Strolling along gravitational vacua

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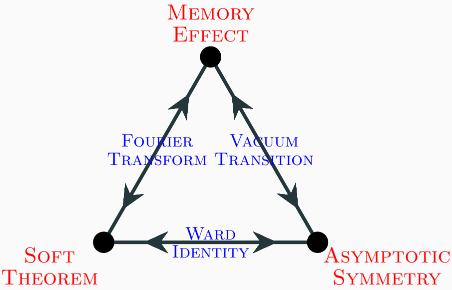
- Strominger's infrared triangle



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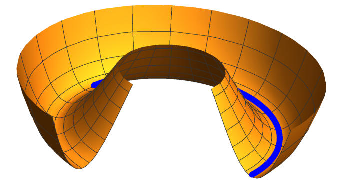

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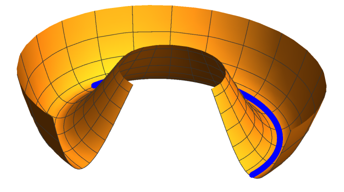

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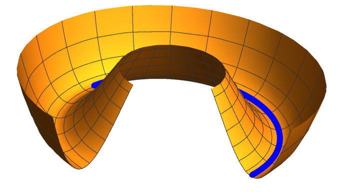

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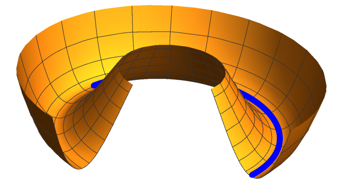

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Warm-up: Particle theory

Particle theory

- Lagrangian

$$L = \frac{1}{2} g_{IJ}(X) \dot{X}^I \dot{X}^J - V(X^I)$$

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- Space of vacua $\mathcal{V} = S^2 = SO(3)/SO(2)$
- Spherical coordinates $(R+\rho, z^\alpha)$

$$\begin{aligned}(R+\rho)\left(\ddot{z}^\alpha + \Gamma^\alpha_{\beta\gamma}\dot{z}^\beta \dot{z}^\gamma\right) + 2\dot{\rho}\dot{z}^\alpha &= 0 \\ \ddot{\rho} + \frac{4}{m}\rho(\rho+2R)(\rho+R) - (R+\rho)g_{\alpha\beta}\dot{z}^\alpha \dot{z}^\beta &= 0\end{aligned}$$

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- Low energy solutions: Geodesics on the space of vacua.

Maxwell theory and its vacua

Configuration space and its metric

- Temporal gauge $A_0 = 0$

$$L[A_i] = \frac{1}{2}\int d^3x \left(g^{ij}\dot{A}_i\dot{A}_j - F_{ij}F^{ij}\right)$$

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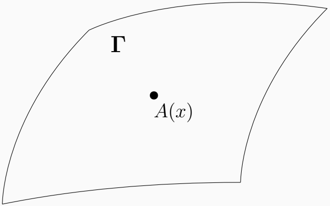

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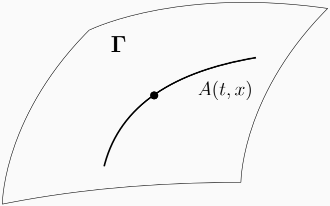

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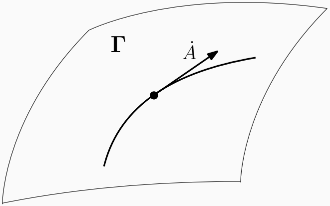

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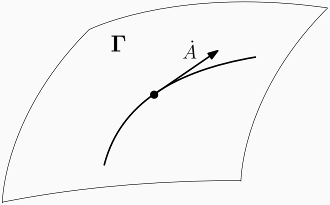

- Metric on the configuration space

$$g(\delta_1 A, \delta_2 A) = \int d^3x \, \mathrm{Tr}\, \delta_1 A_i \, \delta_2 A^i$$

Physical vacua

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- Theorem: $\mathcal{G}_{\text{harmonic}} \cong \mathcal{G}_{\text{boundary}} \cong \mathcal{G}/\mathcal{G}_0$.

