SUPERSYMMETRY AND MECHANISMS OF BREAKING IT

Behnoosh Khavari
Institute for research in Fundamental Sciences

PLAN OF TALK

0-A review on QFT and SM of Particle physics

1-Why Supersymmetry?

2-How to contain Supersymmetry?

3- Why and how to break Supersymmetry?

4-A Simple model for SUSY breaking

SYMMETRY IN PHYSICS

- SYMMETRY ⟷ INVARIANCE
- CLASSICAL MECHANICS
- CLASSICAL ELECTRODYNAMICS

CLASSICAL PHYSICS

- $L = \frac{1}{2} m \dot{X}^2$
- $\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0$
- $m\dot{x} = const.$
- $x : cyclic\ variable$
- SYMMETRY $\Longleftrightarrow$ CONSERVED QUANTITY

ELECTRODYNAMICS

- $L = \frac{-1}{4} F_{\alpha\beta} F^{\alpha\beta} + J^{\alpha} A_{\alpha}$
- $A_{\alpha} \rightarrow A_{\alpha} + \partial_{\alpha} F(x)$

- $\partial_{\alpha} J^{\alpha} = 0$
- $Q = const.$

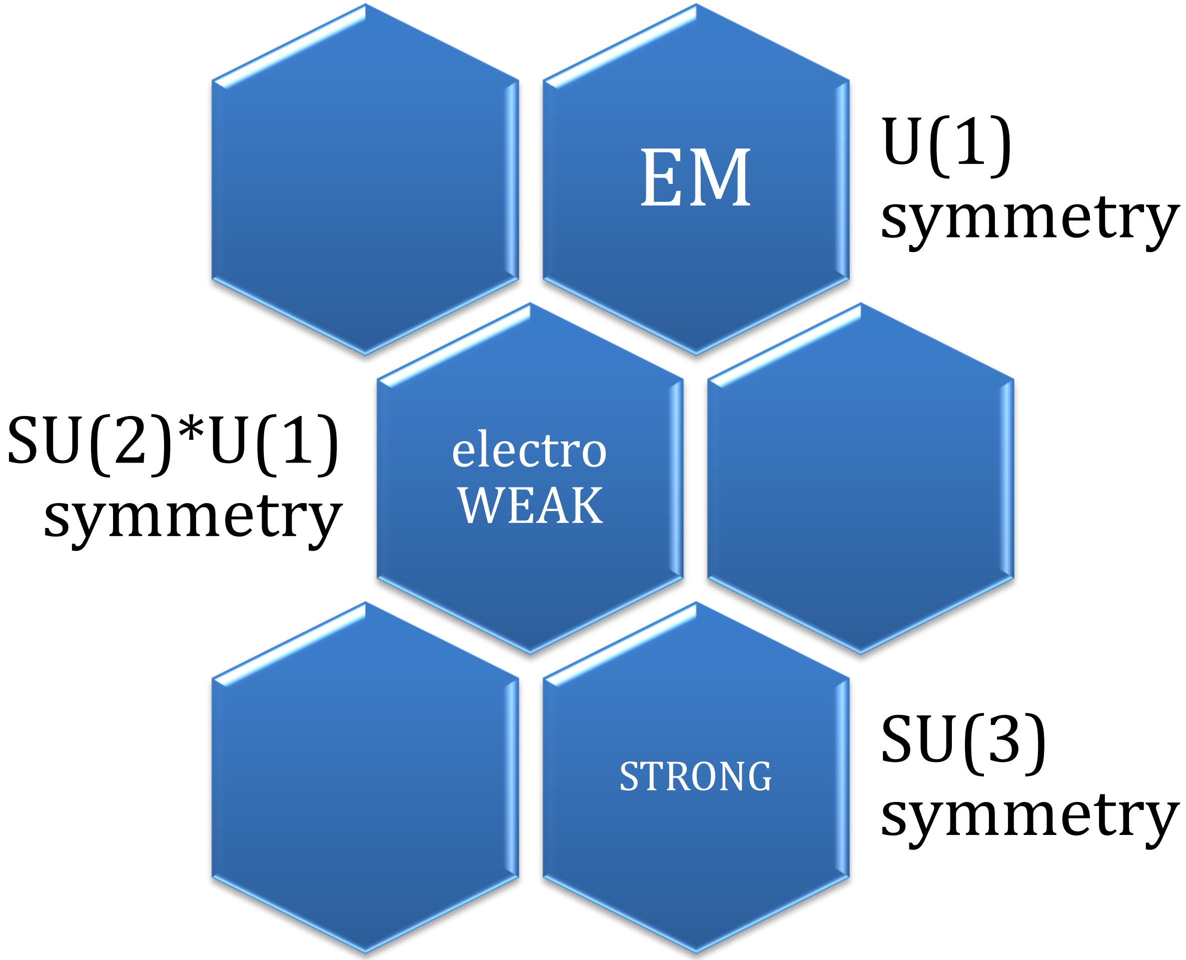
- SYMMETRY $\Longleftrightarrow$ CONSERVED QUANTITY

