & the GCA$_2$ blocks

The four point function in the channel $ij \leftrightarrow mn$

$$\langle \Delta_i, \xi_i | \phi_j(1,0) \phi_m(t,x) | \Delta_n, \xi_n \rangle = \sum_p c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t,x)$$

defines the GCA$_2$-blocks $\mathcal{A}_{mn}^{ji}(p|t,x)$.

4-point functions & the GCA$_2$ blocks

The four point function in the channel $ij \leftrightarrow mn$

$$\langle \Delta_i, \xi_i | \phi_j(1,0) \phi_m(t,x) | \Delta_n, \xi_n \rangle = \sum_p c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t,x)$$

defines the GCA$_2$-blocks $\mathcal{A}_{mn}^{ji}(p|t,x)$. They are

$$A_{mn}^{ji}(p|t,x) = (c_{ji}^p)^{-1} \frac{e^{-\xi_{mnp}\frac{x}{t}}}{t^{\Delta_{mnp}}} \sum_{\{N\},\alpha} \beta_{mn}^{p,\{N\},\alpha} t^{N-\alpha} x^\alpha \langle \Delta_i, \xi_i | \phi_j(1,0) | \Delta_p, \xi_p, \{N\} \rangle$$

$$= \frac{e^{-\xi_{mnp}\frac{x}{t}}}{t^{\Delta_{mnp}}} \sum_{\{N\},\{N'\},\alpha} t^{N-\alpha} x^\alpha \beta_{ji}^{p,\{N'\},0} \mathfrak{M}_{\{N'\},\{N\}} \beta_{mn}^{p,\{N\},\alpha}$$

4-point functions & the GCA$_2$ blocks

The four point function in the channel $ij \leftrightarrow mn$

$$\langle \Delta_i, \xi_i | \phi_j(1,0) \phi_m(t,x) | \Delta_n, \xi_n \rangle = \sum_p c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t,x)$$

defines the GCA$_2$-blocks $\mathcal{A}_{mn}^{ji}(p|t,x)$. They are

$$A_{mn}^{ji}(p|t,x) = (c_{ji}^p)^{-1} \frac{e^{-\xi_{mnp}\frac{x}{t}}}{t^{\Delta_{mnp}}} \sum_{\{N\},\alpha} \beta_{mn}^{p,\{N\},\alpha} t^{N-\alpha} x^\alpha \langle \Delta_i, \xi_i | \phi_j(1,0) | \Delta_p, \xi_p, \{N\} \rangle$$

$$= \frac{e^{-\xi_{mnp}\frac{x}{t}}}{t^{\Delta_{mnp}}} \sum_{\{N\},\{N'\},\alpha} t^{N-\alpha} x^\alpha \beta_{ji}^{p,\{N'\},0} \mathfrak{M}_{\{N'\},\{N\}} \beta_{mn}^{p,\{N\},\alpha}$$

Changing the way of contracting the fields inside the four point function should not change the final result. Instead contracting $in \leftrightarrow mj$ takes $t \to 1-t$ and $x \to -x$ and so

$$\sum_p c_{ji}^p c_{mn}^p A_{mn}^{ji}(p|t,x) = \sum_{p'} c_{ni}^{p'} c_{mj}^{p'} A_{mj}^{ni}(p'|1-t,-x)\,.$$

Global GCA_2-blocks

- Generically it is quite hard (impossible?) to find a closed form of the blocks.

Global GCA$_2$-blocks

- Generically it is quite hard (impossible?) to find a closed form of the blocks.
- In the large central charge limit, though, it is possible to find them by a neat trick.

Global GCA$_2$-blocks

- Generically it is quite hard (impossible?) to find a closed form of the blocks.
- In the large central charge limit, though, it is possible to find them by a neat trick.
- When $c_{L,M} \to \infty$, the only descendents contributing are L_{-1} and M_{-1}.

Global GCA$_2$-blocks

- Generically it is quite hard (impossible?) to find a closed form of the blocks.
- In the large central charge limit, though, it is possible to find them by a neat trick.
- When $c_{L,M} \to \infty$, the only descendents contributing are L_{-1} and M_{-1}.
- The OPE is an eigenfunction of the quadratic Casimirs of the global subalgebra

$$\mathcal{C}_1 = M_0^2 - M_{-1}M_{+1}\,,$$
$$\mathcal{C}_2 = 2L_0M_0 - \frac{1}{2}\left(M_{-1}L_{+1} + M_{+1}L_{-1} + L_{-1}M_{+1} + L_{+1}M_{-1}\right)\,.$$

Global GCA$_2$-blocks

- Generically it is quite hard (impossible?) to find a closed form of the blocks.
- In the large central charge limit, though, it is possible to find them by a neat trick.
- When $c_{L,M} \to \infty$, the only descendents contributing are L_{-1} and M_{-1}.
- The OPE is an eigenfunction of the quadratic Casimirs of the global subalgebra

$$\mathcal{C}_1 = M_0^2 - M_{-1}M_{+1}\,,$$
$$\mathcal{C}_2 = 2L_0M_0 - \frac{1}{2}\left(M_{-1}L_{+1} + M_{+1}L_{-1} + L_{-1}M_{+1} + L_{+1}M_{-1}\right)\,.$$

