Quantum Null Energy Condition

In two dimensions

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Based on 1901.04499

Inequalities in physics

Interesting physical consequences from mathematical inequalities

- Positivity inequalities: probabilities non-negative, $P \geq 0$

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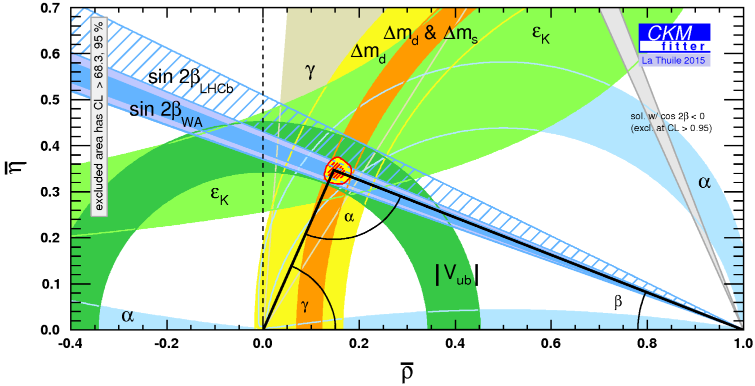

Example: unitarity constraints on physical parameters in quark mixing matrix

if Standard Model correct then measurements must reproduce unitarity triangle

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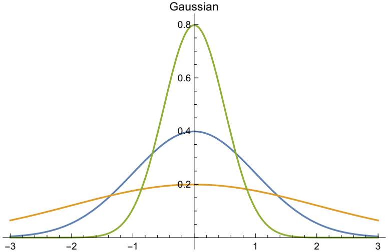

green: localized in coordinate space (x), delocalized in momentum space (p)
blue: mildly (de-)localized in coordinate and momentum space
orange: delocalized in coordinate space (x), localized in momentum space (p)

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  - Definition: (local) inequalities on the stress tensor $T_{\mu\nu}$
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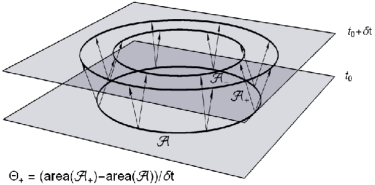
For instance: Penrose singularity theorem from Raychaudhuri eq.

$$\frac{\mathrm{d}^2\text{area}}{\mathrm{d}k^2} = -\Big(\frac{\mathrm{d}\,\text{area}}{\mathrm{d}k}\Big)^2 - \text{shear}^2 - 8\pi G \, T_{kk} \leq -8\pi G \, T_{kk} \overset{\text{NEC}}{\leq} 0$$

If $T_{kk} \geq 0$ (NEC) $\Rightarrow$ focussing!
(negative acceleration of area)

For experts:
$\frac{\mathrm{d}\,\text{area}}{\mathrm{d}k} = \theta$ is null expansion



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However: all classical energy inequalities violated by quantum effects!

NEC violated by Casimir energy, accelerated mirrors, Hawking radiation, ...

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Are there quantum energy conditions?
[How is 2^{nd} law saved?]

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  Faulkner, Leigh, Parrikar and Wang 1605.08072
  Hartman, Kundu and Tajdini 1610.05308

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> Is there a local quantum energy condition?

Quantum null energy condition (QNEC)

Proposed by Bousso, Fisher, Leichenauer and Wall in 1506.02669

QNEC (in $D > 2$) is the following inequality

$$\langle T_{kk}\rangle \geq \frac{\hbar}{2\pi\sqrt{\gamma}}\, S''$$

Physical motivation from focussing properties and second law:
Replace area by area $+ 4G$ (entanglement entropy)
Modified Raychaudhuri eq., schematically:

$$\frac{\mathrm{d}^2\text{area}}{\mathrm{d}k^2} + 4G\, S'' = -8\pi G\, T_{kk} + 4G\, S'' \overset{\text{QNEC}}{\leq} 0$$

requires for focussing property ($= 2^{\text{nd}}$ law) QNEC

fineprint: above we set expansion to zero, $\frac{\mathrm{d\,area}}{\mathrm{d}k} = 0$, and shear to zero; we also set the area to unity, $\sqrt{\gamma} = 1$

thus, QNEC is implied from quantum focussing for special congruences

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Obvious observations:

- if r.h.s. vanishes: semi-classical version of NEC
- if r.h.s. negative: weaker condition than NEC (NEC can be violated while QNEC holds)
- if r.h.s. positive: stronger condition than NEC (if QNEC holds also NEC holds)

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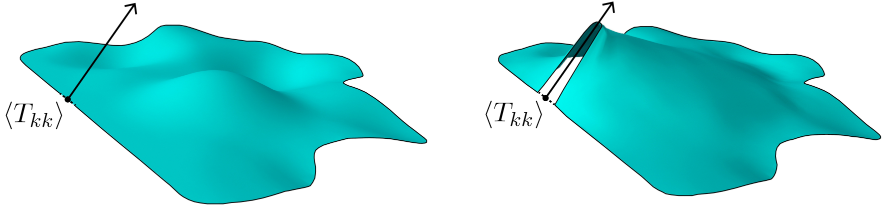