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- Metric on the space of vacua

$$\begin{aligned} \mathrm{g}(\nabla\phi_{\ell m}, \nabla\phi_{\ell' m'}) &= \int_M \nabla_i \phi_{\ell m} \nabla^i \phi_{\ell' m'} \\ &= \oint_{\partial M} \phi_{\ell' m'}\, n \cdot \nabla\phi_{\ell m} = \ell\, \delta_{\ell\ell'}\, \delta_{mm'} \end{aligned}$$

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- Physical vs. pure gauge

$$\mathrm{g}(\nabla\phi_{\mathsf{harm}}, \nabla\lambda_{\mathsf{pure\ gauge}}) = 0, \qquad \lambda_{\mathsf{pure\ gauge}}\big|_{\partial M} = 0$$

Summary so far

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- This is missing in asymptotic symmetry analyses

Gravitational vacua and low energy dynamics

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- Field equations

$$R_{ij} + \frac{1}{2}\ddot{h}_{ij} - \frac{1}{2}h^{kl}\dot{h}_{k[i}\dot{h}_{l]j} = 0 \qquad \textit{Dynamical equation}$$

$$\nabla^i\left(\dot{h}_{ij} - h^{kl}\dot{h}_{kl}h_{ij}\right) = 0 \qquad \textit{Momentum constraint}$$

$$R + \frac{1}{2}h^{ij}h^{kl}\dot{h}_{i[j}\dot{h}_{k]l} = 0 \qquad \textit{Hamiltonian constraint}$$

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- Coordinate system z^α $\qquad \bar{h}(x;z) = g_z \cdot h_{\mathsf{ref}}$

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- Low energy dynamics

$$S[z(t)] = \int dt \frac{1}{2}\bar{\mathrm{g}}_z(\dot{z},\dot{z}) \qquad \bar{\mathrm{g}}_z(\dot{z},\dot{z}) = \mathrm{g}(\bar{\nabla}_{\mathrm{s}}\chi_z, \bar{\nabla}_{\mathrm{s}}\chi_z)$$

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$$\chi_i = \eta_i + \partial_j \phi, \qquad \nabla_i \eta^i = 0$$

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$$\begin{aligned}\chi_i = \eta_i + \partial_j \phi, &\qquad \nabla_i \eta^i = 0 \\ \langle \eta, \eta \rangle_h \geq 0, &\qquad \langle \partial\phi, \partial\phi \rangle_h \leq 0\end{aligned}$$

Imposing constraints

- Momentum constraint

$$\nabla^i \partial_{[i} \chi_{j]} = 0$$

- Metric on the space of vacua

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- Hamiltonian constraint $\qquad \langle \chi_1, \chi_2 \rangle_h = 0$

Bulk extension

- Bulk extension of $\chi\Big|_{\partial M} = \zeta$ is unique up to exact vector fields $\partial_i \phi$.
- This however, does not affect the metric
- Conjecture The Hamiltonian constraint then fix the remaining part uniquely
- All the data is available on the boundary

Conserved momenta

- Conserved momenta

$$P_\zeta \equiv \mathrm{g}_z(\dot{z}, \delta_\zeta h) = \langle \zeta, \chi_z \rangle_{\bar{h}(z)} = \int_{\partial M} \sqrt{\bar{k}} d^2 y \, n^i e_a^j \zeta^a \dot{h}_{ij}$$

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- Surface charges from covariant phase space

$$Q_\chi = \int_M \Theta(\delta_\chi h)$$

$$\Theta(\delta_\chi h) = \int_M d^3x \, \sqrt{h} (\dot{h}_{ij} - h_{ij} h^{kl} \dot{h}_{kl}) \nabla^i \chi^j$$

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$$Q_\chi = \int_{\partial M} d^2 y \sqrt{k} \, n^i \, \chi^j \dot{h}_{ij}$$

- The momenta are equal to Noether surface charges

$$P_\zeta = Q_\chi \quad \text{where} \quad \zeta^a = \chi^a|_{\partial M}$$

Example: Spherical boundary

- Diffeos on the boundary

$$\zeta_a = \sqrt{k}\epsilon_a{}^b\partial_b\tau + \partial_a\rho \qquad \tau = \sum_{\ell,m} a_{\ell m}, \quad \rho = \sum_{\ell,m} b_{\ell m}Y_{\ell m}$$

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- Modeling a generic stationary solution (conjecture)

Summary and outlook

- Adiabatic approximation as a probe of the space of vacua
- It allows to identify the physical symmetries and their bulk extensions
- and to identify a set of solutions of the theory
- Divergences in the large volume limit
- Solve the bulk extension and identify solutions

Thank you for your attention