بسم الله الرحمن الرحیم

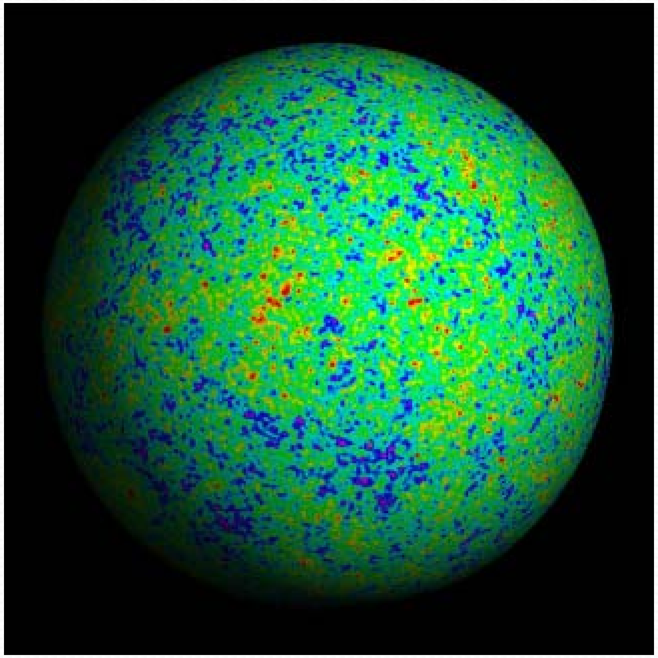
DBI Lifshitz Inflation

M.Alishahiha, H.Firouzjahi, M.H.Namjoo

JCAP 1108, 028 (2011)
[arXiv:1103.2919 [hep-th]]

Outline

- Inflation: A review
- DBI Inflation
- DBI Lifshitz-inflation
- The results



Dynamics of Inflation

Friedmann equations

$$\begin{cases} H^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi}{m_{P\ell}^2}\rho - \frac{K}{a^2} \\ \frac{\ddot{a}}{a} = -\frac{4\pi}{3m_{P\ell}^2}(\rho + 3P) \end{cases}$$

$\longrightarrow$ Accelerating the universe needs extraordinary content

FRW metric:

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= -dt^2 + a^2\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \end{aligned}$$

- Matter content of universe at inflationary phase is inflaton, a scalar field

- Accelerating is possible:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad , \quad P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

conditions for inflation:

- Slow-Roll conditions:

$$\epsilon = \frac{m_{\rm pl}^2}{16\pi}\left(\frac{V_\phi}{V}\right)^2 \ , \ \eta = \frac{m_{\rm pl}^2 V_{\phi\phi}}{8\pi V}$$

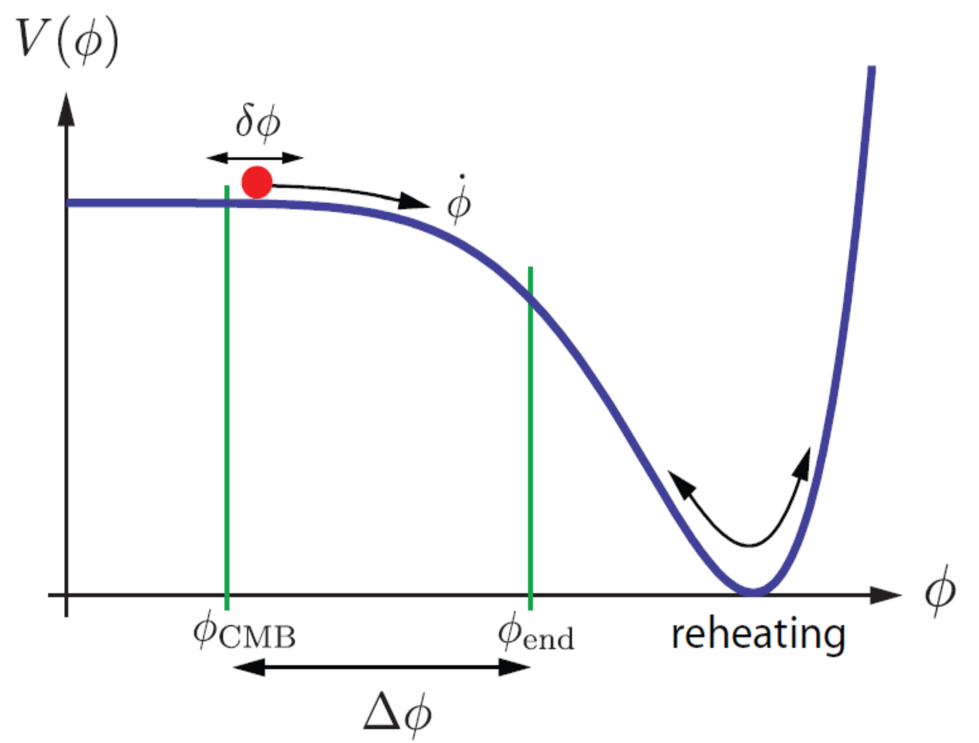
$$\epsilon \ll 1 \text{ and } |\eta| \ll 1$$