- When acting inside the four point function, we can represent the Casimirs as differential operators $\mathcal{D}_1$ and $\mathcal{D}_2$. This leads to

$$\mathcal{D}_1 A^{ji}_{mn}(p|t,x) = \xi_p^2 A^{ji}_{mn}(p|t,x) + \mathcal{O}(\tfrac{1}{c_M},\tfrac{1}{c_L})$$
$$\mathcal{D}_2 A^{ji}_{mn}(p|t,x) = 2\xi_p(\Delta_p - 1)\mathcal{A}^{ji}_{mn}(p|t,x) + \mathcal{O}(\tfrac{1}{c_M},\tfrac{1}{c_L})$$

Global GCA$_2$-blocks

Explicitly, calling the leading order in the $c_{M,L}$ expansion of the blocks at equal external weight $g_{\Delta,\xi}(p|t,x)$ and

$$g_{\Delta,\xi}(p|t,x) = \frac{e^{-2\frac{x}{t}\xi}}{t^{2\Delta}} f(p|t,x)$$

the relevant equations are

$$t^2(1-t)\partial_x^2 f(p|t,x) = \xi_p^2 f(p|t,x)\,,$$

$$\Big[-2t^2\partial_x + (2-3t)x\,t\,\partial_x^2 + 2(1-t)t^2\,\partial_x\partial_t\Big] f(p|t,x) = 2\xi_p(\Delta_p - 1) f(p|t,x)\,.$$

Global GCA$_2$-blocks

Explicitly, calling the leading order in the $c_{M,L}$ expansion of the blocks at equal external weight $g_{\Delta,\xi}(p|t,x)$ and

$$g_{\Delta,\xi}(p|t,x) = \frac{e^{-2\frac{x}{t}\xi}}{t^{2\Delta}} f(p|t,x)$$

the relevant equations are

$$t^2(1-t)\partial_x^2 f(p|t,x) = \xi_p^2 f(p|t,x)\,,$$
$$\Big[-2t^2\partial_x + (2-3t)x\,t\,\partial_x^2 + 2(1-t)t^2\,\partial_x\partial_t\Big] f(p|t,x) = 2\xi_p(\Delta_p - 1) f(p|t,x)\,.$$

And the solutions are the global GCA$_2$-blocks [Bagchi, Gary, Zodinmawia, '16]

$$g_{\Delta,\xi}(p|t,x) = \frac{e^{-2\xi\frac{x}{t}+\xi_p\frac{x}{t\sqrt{1-t}}}}{t^{2\Delta-\Delta_p}\sqrt{1-t}}\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-t}\right)^{2-2\Delta_p}\,,$$

These solutions have been successfully computed holographically in the probe limit [Hijano, Rabideau, '17]

$\mathcal{N}=1$ Super GCA$_2$-blocks

Let's start with one supercharge Q_r in the NS sector

$$
\begin{aligned}
[L_n, L_m] &= (n-m)L_{m+n} + \tfrac{c_L}{12} n(n^2-1)\delta_{m+n,0}\,, \\
[L_n, M_m] &= (n-m)M_{m+n} + \tfrac{c_M}{12} n(n^2-1)\delta_{m+n,0}\,, \\
[L_n, Q_r] &= (\tfrac{n}{2}-r)Q_{r+n}\,, \\
\{Q_r, Q_s\} &= M_{r+s} + \tfrac{c_M}{6}(r^2-\tfrac{1}{4})\delta_{r+s,0}\,,
\end{aligned}
$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Let's start with one supercharge Q_r in the NS sector

$$
\begin{aligned}
[L_n, L_m] &= (n-m)L_{m+n} + \tfrac{c_L}{12} n(n^2-1)\delta_{m+n,0}\,, \\
[L_n, M_m] &= (n-m)M_{m+n} + \tfrac{c_M}{12} n(n^2-1)\delta_{m+n,0}\,, \\
[L_n, Q_r] &= (\tfrac{n}{2} - r)Q_{r+n}\,, \\
\{Q_r, Q_s\} &= M_{r+s} + \tfrac{c_M}{6}(r^2 - \tfrac{1}{4})\delta_{r+s,0}\,,
\end{aligned}
$$

States can now be represented by a primary superfield Φ inserted at the origin of superspace $\{t, x, \theta\}$

$$
\Phi(t,x,\theta) = \phi(t,x) + \theta\,\psi(t,x)\,, \qquad |\Delta_p, \xi_p\rangle = \Phi(0,0,0)|0\rangle
$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Let's start with one supercharge Q_r in the NS sector

$$
\begin{aligned}
[L_n, L_m] &= (n-m)L_{m+n} + \tfrac{c_L}{12} n(n^2-1)\delta_{m+n,0}\,, \\
[L_n, M_m] &= (n-m)M_{m+n} + \tfrac{c_M}{12} n(n^2-1)\delta_{m+n,0}\,, \\
[L_n, Q_r] &= (\tfrac{n}{2} - r)Q_{r+n}\,, \\
\{Q_r, Q_s\} &= M_{r+s} + \tfrac{c_M}{6}(r^2-\tfrac{1}{4})\delta_{r+s,0}\,,
\end{aligned}
$$

States can now be represented by a primary superfield Φ inserted at the origin of superspace $\{t, x, \theta\}$

$$
\Phi(t,x,\theta) = \phi(t,x) + \theta\,\psi(t,x)\,, \qquad |\Delta_p, \xi_p\rangle = \Phi(0,0,0)|0\rangle
$$