QFT AS THE FUNDAMENTAL THEORY

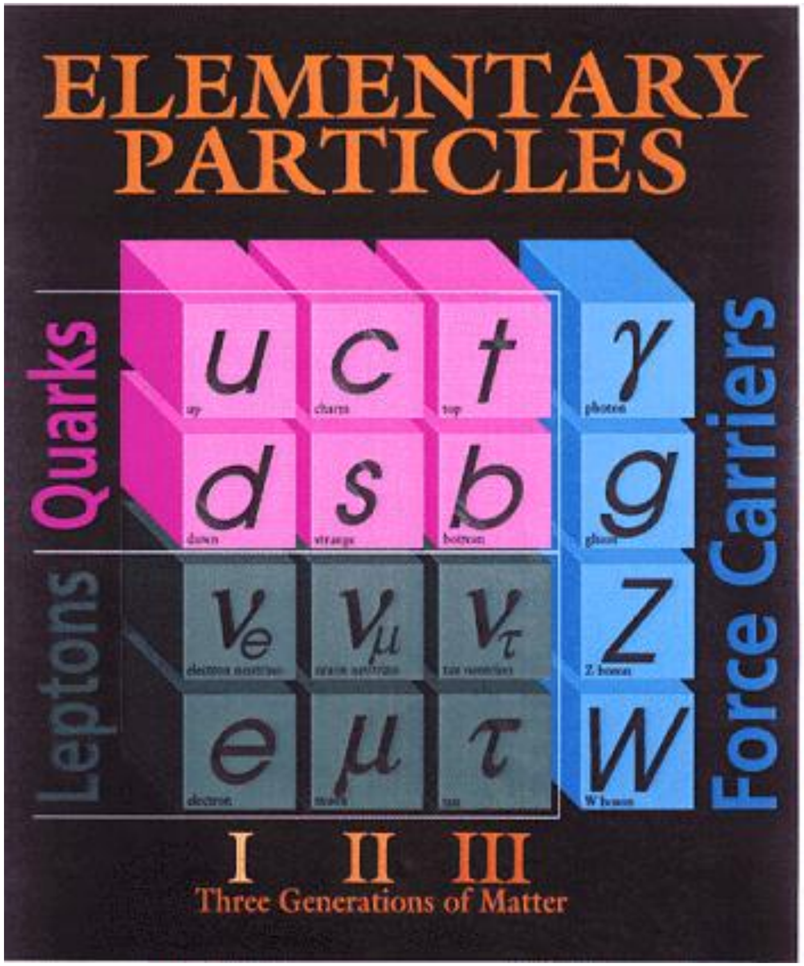
- All fundamental interactions are described through invariances.
- Lorentz invariance is necessary.

- Lagrangian of QFT has all these symmetries.

The standard model describes three fundamental Interactions for elementary particles :



OBSERVED ELEMENTARY PARTICLES UNTIL 2012



http://abyss.uoregon.edu/~js/ast123/lectures/lec07.html

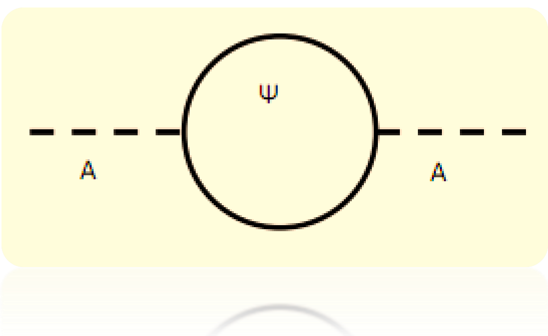
SALAM-WEINBERG MODEL

- PREDICTION : HIGGS BOSON

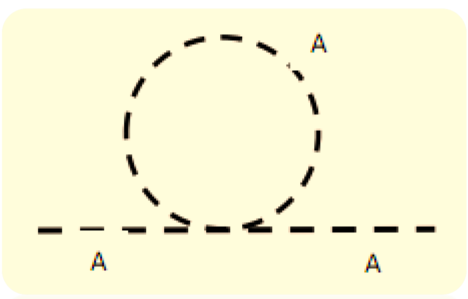
- ANY PROBLEM?
- YES, The gauge hierarchy problem.