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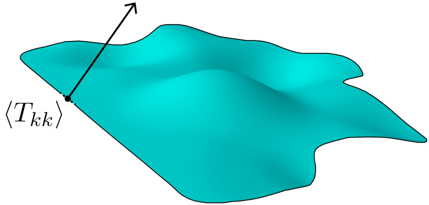

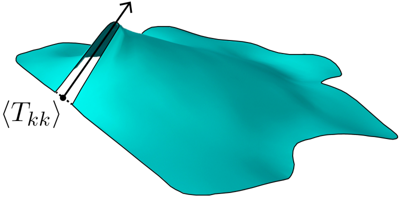

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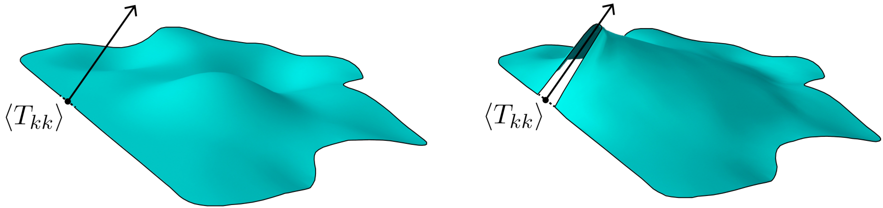

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- $\sqrt{\gamma}$: induced volume form of entangling region (black boundary curve)

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Proofs of QNEC in $D > 2$:

- For free QFTs: Bousso, Fisher, Koeller, Leichenauer and Wall, 1509.02542
- For holographic CFTs: Koeller and Leichenauer, 1512.06109
- For general CFTs: Balakrishnan, Faulkner, Khandker and Wang, 1706.09432
- QNEC from ANEC in QFTs: Ceyhan, Faulkner 1812.04683

QNEC in CFT_2

QNEC (in CFT_2) is the following inequality

$$2\pi \langle T_{kk} \rangle \geq S'' + \frac{6}{c} \left(S'\right)^2$$

$c > 0$ is the central charge of the CFT_2

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Focus here on QNEC in $\mathrm{AdS}_3/\mathrm{CFT}_2$

EE for states dual to Bañados geometries

- Solutions to AdS_3 Einstein gravity with Brown–Henneaux bc's:

$$\mathrm{d}s^2 = \frac{\mathrm{d}z^2 - \mathrm{d}x^+ \, \mathrm{d}x^-}{z^2} + \mathcal{L}^+(x^+)(\mathrm{d}x^+)^2 + \mathcal{L}^-(x^-)(\mathrm{d}x^-)^2 + \mathcal{O}(z^2)$$

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$$S = S^+ + S^- \qquad\qquad S^\pm = \frac{c}{6}\,\ln\big(\ell^\pm(x_1^\pm,\, x_2^\pm)/\epsilon_{\mathrm{UV}}\big)$$

from local diffeo to Poincaré patch AdS_3, $\{z,\, x^\pm\} \to \{z_P,\, x_P^\pm\}$

$x_1^\pm$, $x_2^\pm$: boundary points of entangling region

ϵ_{UV}: cutoff

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- $\ell^\pm(x_1^\pm,\,x_2^\pm) = \psi_1^\pm(x_1^\pm)\psi_2^\pm(x_2^\pm) - \psi_2^\pm(x_1^\pm)\psi_1^\pm(x_2^\pm)$ contain solutions to Hill's equation

$$\psi^{\pm\prime\prime} - \mathcal{L}^\pm\,\psi^\pm = 0$$

  with unit Wronskian $\psi_1^\pm\psi_2^{\pm\prime} - \psi_2^\pm\psi_1^{\pm\prime} = \pm 1$

QNEC saturation

- HEE transforms like anomalous scalar under diffeos (Wall '11)

$$\delta_\xi S = -\xi^\mu \, \partial_\mu S + \frac{c}{12} \, \partial_\mu \xi^\mu$$

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QNEC saturation for all states dual to Bañados geometries

Example: far from equilibrium flow

- QNEC saturation for vacuum-like states obvious from symmetries

  see arguments in Khandker, Kundu, Li '18 for QNEC saturation in absence of bulk matter

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$$\mathcal{L}^+(x) = \mathcal{L}^-(-x) = \pi^2 \left(T_L^2 + \theta(x)\left(T_R^2 - T_L^2\right)\right)$$

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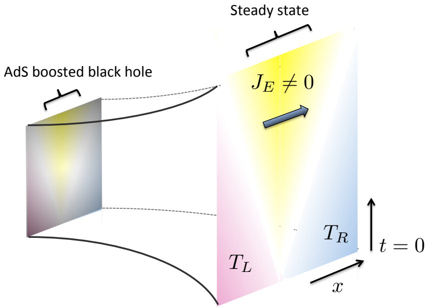
Figure 2 from 1311.3655 (Nature Phys.)