The $\mathcal{N}=1$ generators act as differential operators on $\Phi(t,x,\theta)$

$$
\begin{aligned}
[L_n, \Phi(t,x,\theta)] &= \big[t^{n+1}\partial_t + (n+1)xt^n\partial_x + \tfrac{n+1}{2}t^n\theta\partial_\theta \\
&\qquad + \xi n(n+1)xt^{n-1} + \Delta(n+1)t^n\big]\Phi(t,x,\theta) \\
[M_n, \Phi(t,x,\theta)] &= \big[t^{n+1}\partial_x + \xi(n+1)t^n\big]\Phi(t,x,\theta)\,. \\
[Q_r, \Phi(t,x,\theta)] &= \big[t^{r+\frac{1}{2}}\left(\partial_\theta - \tfrac{1}{2}\theta\partial_x\right) - \xi(r+\tfrac{1}{2})t^{r-\frac{1}{2}}\theta\big]\Phi(t,x,\theta)
\end{aligned}
$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Correlators have x_{kl} replaced by a supersymmetric $\mathbf{x}_{kl}$

$$\mathbf{x}_{kl} = x_k - x_l - \frac{1}{2}\theta_k\theta_l\,.$$

so the crossratios $\mathbf{X}$ will contain fermionic coordinates, while T is unchanged and we now have a Grassmann odd invariant of three points Θ_{ijk}

$$\Theta_{ijk} = \frac{t_{ij}\theta_k + t_{jk}\theta_i + t_{ki}\theta_j}{\sqrt{t_{ij}\,t_{jk}\,t_{ki}}}\,.$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Correlators have x_{kl} replaced by a supersymmetric $\mathbf{x}_{kl}$

$$\mathbf{x}_{kl} = x_k - x_l - \frac{1}{2}\theta_k\theta_l\,.$$

so the crossratios $\mathbf{X}$ will contain fermionic coordinates, while T is unchanged and we now have a Grassmann odd invariant of three points Θ_{ijk}

$$\Theta_{ijk} = \frac{t_{ij}\theta_k + t_{jk}\theta_i + t_{ki}\theta_j}{\sqrt{t_{ij}\,t_{jk}\,t_{ki}}}\,.$$

This implies for the four point function that

$$\begin{aligned}\langle\Phi(t_i,x_i,\theta_i)\Phi(t_j,x_j,\theta_j)\Phi(t_m,x_m,\theta_m)\Phi(t_n,x_n,\theta_n)\rangle \\ = \frac{1}{(t_{ij}\,t_{mn})^{2\Delta}}\,e^{-\frac{2\mathbf{x}_{ij}\xi}{t_{ij}}-\frac{2\mathbf{x}_{mn}\xi}{t_{mn}}}\,F_{\mathrm{SGCA}}(T,X,\Theta_{imj},\Theta_{inj})\,,\end{aligned}$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Correlators have x_{kl} replaced by a supersymmetric $\mathbf{x}_{kl}$

$$\mathbf{x}_{kl} = x_k - x_l - \frac{1}{2}\theta_k\theta_l\,.$$

so the crossratios $\mathbf{X}$ will contain fermionic coordinates, while T is unchanged and we now have a Grassmann odd invariant of three points Θ_{ijk}

$$\Theta_{ijk} = \frac{t_{ij}\theta_k + t_{jk}\theta_i + t_{ki}\theta_j}{\sqrt{t_{ij}\,t_{jk}\,t_{ki}}}\,.$$

We may use the global SGCA subgroup (6 bosonic and 2 fermionic generators) to move the coordinates to

$$\{(t_i,x_i,\theta_i),(t_j,x_j,\theta_j),(t_m,x_m,\theta_m),(t_n,x_n,\theta_n)\} = \{(\infty,0,0),(1,0,0),(t,x,\theta),(0,0,\eta)\}$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Correlators have x_{kl} replaced by a supersymmetric $\mathbf{x}_{kl}$

$$\mathbf{x}_{kl} = x_k - x_l - \frac{1}{2}\theta_k\theta_l\,.$$

so the crossratios $\mathbf{X}$ will contain fermionic coordinates, while T is unchanged and we now have a Grassmann odd invariant of three points Θ_{ijk}

$$\Theta_{ijk} = \frac{t_{ij}\theta_k + t_{jk}\theta_i + t_{ki}\theta_j}{\sqrt{t_{ij}\,t_{jk}\,t_{ki}}}\,.$$

We may use the global SGCA subgroup (6 bosonic and 2 fermionic generators) to move the coordinates to

$$\{(t_i,x_i,\theta_i),(t_j,x_j,\theta_j),(t_m,x_m,\theta_m),(t_n,x_n,\theta_n)\} \\ = \{(\infty,0,0),(1,0,0),(t,x,\theta),(0,0,\eta)\}$$