Hierarchy problem of the Higgs mass

- Quantum corrections in the frame of SM take the mass of Higgs to Planck scale.



Yukawa coupling



H^4

Λ^2 divergence

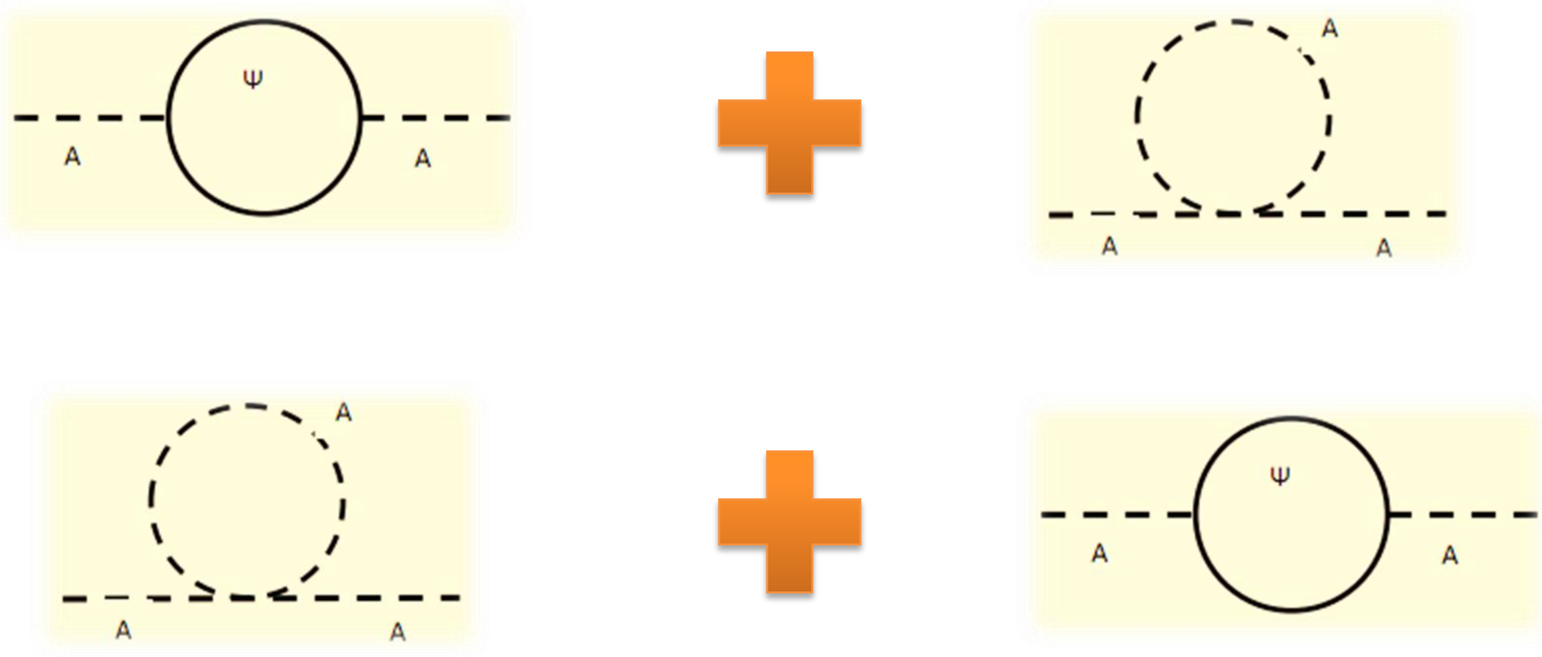
A GOOD MOTIVATION FOR SUSY

What does mean supersymmetry(SUSY)?

- (N=1) type : for each particle in nature there is another one with a difference in spin by one half :
- $Q|fermion\rangle = |boson\rangle$,
- $Q|boson\rangle = |fermion\rangle$.