- In order to inflation occurs and lasts sufficiently long time.

- Minimum number of efolds required:

$$\frac{a_f}{a_i} \equiv e^N$$

$$N > 60$$



B.A.Bassett, S.Tsujikawa and D.Wands, ``Inflation dynamics and reheating,'' Rev. Mod. Phys. 78, 537 (2006) [astro-ph/0507632].

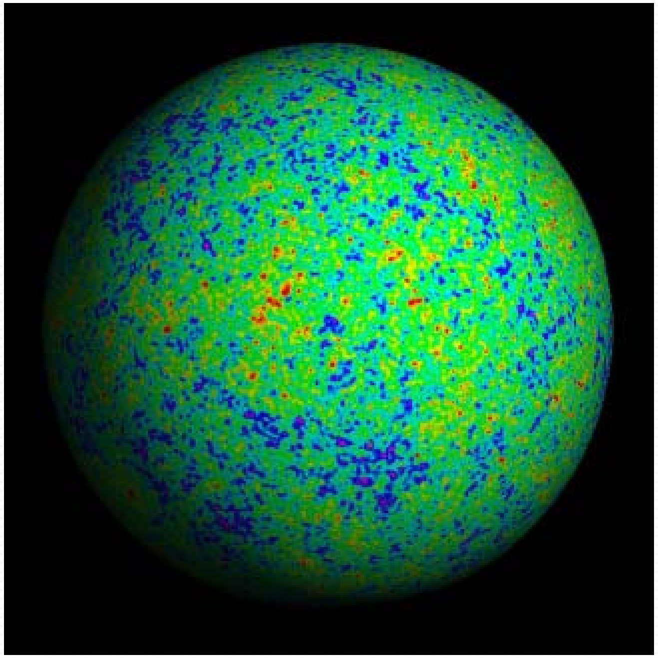
How can we model the CMB data?

Perturbation theory in cosmology

- Perturbed Einstein Equation

$$G^{\alpha}_{\beta} \equiv R^{\alpha}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta} R - \Lambda \delta^{\alpha}_{\beta} = 8\pi G T^{\alpha}_{\beta}$$

$$\phi = \phi_0(t) + \delta\phi(t, x)$$



Curvature perturbation: $\mathcal{R} \equiv \psi - \frac{H}{\rho + P}\delta q$, $\mathcal{R} \equiv \psi + \frac{H}{\dot{\phi}}\delta\phi$

Power spectrum and CMB

Power spectrum of curvature perturbation:

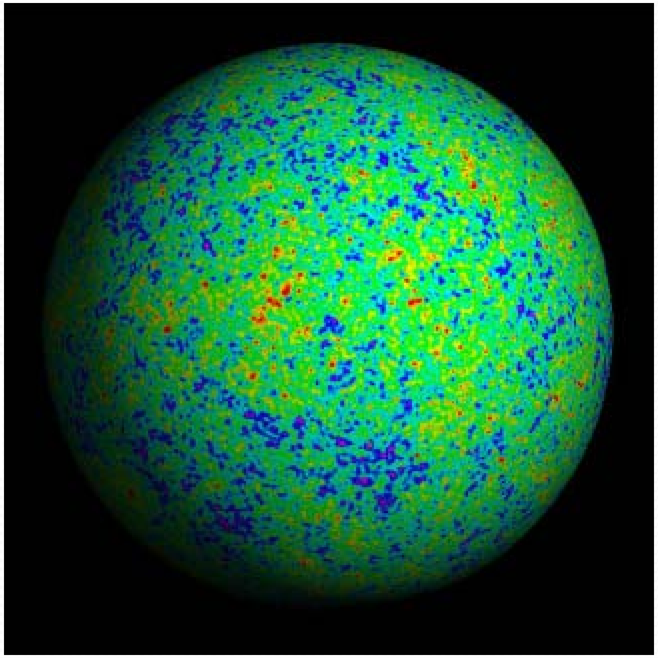
$$< \mathcal{R}_{k_1}\mathcal{R}_{k_2} > \longrightarrow \mathcal{P}_{\mathcal{R}} \equiv \frac{4\pi k^3}{(2\pi)^3}|\mathcal{R}^2|$$