At these points we have

$$T\to t\,,\qquad \mathbf{X}\to\mathbf{x}=x-\frac{1}{2}\theta\eta\,,\qquad \sqrt{1-T}\,\Theta_{imj}\Theta_{inj}\to\theta\eta\,.$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Correlators have x_{kl} replaced by a supersymmetric $\mathbf{x}_{kl}$

$$\mathbf{x}_{kl} = x_k - x_l - \frac{1}{2}\theta_k\theta_l\,.$$

so the crossratios $\mathbf{X}$ will contain fermionic coordinates, while T is unchanged and we now have a Grassmann odd invariant of three points Θ_{ijk}

$$\Theta_{ijk} = \frac{t_{ij}\theta_k + t_{jk}\theta_i + t_{ki}\theta_j}{\sqrt{t_{ij}\,t_{jk}\,t_{ki}}}\,.$$

We may use the global SGCA subgroup (6 bosonic and 2 fermionic generators) to move the coordinates to

$$\{(t_i,x_i,\theta_i),(t_j,x_j,\theta_j),(t_m,x_m,\theta_m),(t_n,x_n,\theta_n)\} \\ = \{(\infty,0,0),(1,0,0),(t,x,\theta),(0,0,\eta)\}$$

The four point function at equal external weights is now

$$\langle\Delta,\xi|\Phi(1,0,0)\Phi(t,x,\theta)\Phi(0,0,\eta)|0\rangle = t^{-2\Delta}e^{-2\frac{\mathbf{x}}{t}\xi}(F_0(t,x)+\theta\eta F_\theta(t,x))$$

$\mathcal{N} = 1$ Super GCA$_2$-blocks

Write the four point function as

$$\langle \Delta_i, \xi_i | \Phi_j(1,0,0) \Phi_m(t,x,\theta) \Phi_n(0,0,\eta) | 0 \rangle = \sum_p c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t,x,\theta,\eta)$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Write the four point function as

$$\langle \Delta_i, \xi_i | \Phi_j(1,0,0)\Phi_m(t,x,\theta)\Phi_n(0,0,\eta)|0\rangle = \sum_p c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t,x,\theta,\eta)$$

We can use the OPE

$$\Phi_m(t,x,\theta)\Phi_n(0,0,0) = \sum_p \frac{c_{mn}^p}{t^{\Delta_{mnp}}} e^{-\frac{x}{t}\xi_{mnp}} \sum_{\{N\},\alpha} \beta_{mn}^{p,\{N\},\alpha}\, t^{N-\alpha-\frac{q}{2}} x^\alpha \theta^q\, \Phi_p^{\{N\}}(0,0,0)\,,$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

Write the four point function as

$$\langle \Delta_i, \xi_i | \Phi_j(1,0,0)\Phi_m(t,x,\theta)\Phi_n(0,0,\eta)|0\rangle = \sum_p c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t,x,\theta,\eta)$$

We can use the OPE

$$\Phi_m(t,x,\theta)\Phi_n(0,0,0) = \sum_p \frac{c_{mn}^p}{t^{\Delta_{mnp}}} e^{-\frac{x}{t}\xi_{mnp}} \sum_{\{N\},\alpha} \beta_{mn}^{p,\{N\},\alpha}\, t^{N-\alpha-\frac{q}{2}} x^\alpha \theta^q\, \Phi_p^{\{N\}}(0,0,0)\,,$$

where now $\Phi_p^{\{N\}}(t,x,\theta) = L_{-k_1}\dots L_{-k_n} M_{-l_1}\dots L_{-l_m} Q_{-r_1}\dots Q_{-r_s}\Phi_p(t,x,\theta)$, to obtain the Supersymmetric GCA blocks

$$\begin{aligned}\mathcal{A}_{mn}^{ji}(p|t,x,\theta,\eta) =& (c_{ji}^p)^{-1} \sum_{\{N\},\alpha} t^{-\Delta_{mnp}+N-\alpha-\frac{q}{2}} e^{-\frac{\mathbf{x}}{t}\xi_{mnp}} \mathbf{x}^\alpha (\theta-\eta)^q \beta_{mn}^{p,\{N\},\alpha} \\ & \times \langle \Delta_i, \xi_i | \Phi_j(1,0,0)\Phi_p^{\{N\}}(0,0,\eta)|0\rangle\,.\end{aligned}$$

$\mathcal{N} = 1$ Super GCA_2-blocks

We can find a closed form expression for the large $c_{M/L}$ limit of the blocks

- In this limit the descendants of the global subgroup give the leading contributions

$\mathcal{N}=1$ Super GCA$_2$-blocks

We can find a closed form expression for the large $c_{M/L}$ limit of the blocks

- In this limit the descendants of the global subgroup give the leading contributions
- The four point function is hence an eigenstate of the quadratic Casimirs

$$\begin{aligned}\mathcal{C}_1 &= M_0^2 - M_{-1}M_{+1}\,,\\ \mathcal{C}_2 &= 2L_0M_0 - \tfrac{1}{2}\left(M_{-1}L_{+1} + M_{+1}L_{-1} + L_{-1}M_{+1} + L_{+1}M_{-1}\right)\\ &\qquad + \tfrac{1}{2}\left(Q_{+1/2}Q_{-1/2} - Q_{-1/2}Q_{+1/2}\right)\end{aligned}$$