How does SUSY solve Hierarchy problem?

- SUSY introduces new interactions due to new particles.



How to incorporate SUSY into QFT ?

A review on non-supersymmetric QFT

- Poincare symmetry (space-time and Lorentz transformation)
- Internal symmetry

$$G_{\mathrm{SM}} = \underbrace{SU(3)_C}_{\text{strong}} \otimes \underbrace{SU(2)_L \otimes U(1)}_{\text{electroweak}}$$

- CPT symmetry

Poincare algebra

- $[P^\mu, P^\nu] = 0$
- $[M^{\mu\nu}, P^\sigma] = i(P^\mu \eta^{\nu\sigma} - P^\nu \eta^{\mu\sigma})$
- $[M^{\mu\nu}, M^{\rho\sigma}]$

$$= i(M^{\mu\sigma}\eta^{\nu\rho} + M^{\nu\rho}\eta^{\mu\sigma} - M^{\mu\rho}\eta^{\nu\sigma} - M^{\nu\sigma}\eta^{\mu\rho})$$

Coleman-mandula theorem

- Limiting algebra to contain only commutation relations ⇒ Only QFT symmetries
- Loosing the constraint "only commutation relations" ⇒ expanding algebra by anti commutation relations

supercharge

- $Q_\alpha \begin{cases} operator\ (generator\ of\ SUSY) \\ representation\ of\ Lorentz\ group \end{cases}$
- $\{Q_\alpha, Q_\beta\} = 0$, $\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu$
- $[Q_\alpha, P^\mu] = 0$ $[Q_\alpha, M^{\mu\nu}] = (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$
- $[Q_\alpha, T] = 0$

Massless case (N=1)

- $Q_\alpha|p,\lambda\rangle_{initial} = 0$ $\quad\quad$ $|p,\lambda\rangle_{initial}=|\Omega\rangle$
- $|p,\lambda\rangle$, $\bar{Q}_{\dot{2}}|p,\lambda\rangle = \left|p,\lambda-\frac{1}{2}\right\rangle$

$\lambda = 0$ scalar	$\lambda = \frac{1}{2}$ fermion
squark	quark
slepton	lepton
Higgs	Higgsino

$\lambda = \frac{1}{2}$ fermion	$\lambda = 1$ boson
photino	photon
gluino	gluon
Wino, Zino	W, Z

$\lambda = \frac{3}{2}$ fermion	$\lambda = 2$ boson
gravitino	graviton

How to incorporate N=1 SUSY into QFT ?

- Generalization:

- Space → superspace : $x^{\mu} \rightarrow x^{\mu}\ ,\theta_{\alpha}\ ,\bar{\theta}_{\dot{\alpha}}$
- Field(x^{μ}) → superfield$(x^{\mu}\ ,\theta_{\alpha}\ ,\bar{\theta}_{\dot{\alpha}})$
- $\theta_{\alpha}\ ,\bar{\theta}_{\dot{\alpha}}$: fermionic coordinates

Chiral superfield

- $S(x^{\mu},\theta,\bar{\theta}) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\partial_{\mu}\varphi(x)\theta\sigma^{\mu}\bar{\theta} - \frac{i}{\sqrt{2}}\theta\theta\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta} - \frac{1}{4}\partial_{\mu}\partial^{\mu}\varphi(x)\theta\theta\bar{\theta}\bar{\theta}$

- It describes quarks and squarks or leptons and sleptons.

How to make a supersymmetric lagrangian ?

$$\delta\varphi = \sqrt{2}\xi\psi$$

$$\delta\psi = \sqrt{2}\xi F - \sqrt{2}\partial_\mu\varphi\sigma^\mu\bar{\xi}$$

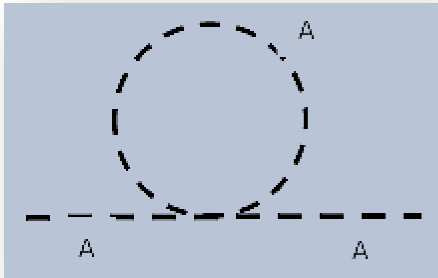
$$\delta F = i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\xi}$$

- $\mathcal{L} = (\phi^\dagger\phi)\big|_D + (W(\Phi)|_F + H.C)$
- $+\left(\frac{1}{4}(W^\alpha W_\alpha)|_F + H.C.\right)$