Spectral index:

$$n_{\mathcal{R}} - 1 \equiv \left.\frac{d\ln \mathcal{P}_{\mathcal{R}}}{d\ln k}\right|_{k=aH}$$

Non-Gaussianity:

$$< \mathcal{R}_{k_1}\mathcal{R}_{k_2}\mathcal{R}_{k_3} >= -6/5 f_{NL}(< \mathcal{R}_{k_1}\mathcal{R}_{k_2} > +2perm.)$$



COBE normalization: $\mathcal{P}_R \simeq 10^{-10}$

From WMAP data: $|f_{NL}| \lesssim 100$

Planck will have more to say about non-Gaussianity

Brane Inflation

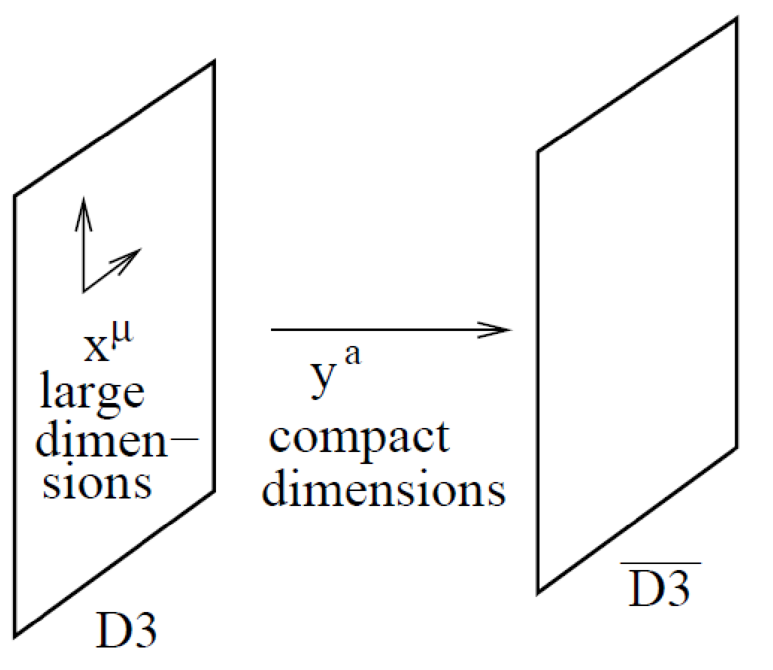
- Geometrical interpretation of inflaton field:

  The distance between brane-anti branes

A natural reheating:
Brane anti-brane annihilation

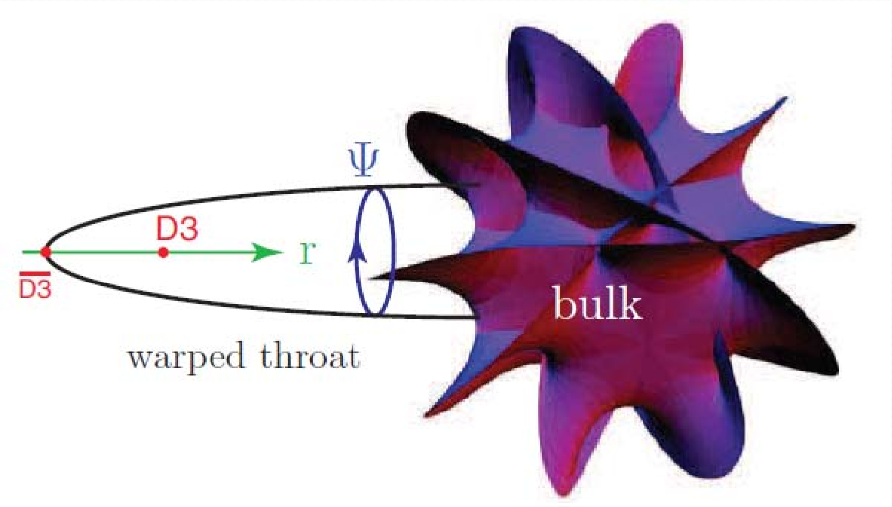
The Eta problem

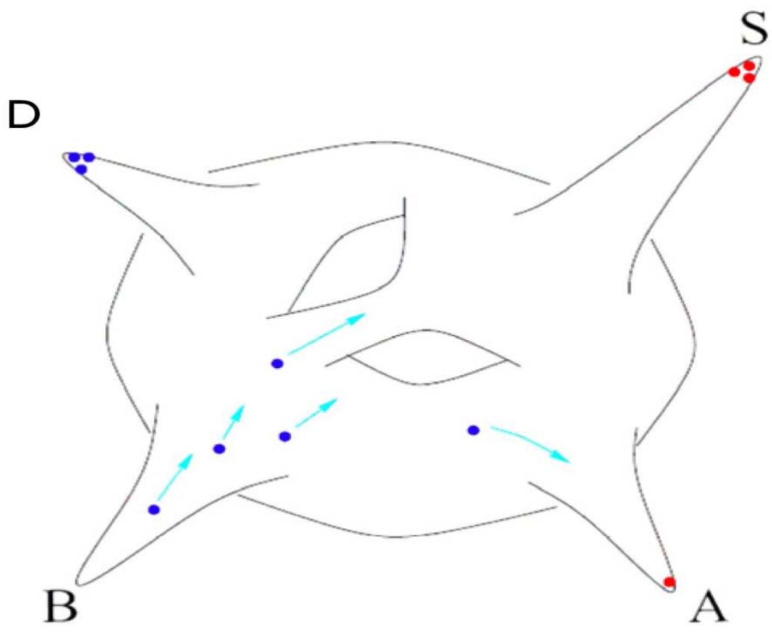
$$\eta = -\frac{10}{\pi^3}(L/r)^6$$



Brane Inflation

A solution for the problem:
Warped geometry





DBI Inflation

- The action in Ads background:

M.Alishahiha, E.Silverstein and D.Tong, ``DBI in the sky,'' Phys. Rev.\D 70, 123505 (2004) [hep-th/0404084].

Canonically normalized field: $\phi = \sqrt{T_3} r$

$$S_{DBI} = \int dx^4 \sqrt{-g} f(\phi)^{-1} \left(\sqrt{1 + f(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} - 1 \right)$$

$$\mathrm{d}S^2 = -\left(\frac{r}{L}\right)^2 \mathrm{d}t^2 + \left(\frac{r}{L}\right)^2 \mathrm{d}\mathbf{x}^2 + \left(\frac{L}{r}\right)^2 \mathrm{d}r^2$$

Warp factor: $f(\phi) = \frac{\lambda}{\phi^4}$

- Non standard kinetic term and "Lorentz factor": $\gamma \equiv \frac{1}{\sqrt{1 - f\dot{\phi}^2}}$

DBI limit: $\gamma \gg 1$

$$|f_{NL}| \simeq 0.32\gamma^2$$

$$n_s - 1 = 0$$

Lifshitz Symmetry
Lifshitz Background

- Conformal symmetry: $t \to \lambda t, \qquad x_i \to \lambda x_i$
- Lifshitz symmetry: $t \to \lambda^z t, \qquad x_i \to \lambda x_i$

- Lifshitz background(?):

$$ds^2 = -\left(\frac{r}{L}\right)^{2z} \mathrm{d}t^2 + \left(\frac{r}{L}\right)^2 \mathrm{d}\mathbf{x}^2 + \left(\frac{L}{r}\right)^2 \mathrm{d}r^2$$

For DBI in Ads background: z=1

The reduction

$$S_G = \kappa^{-2} \int d^5x \sqrt{-G}\ {}^{(5)}R$$

$$ds^2 = g_{00}\left(\frac{r}{L}\right)^{2z} \mathrm{d}t^2 + g_{ij}\left(\frac{r}{L}\right)^{2} \mathrm{d}x^i \mathrm{d}x^j + 2g_{0i}\left(\frac{r}{L}\right)^{z+1} \mathrm{d}t\, \mathrm{d}x^i + \left(\frac{L}{r}\right)^2 \mathrm{d}r^2$$