$\mathcal{N}=1$ Super GCA$_2$-blocks

We can find a closed form expression for the large $c_{M/L}$ limit of the blocks

- In this limit the descendants of the global subgroup give the leading contributions
- The four point function is hence an eigenstate of the quadratic Casimirs

$$\begin{aligned}\mathcal{C}_1 &= M_0^2 - M_{-1}M_{+1}\,,\\ \mathcal{C}_2 &= 2L_0M_0 - \tfrac{1}{2}\left(M_{-1}L_{+1} + M_{+1}L_{-1} + L_{-1}M_{+1} + L_{+1}M_{-1}\right)\\ &\qquad + \tfrac{1}{2}(Q_{+1/2}Q_{-1/2} - Q_{-1/2}Q_{+1/2})\end{aligned}$$

- These act on the four-point function as differential operators

$\mathcal{N}=1$ Super GCA$_2$-blocks

We can find a closed form expression for the large $c_{M/L}$ limit of the blocks

- In this limit the descendants of the global subgroup give the leading contributions
- The four point function is hence an eigenstate of the quadratic Casimirs

$$\begin{aligned}\mathcal{C}_1 &= M_0^2 - M_{-1}M_{+1}\,,\\ \mathcal{C}_2 &= 2L_0M_0 - \tfrac{1}{2}\left(M_{-1}L_{+1} + M_{+1}L_{-1} + L_{-1}M_{+1} + L_{+1}M_{-1}\right)\\ &\qquad + \tfrac{1}{2}(Q_{+1/2}Q_{-1/2} - Q_{-1/2}Q_{+1/2})\end{aligned}$$

- These act on the four-point function as differential operators
- Solving the differential equations at equal external weights Δ and ξ we have

$$g_{\Delta,\xi}(p|t,x,\theta,\eta) = \frac{e^{-(2\xi-\xi_p)\frac{x}{t}+\xi_p\frac{x}{t}\frac{1-\sqrt{1-t}}{\sqrt{1-t}}}}{t^{2\Delta-\Delta_p}\sqrt{1-t}}\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-t}\right)^{2-2\Delta_p}$$

$\mathcal{N} = 2$ "Democratic"-blocks

The computations for the $\mathcal{N} = 2$ Democratic case are a relatively straightforward generalization from the $\mathcal{N} = 1$ case, besides one complication

$\mathcal{N} = 2$ "Democratic"-blocks

The computations for the $\mathcal{N} = 2$ Democratic case are a relatively straightforward generalization from the $\mathcal{N} = 1$ case, besides one complication

- Now there are two independent fermionic invariants $\Theta^{\pm}_{ijk}$

$\mathcal{N}=2$ "Democratic"-blocks

The computations for the $\mathcal{N}=2$ Democratic case are a relatively straightforward generalization from the $\mathcal{N}=1$ case, besides one complication

- Now there are two independent fermionic invariants $\Theta^{\pm}_{ijk}$
- This implies that the three-point function can depend on not one, but two arbitrary coefficients: c_{ijn} and $\tilde{c}_{ijn}$

$$\langle \Phi_i \Phi_j \Phi_n \rangle = \frac{c_{ijn} + \tilde{c}_{ijn} \Theta^{+}_{inj} \Theta^{-}_{inj}}{t_{ij}^{\Delta_{ijn}} t_{jn}^{\Delta_{jni}} t_{in}^{\Delta_{inj}}} e^{-\frac{x_{ij}\xi_{ijn}}{t_{ij}} - \frac{x_{jn}\xi_{jni}}{t_{jn}} - \frac{x_{in}\xi_{inj}}{t_{in}}},$$

$\mathcal{N}=2$ "Democratic"-blocks

The computations for the $\mathcal{N}=2$ Democratic case are a relatively straightforward generalization from the $\mathcal{N}=1$ case, besides one complication

- Now there are two independent fermionic invariants $\Theta^{\pm}_{ijk}$
- This implies that the three-point function can depend on not one, but two arbitrary coefficients: c_{ijn} and $\tilde{c}_{ijn}$

$$\langle \Phi_i \Phi_j \Phi_n \rangle = \frac{c_{ijn} + \tilde{c}_{ijn}\Theta^{+}_{inj}\Theta^{-}_{inj}}{t_{ij}^{\Delta_{ijn}} t_{jn}^{\Delta_{jni}} t_{in}^{\Delta_{inj}}} e^{-\frac{x_{ij}\xi_{ijn}}{t_{ij}} - \frac{x_{jn}\xi_{jni}}{t_{jn}} - \frac{x_{in}\xi_{inj}}{t_{in}}},$$

- This also implies that the OPE has two branches with coefficients $\beta^{p,\{N\},\alpha}_{mn}$ and $\tilde{\beta}^{p,\{N\},\alpha}_{mn}$

$$\Phi_m(t,x,\theta^{\pm})\Phi_n(\mathbf{0}) = \sum_{p,\{N\},\alpha} \frac{e^{-\frac{x}{t}\xi_{mnp}} x^{\alpha}}{t^{\Delta_{mnp}-N+\alpha}} \Big(c^{p}_{mn}\beta^{p,\{N\},\alpha}_{mn} t^{-\frac{R+S}{2}} (\theta^{+})^{R}(\theta^{-})^{S} + \tilde{c}^{p}_{mn}\tilde{\beta}^{p,\{N\},\alpha}_{mn} t^{-1+\frac{R+S}{2}} (\theta^{+})^{1-R}(\theta^{-})^{1-S} \Big) L_{-\{k\}} M_{-\{l\}} Q^{+}_{-\{r\}} Q^{-}_{-\{s\}} \Phi_p(\mathbf{0}).$$

