Gauge/gravity duality: unification of ideas

Jakub Jankowski

Faculty of Physics, University of Warsaw

"On the other hand, it might be helpful to explore a theory that deviates from the unknown truth in the opposite direction from that of the conventional theory."

Enrico Fermi

NARODOWE CENTRUM NAUKI

UNIVERSITY OF WARSAW

Bekenstein bound

The maximum entropy in a region of space is

$$S_{\mathrm{max}} = \frac{A}{4G_N}$$

where A is area of the boundary of the region

Argument: Suppose we have a state with $S > S_{\mathrm{max}}$. Throwing in more and more stuff we produce a black hole contained in the chosen region. But

$$S_{\mathrm{BH}} = \frac{A_{\mathrm{BH}}}{4G_N} < S_{\mathrm{max}}$$

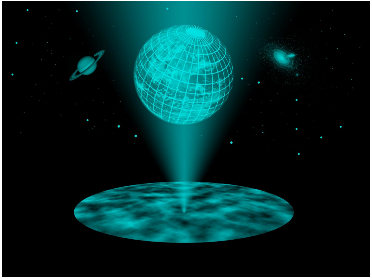
J. D. Bekenstein, Phys. Rev. D **49**, 1912 (1994)

The meaning of the bound

- The maximal entropy determines the number of degrees of freedom of a theory (dof)
- The Bekenstein bound implies that the number of degrees of freedom inside some region grows as the area of the boundary of a region and not like the volume of the region
- Is it more than just degrees of freedom ... ?

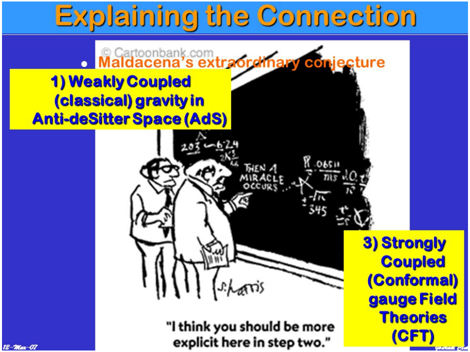
Holographic principle

In a quantum gravity theory physics within some volume can be described in terms of a theory on the boundary which has less than one degree of freedom per Planck area



G. 't Hooft, Conf. Proc. C **930308**, 284 (1993)
L. Susskind, J. Math. Phys. **36**, 6377 (1995)

AdS/CFT correspondence



$\mathcal{N} = 4$ $U(N)$ super-Yang-Mills theory in $3+1$ dimensions is dual to type IIB superstring theory on $AdS_5 \times S_5$

J. M. Maldacena, Int. J. Theor. Phys. **38**, 1113 (1999)

When is gravity classical?

- **Large N-limit**: number of black hole microstates is

$$N_{\rm BH} \sim \exp(M^2/M_{\rm Pl}^2)$$

where $M_{\rm Pl} \sim 10^{-5}$ g is the Planck Mass. For $M \sim 10^{33}$ g we have $N_{\rm BH} \sim 10^{10^{76}} \longrightarrow$ we need a large number of dof

- **Strong coupling**: classical gravity can emerge in the limit where gauge fields are are strongly quantum mechanical, and the gravitational degrees of freedom arise as effective classical fields

- **Supersymmetry**: provides stability properties at strong coupling because $H = Q^\dagger Q$ is bounded from below. It's a technical assumption

G. T. Horowitz, J. Polchinski, gr-qc/0602037

Some consequences

- Spacetime is an emergent concept: can be reconstructed from QFT's quantum information data

D. Harlow, arXiv:1802.01040 [hep-th]

- Operational definition of strongly coupled QFT: dynamics of Yang-Mills theories mapped to the black hole dynamics

O. DeWolfe, arXiv:1802.08267 [hep-th]

Non-conformal holographic systems

- Top-down construction

  $\mathcal{N}=4$ broken to $\mathcal{N}=2^*$ SUSY theory. Known, but complicated dual gravity description

  A. Buchel, S. Deakin, P. Kerner, J. T. Liu, Nucl. Phys. B **784**, 72 (2007)

- Bottom-up construction

  Assuming AdS/CFT dictionary, try to model gravity+matter background to approach as closely as possible to your favourite physics

  U. Gürsoy, *et.al.* JHEP **0905**, 033 (2009)
  S. S. Gubser, A. Nellore, Phys. Rev. D **78** (2008) 086007

A perspective on QFT

- Lattice methods do not reach real time dynamics easily
- Use a string theory based approach to formulate models at strong coupling
- Compute the spectrum of linearized perturbations
- Compute transport coefficients and non-hydrodynamic modes
- Compute the non-linear time evolution