A chiral model : Wess-Zumino

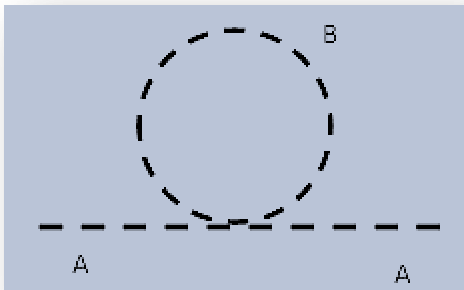
$$W = \frac{1}{2} m\Phi^2 + \frac{1}{3} g\Phi^3$$

- $\mathcal{L} = \frac{1}{2}\partial_\mu A \partial^\mu A - \frac{1}{2}m^2 A^2 + \frac{1}{2}\partial_\mu B \partial^\mu B - \frac{1}{2}m^2 B^2$
- $+\frac{1}{2}\overline{\Psi}(i\eth - m)\Psi - \frac{mg}{\sqrt{2}}A(A^2 + B^2)$
- $-\frac{g^2}{4}(A^4 + B^4 + 2A^2B^2) - \frac{g}{\sqrt{2}}\overline{\Psi}(A - iB\Upsilon^5)\Psi$



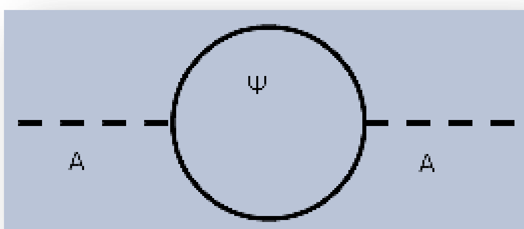
$$= -i\frac{g^2}{4}4\,.\,3\int\frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}$$

$$= 3g^2\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2}$$



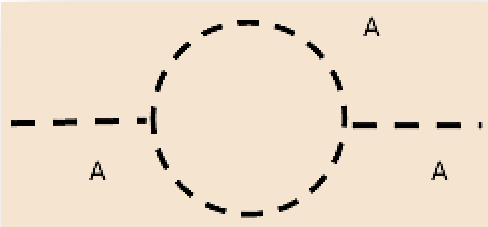
$$= -i\frac{g^2}{2}\;2\int\frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}$$

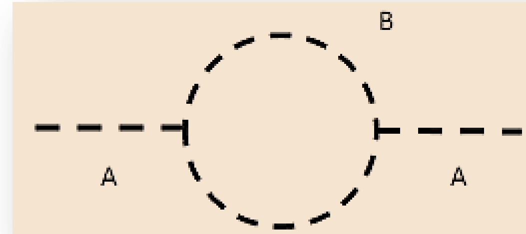
$$= g^2\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2}$$



$$= -2g^2\Big(\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2}+\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k-p)^2-m^2}$$

$$+\int\frac{d^4k}{(2\pi)^4}\,\frac{4m^2-p^2}{(k^2-m^2)\,((k-p)^2-m^2)}\Big)$$





$log\Lambda$ $log\Lambda$

There remains no Λ^2 divergence.

Take a look at nature !

Selectron, Squark have not been observed.

SUSY must be broken : ~~SUSY~~

SUPERSYMMETRY BREAKING

HOW TO BREAK SUSY?

Spontaneous SUSY breaking :

$\mathcal{L}$ is supersymmetric, vacuum is not.

$Q_\alpha|0\rangle \neq 0$

F-term SUSY breaking

Chiral superfield transformation:

$$\delta\varphi = \sqrt{2}\xi\psi$$

$$\delta\psi = \sqrt{2}\xi F - \sqrt{2}\partial_\mu\varphi\sigma^\mu\bar{\xi}$$

$$\delta F = i\sqrt{2}\partial_\mu\psi\sigma^\mu\bar{\xi}$$

Only $\langle\delta\psi\rangle \neq 0$ respects Lorentz invariance.