$\mathcal{N}=2$ "Democratic"-blocks

- The blocks are essentially the four point functions expanded by the OPE twice, so they now split into four pieces

$$\langle \Delta_i, \xi_i | \Phi_j(1,\mathbf{0}) \Phi_m(t,x,\theta^{\pm}) \Phi_n(\mathbf{0},\eta^{\pm}) |0\rangle = \sum_p \Big[c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t,x,\theta^{\pm},\eta^{\pm}) + \tilde{c}_{ji}^p c_{mn}^p \mathcal{B}_{mn}^{ji}(p|t,x,\theta^{\pm},\eta^{\pm}) + c_{ji}^p \tilde{c}_{mn}^p \mathcal{C}_{mn}^{ji}(p|t,x,\theta^{\pm},\eta^{\pm}) + \tilde{c}_{ji}^p \tilde{c}_{mn}^p \mathcal{D}_{mn}^{ji}(p|t,x,\theta^{\pm},\eta^{\pm}) \Big]$$

$\mathcal{N} = 2$ "Democratic"-blocks

- The blocks are essentially the four point functions expanded by the OPE twice, so they now split into four pieces

$$\langle \Delta_i, \xi_i | \Phi_j(1, \mathbf{0}) \Phi_m(t, x, \theta^{\pm}) \Phi_n(\mathbf{0}, \eta^{\pm}) | 0 \rangle = \sum_p \Big[c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + \tilde{c}_{ji}^p c_{mn}^p \mathcal{B}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + c_{ji}^p \tilde{c}_{mn}^p \mathcal{C}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + \tilde{c}_{ji}^p \tilde{c}_{mn}^p \mathcal{D}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) \Big]$$

- In the limit of large central charge, each of the blocks is an independent solution to the Casimir eigenequations written as differential equations

$\mathcal{N} = 2$ "Democratic"-blocks

- The blocks are essentially the four point functions expanded by the OPE twice, so they now split into four pieces

$$\langle \Delta_i, \xi_i | \Phi_j(1, \mathbf{0}) \Phi_m(t, x, \theta^{\pm}) \Phi_n(\mathbf{0}, \eta^{\pm}) | 0 \rangle = \sum_p \Big[c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + \tilde{c}_{ji}^p c_{mn}^p \mathcal{B}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + c_{ji}^p \tilde{c}_{mn}^p \mathcal{C}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + \tilde{c}_{ji}^p \tilde{c}_{mn}^p \mathcal{D}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) \Big]$$

- In the limit of large central charge, each of the blocks is an independent solution to the Casimir eigenequations written as differential equations
- Explicitly expanding in the Grassmann variables gives eight coupled differential equations for eight independent functions of t and x

$\mathcal{N} = 2$ "Democratic"-blocks

- The blocks are essentially the four point functions expanded by the OPE twice, so they now split into four pieces

$$\langle \Delta_i, \xi_i | \Phi_j(1, \mathbf{0}) \Phi_m(t, x, \theta^{\pm}) \Phi_n(\mathbf{0}, \eta^{\pm}) | 0 \rangle = \sum_p \Big[c_{ji}^p c_{mn}^p \mathcal{A}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + \tilde{c}_{ji}^p c_{mn}^p \mathcal{B}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + c_{ji}^p \tilde{c}_{mn}^p \mathcal{C}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) + \tilde{c}_{ji}^p \tilde{c}_{mn}^p \mathcal{D}_{mn}^{ji}(p|t, x, \theta^{\pm}, \eta^{\pm}) \Big]$$

- In the limit of large central charge, each of the blocks is an independent solution to the Casimir eigenequations written as differential equations
- Explicitly expanding in the Grassmann variables gives eight coupled differential equations for eight independent functions of t and x
- By comparing the solutions to the expansion of the four point function using the OPE we can fix the integration constants for each of the blocks separately

$\mathcal{N}=2$ "Democratic"-blocks

The solutions in the limit of large central charges are

$$\mathcal{A}(p|t,x,\theta^{\pm},\eta^{\pm}) = e^{-\frac{x}{t}(2\xi_i-\xi_p)}\frac{e^{\frac{(1-\sqrt{1-t})}{\sqrt{1-t}}\frac{x}{t}\xi_p}}{t^{2\Delta_i-\Delta_p}\sqrt{1-t}}\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-t}\right)^{2-2\Delta_p}.$$

$$\mathcal{B}(p|t,x,\theta^{\pm},\eta^{\pm}) = \frac{2^{2\Delta_p-2}e^{-\frac{x}{t}(2\xi_i-\frac{\xi_p}{\sqrt{1-t}})}}{t^{2\Delta_i-\Delta_p}(1-t)\left(1+\sqrt{1-t}\right)^{2\Delta_p}} \times(\theta^+ + \sqrt{1-t}\,\eta^+)(\theta^- + \sqrt{1-t}\,\eta^-).$$

$$\mathcal{C}(p|t,x,\theta^{\pm},\eta^{\pm}) = \frac{2^{2\Delta_p-2}e^{-\frac{x}{t}(2\xi_i-\frac{\xi_p}{\sqrt{1-t}})}}{t^{2\Delta_i-\Delta_p}t(1-t)\left(1+\sqrt{1-t}\right)^{2\Delta_p-2}} \times(\theta^+ - \sqrt{1-t}\,\eta^+)(\theta^- - \sqrt{1-t}\,\eta^-).$$

$$\mathcal{D}(p|t,x,\theta^{\pm},\eta^{\pm}) = e^{-\frac{x}{t}(2\xi_i+\xi_p)}\frac{e^{\frac{(1+\sqrt{1-t})}{\sqrt{1-t}}\frac{x}{t}\xi_p}}{4\xi_p^2\,t^{2\Delta_i-\Delta_p-1}\sqrt{1-t}}\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-t}\right)^{-2\Delta_p}.$$