SYM vs. QCD plasma

- **Similarities**
  → deconfined phase
  → strongly coupled
  → no SUSY at finite T
  → at weak coupling similar to pQCD plasma

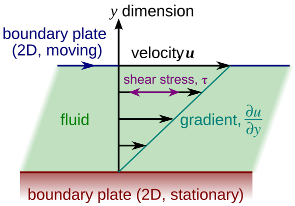
A. Czajka, S. Mrówczyński, Phys. Rev. D **86**, 025017 (2012)

- **Differences**
  → no running coupling
  → no confinement-deconfinement phase transition
  → exactly conformal EoS

- **Perspective**
  first-principles calculation of real time dynamics in a specific strongly coupled gauge theory

- Shear viscosity

$$\frac{F}{A} = \eta \frac{\partial u}{\partial y}$$



- Kinetic theory $\eta \sim n\langle p\rangle l_{\mathrm{mfp}}$ and $s \sim k_{\mathrm{B}} n$

$$\frac{\eta}{s} \simeq \frac{\langle p\rangle l_{\mathrm{mfp}}}{k_{\mathrm{B}}} \geq \frac{\hbar}{k_{\mathrm{B}}}$$

- Weakly coupled QFT e.g. $\lambda\phi^4$ theory

$$\frac{\eta}{s} \simeq \frac{1}{\lambda^2} \frac{\hbar}{k_{\mathrm{B}}} \gg \frac{\hbar}{k_{\mathrm{B}}}$$

Holographic computation of viscosity

- For systems dual to a black hole

$$\eta = \frac{\sigma_{\mathrm{abs}}(\omega = 0)}{16\pi G_5} = \frac{\mathrm{Area}}{16\pi G_5}$$

which results in

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_{\mathrm{B}}} \approx 6.08 \quad \mathrm{Ks}$$

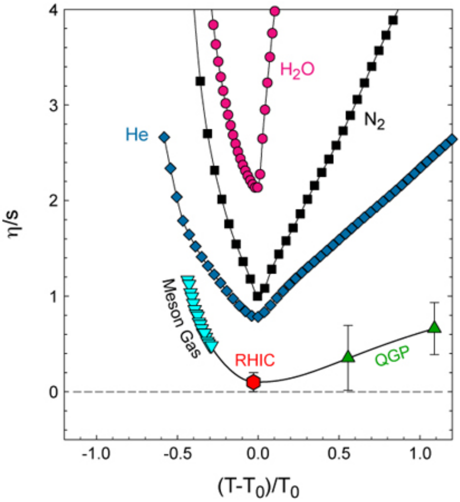
- *Viscosity is a consequence of a universal absorption by the event horizon*
- The result still holds at non-linear level
- Possible bound?

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_{\mathrm{B}}}$$

P. Kovtun, D. T. Son, A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005)

R. A. Janik, Phys. Rev. Lett. **98**, 022302 (2007)

Experimental data



R. A. Lacey *et al.*, Phys. Rev. Lett. **98** (2007) 092301

Experimental data

- Water

$$\frac{\eta}{s} \simeq 380 \frac{\hbar}{4\pi k_{\mathrm{B}}}$$

- Liquid helium He^4 at $P = 0.5$ MPa and $T = 5$ K

$$\frac{\eta}{s} \simeq 9 \frac{\hbar}{4\pi k_{\mathrm{B}}}$$

- QGP at the LHC Pb-Pb @ $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV

$$\frac{\eta}{s} \simeq 2 \frac{\hbar}{4\pi k_{\mathrm{B}}}$$

M. Alqahtani *et al.* Phys. Rev. Lett. **119**, no. 4, 042301 (2017)

Let's go simple: Bjorken flow

- Boost invariant metric $ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$
- Energy momentum tensor is diagonal

$$T_{\mu\nu} = \mathrm{diag}\{\epsilon(\tau), P_L(\tau), P_T(\tau), P_T(\tau)\}$$

- Conditions: $\nabla_\mu T^{\mu\nu} = 0$ and $T^\mu_\mu = 0$ imply

$$P_L = -\epsilon - \tau\dot{\epsilon}\ , \quad P_T = \epsilon + \frac{1}{2}\tau\dot{\epsilon}$$

- Evolution of the system is captured by a single function $\epsilon(\tau)$
- Strict for an infinite energy collision of infinitely large nuclei