$$\langle\delta\psi\rangle \propto \langle F\rangle \neq 0$$

Scalar potential

$$V = V_F(\varphi) = \left(\frac{\partial W}{\partial \varphi}\right)^2$$

$$F_i{}^{\dagger} = -\frac{\partial W(\varphi)}{\partial \varphi_i}$$

Energy criterion

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$

$$H = P^0 = \frac{1}{4}(Q_1\bar{Q}_{\dot{1}} + \bar{Q}_{\dot{1}}Q_1 + Q_2\bar{Q}_{\dot{2}} + \bar{Q}_{\dot{2}}Q_2)$$

$$\langle \boldsymbol{H} \rangle = \frac{1}{4}\langle \boldsymbol{0} \big| \boldsymbol{Q}_1\bar{\boldsymbol{Q}}_{\dot{1}} + \bar{\boldsymbol{Q}}_{\dot{1}}\boldsymbol{Q}_1 + \boldsymbol{Q}_2\bar{\boldsymbol{Q}}_{\dot{2}} + \bar{\boldsymbol{Q}}_{\dot{2}}\boldsymbol{Q}_2 \big| \boldsymbol{0} \rangle$$

~~SUSY~~ $\boldsymbol{E_{vac} = 0}$

SUSY $\boldsymbol{E_{vac} > 0}$

A prototype model for ~~susy~~

$$W(\Phi_1, \Phi_2, \Phi_3) = \mu\Phi_2\Phi_3 + \lambda_1\Phi_1\left({\Phi_3}^2 - M^2\right)$$

$$-{F_1}^\dagger = \frac{\partial W}{\partial \varphi_1} = \lambda_1({\varphi_3}^2 - M^2)$$

$$-{F_2}^\dagger = \frac{\partial W}{\partial \varphi_2} = \mu\varphi_3$$

$$-{F_3}^\dagger = \frac{\partial W}{\partial \varphi_3} = \mu\varphi_2 + 2\lambda_1\varphi_1\varphi_3$$

$$V = \sum_{i=1}^{3} |F_i|^2 = {\lambda_1}^2 |{\varphi_3}^2 - M^2|^2 + \mu^2 \, |\varphi_3|^2$$

$$+|\mu\varphi_2 + 2\lambda_1\varphi_1\varphi_3|^2$$

$$m_0{}^2 = \begin{pmatrix} \bar{W}^{ac}W_{cb} & \bar{W}^{abc}W_c \\ W_{abc}\bar{W}^{c} & W_{ac}\bar{W}^{cb} \end{pmatrix}$$

$$m_{\frac{1}{2}}{}^2 = \begin{pmatrix} \bar{W}^{ac}W_{cb} & 0 \\ 0 & W_{ac}\bar{W}^{cb} \end{pmatrix}$$

$$m_0{}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |\mu|^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & |\mu|^2 & 0 & 0 & -2M^2|\lambda_1|^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & |\mu|^2 & 0 \\ 0 & 0 & -2M^2|\lambda_1|^2 & 0 & 0 & |\mu|^2 \end{pmatrix}$$

$$I_1 = I_2 = 0 \qquad I_5 = |\mu|^2 - 2M^2|\lambda_1|^2$$

$$I_3 = I_4 = |\mu|^2 \qquad I_6 = |\mu|^2 + 2M^2|\lambda_1|^2$$

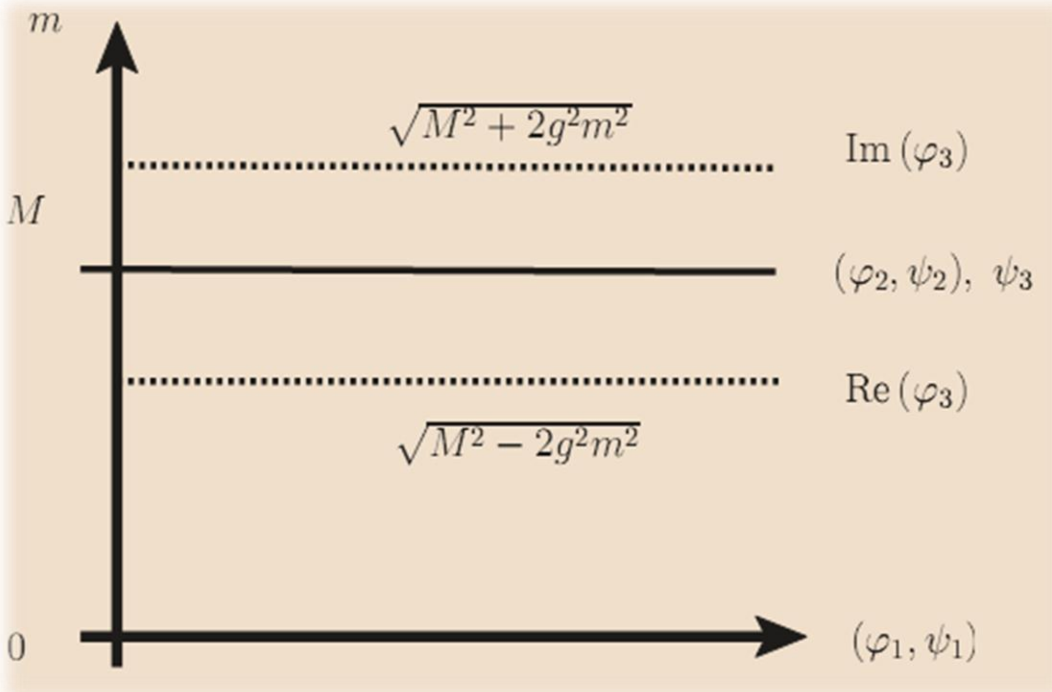
$$m_{\frac{1}{2}}{}^2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & |\mu|^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & |\mu|^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & |\mu|^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & |\mu|^2 \end{pmatrix}$$