$\mathcal{N} = 2$ "Despotic"-blocks

The computations for the $\mathcal{N} = 2$ Despotic case are a relatively straightforward generalization from the $\mathcal{N} = 2$ Democratic case, besides one complication

$\mathcal{N} = 2$ "Despotic"-blocks

The computations for the $\mathcal{N} = 2$ Despotic case are a relatively straightforward generalization from the $\mathcal{N} = 2$ Democratic case, besides one complication

- Now the t coordinates receive contributions from Grassmann coordinates (θ_i, χ_i)

$$\mathbf{t}_{ij} = t_i - t_j - \theta_i\theta_j\,, \qquad \mathbf{x}_{ij} = x_i - x_j - \theta_i\chi_j - \chi_i\theta_j$$

$\mathcal{N}=2$ "Despotic"-blocks

The computations for the $\mathcal{N}=2$ Despotic case are a relatively straightforward generalization from the $\mathcal{N}=2$ Democratic case, besides one complication

- Now the t coordinates receive contributions from Grassmann coordinates (θ_i, χ_i)

$$\mathbf{t}_{ij} = t_i - t_j - \theta_i\theta_j\,, \qquad \mathbf{x}_{ij} = x_i - x_j - \theta_i\chi_j - \chi_i\theta_j$$

- Hence the cross-ratios both involve fermionic coordinates

$$\mathbf{T} = \frac{\mathbf{t}_{ij}\mathbf{t}_{mn}}{\mathbf{t}_{im}\mathbf{t}_{jn}} \qquad \frac{\mathbf{X}}{\mathbf{T}} = \frac{\mathbf{x}_{ij}}{\mathbf{t}_{ij}} + \frac{\mathbf{x}_{mn}}{\mathbf{t}_{mn}} - \frac{\mathbf{x}_{im}}{\mathbf{t}_{im}} - \frac{\mathbf{x}_{jn}}{\mathbf{t}_{jn}}\,.$$

- The fermionic invariants also become more involved:

$$\Theta_{ijk} = \frac{t_{ij}\theta_k + t_{jk}\theta_i + t_{ki}\theta_j - \frac{1}{2}\theta_i\theta_j\theta_k}{\sqrt{t_{ij}\,t_{jk}\,t_{ki}}}$$

$$\Xi_{ijk} = \frac{1}{2\sqrt{t_{ij}\,t_{jk}\,t_{ki}}}\left(2t_{ij}\chi_k + x_{ij}\theta_k - \theta_i\theta_j\chi_k \right.$$
$$\left. - t_{ij}\theta_k\left(\frac{x_{jk}}{t_{jk}} + \frac{x_{ki}}{t_{ki}}\right) + \frac{1}{2}\theta_i\theta_j\theta_k\frac{x_{ij}}{t_{ij}} + \text{perm.}\right)$$

$\mathcal{N} = 2$ "Despotic"-blocks

Besides this complication the computations carry through analogously:

$\mathcal{N} = 2$ "Despotic"-blocks

Besides this complication the computations carry through analogously:

- The correlation functions are the same, with t and x replaced by $\mathbf{t}$ and $\mathbf{x}$

$\mathcal{N} = 2$ "Despotic"-blocks

Besides this complication the computations carry through analogously:

- The correlation functions are the same, with t and x replaced by $\mathbf{t}$ and $\mathbf{x}$
- The OPE still has two branches with coefficients $\beta_{mn}^{p,\{N\},\alpha}$ and $\tilde{\beta}_{mn}^{p,\{N\},\alpha}$

$\mathcal{N} = 2$ "Despotic"-blocks

Besides this complication the computations carry through analogously:

- The correlation functions are the same, with t and x replaced by $\mathbf{t}$ and $\mathbf{x}$
- The OPE still has two branches with coefficients $\beta_{mn}^{p,\{N\},\alpha}$ and $\tilde{\beta}_{mn}^{p,\{N\},\alpha}$
- The blocks therefor again split into four parts

$\mathcal{N}=2$ "Despotic"-blocks

Besides this complication the computations carry through analogously:

- The correlation functions are the same, with t and x replaced by **t** and **x**
- The OPE still has two branches with coefficients $\beta_{mn}^{p,\{N\},\alpha}$ and $\tilde{\beta}_{mn}^{p,\{N\},\alpha}$
- The blocks therefor again split into four parts
- In the global limit ($c_{M/L} \to \infty$) the blocks can be found by solving the differential equations resulting from the Casimir eigenequations, where now the Casimirs are slightly more involved:

$$\mathcal{C}_1 = M_0^2 - M_{-1}M_{+1} + \frac{1}{2}H_{1/2}H_{-1/2}\,,$$

$$\mathcal{C}_2 = 2L_0M_0 - \tfrac{1}{2}\left(M_{-1}L_{+1} + M_{+1}L_{-1} + L_{-1}M_{+1} + L_{+1}M_{-1}\right) + \tfrac{1}{4}\left(H_{+1/2}G_{-1/2} - G_{-1/2}H_{+1/2} + G_{+1/2}H_{-1/2} - H_{-1/2}G_{+1/2}\right)$$