J. D. Bjorken, Phys. Rev. D **27**, 140 (1983)

Gravity dual of Bjorken flow

- Using gauge gravity duality one can construct the dual geometry
- For the simplest case the $d = 5$ geometry is determined by

$$R_{ab} + 4g_{ab} = 0$$

- Can be embedded into $d = 10$ SUGRA
- Detailed studies of dynamics of SYM plasma

M. P. Heller, R. A. Janik, P. Witaszczyk, Phys. Rev. Lett. **108**, 201602 (2012)

J. J, G. Plewa and M. Spaliński, JHEP **1412**, 105 (2014)

R. A. Janik, Lect. Notes Phys. **828**, 147 (2011)

Entropy beyond the equilibrium

- Non-equilibrium states $\longleftrightarrow$ geometries with *horizons*

- Out-of-equilibrium entropy is defined by

$$S = \frac{a_{\mathrm{AH}}}{\pi}$$

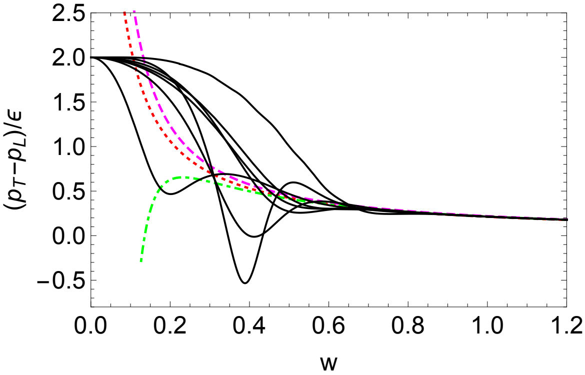
  where a_{AH} - is the apparent horizon area

- S is non-decreasing and agrees with hydrodynamic entropy for late times

Booth, I *et al.* Phys.Rev. D80 (2009) 126013

Thermalization $\neq$ Hydrodynamization

$$\Delta p := \frac{p_L - p_T}{\epsilon} = 1 - 3\frac{p_L}{\epsilon} \sim 0.7$$



J. J, G. Plewa and M. Spaliński, JHEP **1412**, 105 (2014)

High order transport coefficients

- At large proper times energy density admits an asymptotic gradient expansion

$$\epsilon_{\mathrm{hydro}}(\tau) \sim \frac{\Lambda}{(\Lambda\tau)^{4/3}} \sum_{n=0}^{\infty} \epsilon_n^{(0)} (\Lambda\tau)^{-2n/3}$$

- Coefficients $\epsilon_n^{(0)} \sim \Gamma(n+\beta) A^{-n-\beta}$ for $n \gg 1$
- $A \sim \omega$ is the frequency of the transient mode
- $\mathrm{Im}\ \omega \sim \tau_0^{-1}$ where τ_0 is the equilibration time
- At RHIC and LHC $\tau_0 \sim 0.5 - 1\ \mathrm{fm/c}$
- All information is stored in the transport coefficients: resurgence property

M. P. Heller *et al.* Phys. Rev. Lett. **110**, no. 21, 211602 (2013)

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Transient modes

- Exponentially damped modes

$$\epsilon(\tau) \sim \epsilon_{\mathrm{hydro}}(\tau) + \frac{\Lambda}{(\Lambda\tau)^{4/3}} \sum_{n=0}^{\infty} \epsilon_n^{(1)} (\Lambda\tau)^{-2n/3} e^{-A(\Lambda\tau)^{2/3}}$$

- Series $\epsilon_n^{(1)} \sim \Gamma(n)$ is divergent and can be obtained from $\epsilon_n^{(0)}$
- $A \sim \omega$ is determined by the black hole quasinormal mode frequency
- First, strong evidence for resurgence in strongly coupled QFT

M. P. Heller *et al.* Phys. Rev. Lett. **110**, no. 21, 211602 (2013)

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Borel transform

- The Borel transform is defined

$$\mathcal{B}\left[\epsilon_{\text{hydro}}\right](\xi) = \xi^{\beta-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{\epsilon_n^{(0)}}{\Gamma\left(n+\beta+\frac{1}{2}\right)} \xi^n$$

- Has finite radius of convergence
- First singularities appear at $\xi = A$ and $\xi = \bar{A}$
- Singularity encodes the coefficients $\epsilon_n^{(1)}$
- Analyze numerically by Borel-Padé approximant