- Far from equilibrium transport in strongly coupled CFT
- Long-time energy transport universally via steady-state
- In AdS_3/CFT_2: specific Bañados geometry with step function
- Our results imply QNEC saturation at all times

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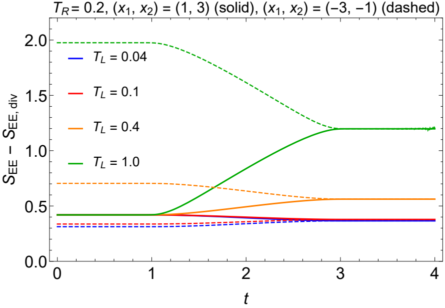

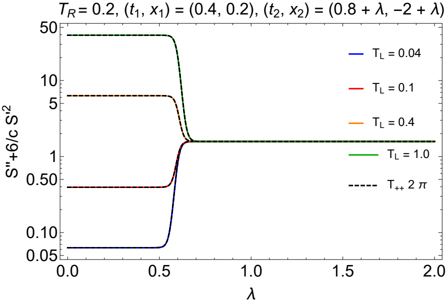

Left: HEE

Right: QNEC saturation

QNEC in dual of AdS$_3$-Vaidya

- Vaidya = simple model for bulk matter; mass function $M(t)$

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- Can be derived perturbatively for $M(t) = \epsilon\theta(t)$ with $\epsilon \ll 1$

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- If time is much larger than entangling region we find QNEC saturation

$$\lim_{t_0\gg l}\frac{S'' + \frac{6}{c}\,(S')^2}{2\pi\,\langle T_{kk}\rangle} = 1 + \mathcal{O}(\epsilon)$$

Finite-c corrections to EE and QNEC

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NOT a Bañados geometry

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- Asymptotically conical defect, $2\pi\,\langle T_{\pm\pm}\rangle = -\frac{c}{24} + h + \mathcal{O}(h^2/c)$

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$$\delta S_{\text{bulk}} = 2\pi\,\langle\delta H_{\text{bulk}}\rangle + \mathcal{O}(\delta^2)$$

δ: small perturbation of the vacuum

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$\alpha_{\omega,\,k}, \beta_{\omega,\,k}$: Bogoliubov coefficients

calculated in appendix of Belin, Iqbal, Lokhande '18

map entanglement wedge to Rindler space (see Casini, Huerta, Myers '11) and expand scalar field ϕ in Rindler modes

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QNEC holds. QNEC saturation up to $\mathcal{O}(\Delta\varphi^{4h})$

Half-interval

- Pragmatically, need small parameter for S_{bulk}

  reason: for Renyi entropies need reduced density matrix
  ρ: bulk density matrix
  ρ_1: reduced density matrix
  ρ_0: reduced density matrix for vacuum
  $\delta\rho = \rho_1 - \rho_0$: small for small interval

  e.g. small interval limit: $\Delta\varphi$ is small parameter

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unexpected!

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Recovered from CFT_2 calculation

$$\delta S_n = \frac{1}{1-n}\,\ln\left[\left(\frac{1}{n}\,\sin(\Delta\varphi/2)\right)^{4nh}\text{Hf}\right]$$

by taking suitable limit of Hafnian for $\Delta\varphi = \pi - \epsilon$

$$\text{Hf} = \text{Hf}_0 + (n-1)\,\text{Hf}_1 + \dots$$

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QNEC non-saturation at half-interval:

$$2\pi\,\langle T_{\pm\pm}\rangle - S'' - \frac{6}{c}\,(S')^2\big|_{\Delta\varphi=\pi,\,c\gg h\gg 1} = \frac{1}{4}\,h$$

Loose ends

We scratched the surface of QNEC in $\text{AdS}_3/\text{CFT}_2$

- QNEC saturates for vacuum-like states
- Curious half-saturation for states dual to AdS_3-Vaidya
- Leading $1/c$ correction
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- QNEC as constraint for semi-classical model building (black holes)
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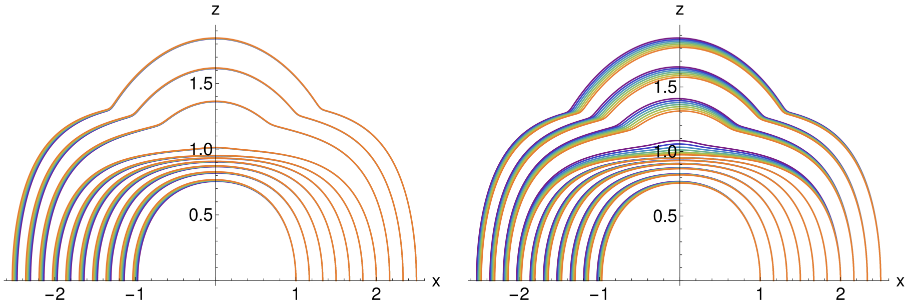
- QNEC-like inequalities for QFTs that are not CFTs?
  - Galilean/ultrarelativistic CFTs and flat space holography? Bagchi et al '09–'18
  - warped CFTs (and their holographic duals)? Detournay, Hartman, Hofman '12
  - other non-standard QFTs?

Much to be learned about QNEC and its potential applications



Families of geodesics used in AdS_3-Vaidya for numerical QNEC determination

Thanks for your attention!