$\mathcal{N} = 2$ "Despotic"-blocks

Besides this complication the computations carry through analogously:

- The correlation functions are the same, with t and x replaced by **t** and **x**
- The OPE still has two branches with coefficients $\beta_{mn}^{p,\{N\},\alpha}$ and $\tilde{\beta}_{mn}^{p,\{N\},\alpha}$
- The blocks therefor again split into four parts
- In the global limit ($c_{M/L} \to \infty$) the blocks can be found by solving the differential equations resulting from the Casimir eigenequations, where now the Casimirs are slightly more involved:

$$\mathcal{C}_1 = M_0^2 - M_{-1}M_{+1} + \frac{1}{2}H_{1/2}H_{-1/2}\,,$$

$$\mathcal{C}_2 = 2L_0M_0 - \tfrac{1}{2}\left(M_{-1}L_{+1} + M_{+1}L_{-1} + L_{-1}M_{+1} + L_{+1}M_{-1}\right) + \tfrac{1}{4}\left(H_{+1/2}G_{-1/2} - G_{-1/2}H_{+1/2} + G_{+1/2}H_{-1/2} - H_{-1/2}G_{+1/2}\right)$$

- The solutions exist and will be communicated to the world in a forthcoming publication, right after we massage them into a presentable form

Conclusions & Outlook

- We have studied the supersymmetric $\mathrm{GCA}_2/\mathrm{BMS}_3$ blocks for $\mathcal{N}=1$ supersymmetry and for two different possible $\mathcal{N}=2$ extensions of $\mathrm{GCA}_2/\mathrm{BMS}_3$.

Conclusions & Outlook

- We have studied the supersymmetric $\mathrm{GCA}_2/\mathrm{BMS}_3$ blocks for $\mathcal{N}=1$ supersymmetry and for two different possible $\mathcal{N}=2$ extensions of $\mathrm{GCA}_2/\mathrm{BMS}_3$.
- For generic central charge the blocks can be computed as a sum over operators (primaries and their descendants) in the exchange channel

Conclusions & Outlook

- We have studied the supersymmetric $\mathrm{GCA}_2/\mathrm{BMS}_3$ blocks for $\mathcal{N}=1$ supersymmetry and for two different possible $\mathcal{N}=2$ extensions of $\mathrm{GCA}_2/\mathrm{BMS}_3$.
- For generic central charge the blocks can be computed as a sum over operators (primaries and their descendants) in the exchange channel
- In the large $c_{M/L}$ limit, a closed form expression for the blocks can be found by solving the differential equations obtained from the Casimirs of the global subalgebra

Conclusions & Outlook

- We have studied the supersymmetric $\mathrm{GCA}_2/\mathrm{BMS}_3$ blocks for $\mathcal{N}=1$ supersymmetry and for two different possible $\mathcal{N}=2$ extensions of $\mathrm{GCA}_2/\mathrm{BMS}_3$.
- For generic central charge the blocks can be computed as a sum over operators (primaries and their descendants) in the exchange channel
- In the large $c_{M/L}$ limit, a closed form expression for the blocks can be found by solving the differential equations obtained from the Casimirs of the global subalgebra
- It will be interesting to compute the global blocks holographically in 3d flat space supergravity and compare.

Conclusions & Outlook

- We have studied the supersymmetric $\mathrm{GCA}_2/\mathrm{BMS}_3$ blocks for $\mathcal{N}=1$ supersymmetry and for two different possible $\mathcal{N}=2$ extensions of $\mathrm{GCA}_2/\mathrm{BMS}_3$.
- For generic central charge the blocks can be computed as a sum over operators (primaries and their descendants) in the exchange channel
- In the large $c_{M/L}$ limit, a closed form expression for the blocks can be found by solving the differential equations obtained from the Casimirs of the global subalgebra
- It will be interesting to compute the global blocks holographically in 3d flat space supergravity and compare.
- So far we have relied on the (non-unitary) highest-weight representations. Can we generalize the BMS_3 bootstrap using induced representations? What would change?

Conclusions & Outlook

- We have studied the supersymmetric $\mathrm{GCA}_2/\mathrm{BMS}_3$ blocks for $\mathcal{N}=1$ supersymmetry and for two different possible $\mathcal{N}=2$ extensions of $\mathrm{GCA}_2/\mathrm{BMS}_3$.
- For generic central charge the blocks can be computed as a sum over operators (primaries and their descendants) in the exchange channel
- In the large $c_{M/L}$ limit, a closed form expression for the blocks can be found by solving the differential equations obtained from the Casimirs of the global subalgebra
- It will be interesting to compute the global blocks holographically in 3d flat space supergravity and compare.
- So far we have relied on the (non-unitary) highest-weight representations. Can we generalize the BMS_3 bootstrap using induced representations? What would change?
- Ultimately: can we derive bounds on the operator spectrum and structure constants for BMS invariant theories?

The End

Thank you for your attention