DBI-Lifshitz: The model

$$\mathcal{L} = -f^{-1}\left[\left(1 + f\, g^{00}\dot{\phi}^2 + h g^{ij}\partial_i\phi\partial_j\phi + 2l g^{0i}\dot{\phi}\partial_i\phi\right)^{1/2} - 1\right] - V$$

$$f(\phi) \equiv T_3^{-1}\left(\frac{\mu_z}{\phi}\right)^{\alpha} \quad , \quad h(\phi) \equiv T_3^{-1}\left(\frac{\mu_z}{\phi}\right)^{\alpha'} \qquad z \neq 3$$

$$f(\phi) \equiv T_3^{-1}e^{-\frac{6\phi}{\mu_3}} \quad , \quad h(\phi) \equiv T_3^{-1}e^{-\frac{2\phi}{\mu_3}} \quad , \quad z = 3$$

$$\alpha \equiv \frac{2(3+z)}{3-z} \quad , \quad \alpha' \equiv \frac{2(5-z)}{3-z}$$

- Canonically normalized field:

$$\phi = \begin{cases} \mu_z\left(\frac{r}{L}\right)^{\frac{3-z}{2}} & \text{if } z \neq 3 \\ \mu_3 \ln\left(\frac{r}{L}\right) & \text{if } z = 3 \end{cases}$$

$$\mu_3 \equiv \sqrt{T_3}L \quad , \quad \mu_z \equiv \frac{2\mu_3}{3-z} = \frac{2\sqrt{T_3}}{3-z}L$$

Z=3 is special

DBI-Lifshitz: The model

- The potential:

$$V \sim r^n$$

$$V(\phi) = \begin{cases} V_0 \left(\frac{\phi}{\mu_z}\right)^p & \text{if } z \neq 3 \\ V_0\, e^{\frac{n\phi}{\mu_3}} & \text{if } z = 3 \end{cases}$$

$$p \equiv \frac{2n}{3-z}$$

- The symmetry and degrees of freedom:

$$t \to \tilde{t}(t) \quad , \quad \mathbf{x} \to \tilde{\mathbf{x}}(\mathbf{x})$$

We have five physical degrees of freedom

The speeds

- We have three distinct speeds:

- Both gravity and matter contribute to the sound speed

- Superluminal propagation

- Decoupling of scalar field perturbations

$$(c_{gr}^2 - c_\gamma^2(\phi))\nabla^2\delta\phi = 0$$

$$c_\gamma^2 = \left(\frac{r_O}{L}\right)^{2(z-1)}$$

$$c_{gr}^2 = 1 + \beta_0$$

$$c_s^2 = \frac{1+\beta_0}{\gamma^2}$$

$$\beta_0 \equiv -1 + \frac{\int_V dr\, \left(\frac{r}{L}\right)^z}{\int_V dr\, \left(\frac{r}{L}\right)^{2-z}}$$

The results of the model

- Number of efolds:

$$N \simeq \sqrt{\frac{4V_0\mu_3^2}{3M_P^2T_3}}\frac{1}{n-2z}\left(\frac{r}{L}\right)^{\frac{n-2z}{2}}\Big|_{r_f}^{r_i} \qquad \left(z \neq \frac{n}{2}\right)$$

$$N \simeq \sqrt{\frac{4V_0\mu_3^2}{3M_P^2T_3}}\ln\left(\frac{r_i}{r_f}\right) \qquad (z = 2n)$$

- Power spectrum:

$$\mathcal{P}_{\mathcal{R}} \simeq (1+\beta_0)^{-3/2}\left(\frac{H^2}{2\pi\dot{\phi}}\right)^2$$

- Spectral index: $n_s - 1 \simeq \frac{2\epsilon_D}{n}(3+z-2n)$

Conclusion and outlook

- DBI inflation in Lifshitz background
- Different speeds for different components. Superluminal propagation is possible and natural. The scalar field decouples.
- The model satisfies the observational bounds.
- The non-Gaussianity and the exact relation with CMB are open interesting questions

متشکرم

Thank you