I. Aniceto, G. Basar and R. Schiappa, arXiv:1802.10441 [hep-th]

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Large order relations

- For the hydrodynamic series at the leading singularity

$$\varepsilon_n^{(0)} \sim -\frac{S_{0\to 1}}{2\pi \mathrm{i}} \frac{\Gamma(n+\beta)}{A_1^{n+\beta}} \left(\varepsilon_0^{(1)} + \frac{A_1 \varepsilon^{(1)}}{n+\beta-1} + \right.$$

$$\left. + \frac{A_1^2 \varepsilon_2^{(1)}}{(n+\beta-1)(n+\beta-2)} + \cdots \right) + \mathrm{c.c.} + \cdots$$

where $S_{0\to 1}$ is the Stokes constant $(\beta = \beta_{\mathbf{0}} - \beta_1)$

- Large order relations contain contributions from *all* sectors and couplings between them

- Every sector has an independent Stokes constant $S_{0\to e_k}$

I. Aniceto, G. Basar and R. Schiappa, arXiv:1802.10441 [hep-th]

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Numerical analysis and Borel-Pade approximant

- Provided limited number of coefficients one uses Borel-Pade approximant to analytically continue the Borel transform

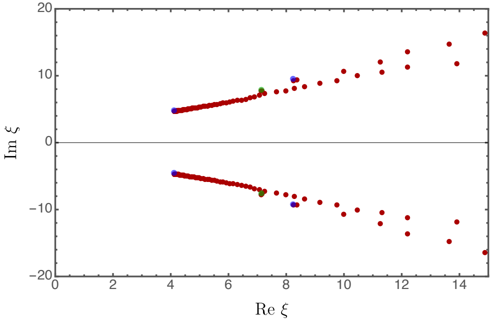
$$\mathrm{BP}_N\left[\Phi\right](s) = \frac{P_N(s)}{Q_N(s)}$$

  where $P_N(s)$ and $Q_N(s)$ are polynomials

- Having coefficients in different sectors, and assuming resurgence, one can obtain and check the large order relations

- In consequence the above procedure determines the Stokes constants

Borel plane

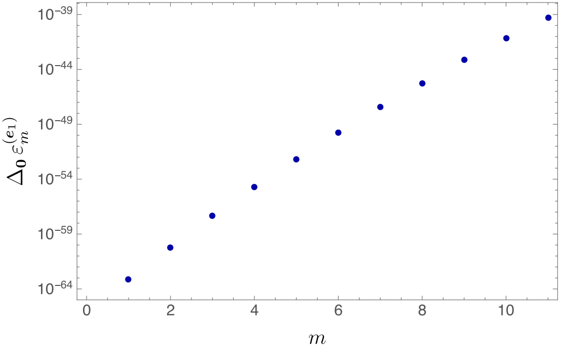


Poles of the Borel-Padé approximant $\mathrm{BP}_{189}\left[\epsilon_{\text{hydro}}\right]$, in the complex ξ−plane $\xi = A_1, \overline{A_1}, 2A_1, 2\overline{A_1}$ $\quad \xi = A_2, \overline{A_2}$

$$S_{0\to \boldsymbol{e_1}} = 0.01113\cdots - i0.03050\cdots$$

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Numerical check of resurgence



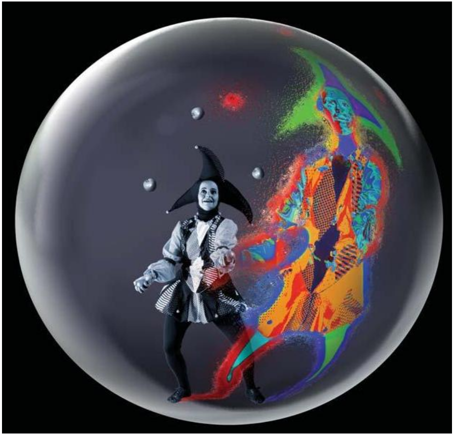
$$\Delta_{\boldsymbol{n}}\varepsilon_k^{(\boldsymbol{m})} \equiv \frac{\varepsilon_k^{(\boldsymbol{m})}\big|_{\boldsymbol{n}-\text{predicted}} - \varepsilon_k^{(\boldsymbol{m})}\big|_{\text{numerical}}}{\varepsilon_k^{(\boldsymbol{m})}\big|_{\text{numerical}}}, \quad k \geq 1,$$

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

What can we learn?

- Strongly coupled QFTs are understood in terms of classical General Relativity
- Nature of the hydrodynamic expansion and the meaning of hydrodynamic approximation
- General principles supported with holographic methods provide new insights into physics
- Holography provides semi-quantitative results so far ...

Thank you!



J. M. Maldacena, Spektrum Wiss. **2006N3**, 36 (2006)