$$I_1 = I_2 = 0$$

$$I_3 = I_4 = I_5 = I_6 = |\mu|^2$$

ψ_1 is the Goldstino

$$\varphi_3 = \frac{a_3 + ib_3}{\sqrt{2}} \qquad m_{a_3}{}^2 = \mu^2 - 2\lambda_1{}^2 M^2 \qquad m_{b_3}{}^2 = \mu^2 + 2\lambda_1{}^2 M^2$$



Cambridge Lectures on Supersymmetry and Extra Dimensions,
Lectures by: Fernando Quevedo, Notes by: Sven Krippendorf, Oliver Schlotterer

SUSY breaking is apparent.

Some details of SUSY breaking

A generic criterion for ~~susy~~

$W = \mathrm{W}(\Phi_i)$

$\partial_a \mathrm{W}(\Phi_i) = 0, \qquad a = 1, \ldots n\,, i = 1, \ldots, n$

Generically there is a solution.

Let's invoke to R-symmetry

$[Q_\alpha, R] = Q_\alpha$

$\rightarrow Q_\alpha R|fermion\rangle - RQ_\alpha|fermion\rangle = Q_\alpha|fermion\rangle$
$\rightarrow R_F|boson\rangle - R_B|boson\rangle = |boson\rangle \rightarrow R_F = R_B + 1$
$R(\theta) = -1$
$\rightarrow R(W) = 2 \rightarrow \mathrm{W} = \Phi_1{}^{\frac{2}{\mathrm{r}_1}} \mathrm{f}\left(t^a = \Phi^a \Phi_1{}^{-\frac{\mathrm{r}_a}{\mathrm{r}_1}}\right)$

$n + 1\ equations\ , n\ unknowns$

Generically there is no solution.

Now consider some models!

K.Intriligator and N. Seiberg,Lectures on supersymmetry Breaking
arXiv:hep-ph/0702069v3

First example

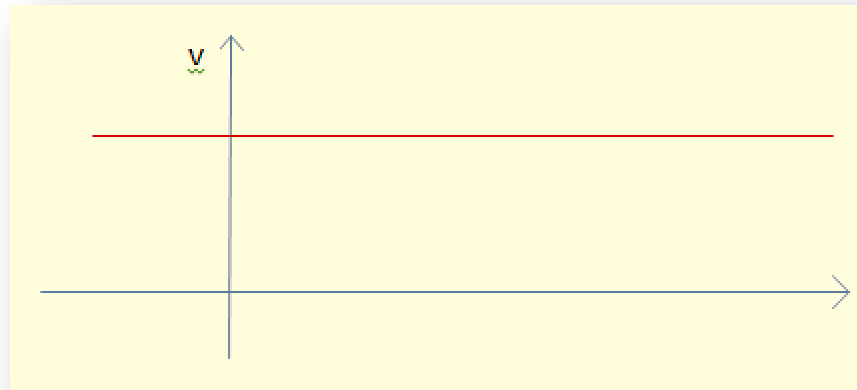
$$W = fX\,, \quad R(X) = 2$$

$$K = X\bar{X}$$

$$V = g^{a\bar{a}}\partial_a W \partial_{\bar{a}} \bar{W}$$

$$g_{a\bar{a}} = \partial_a \partial_{\bar{a}} K$$

$$V = f\bar{f} = |f|^2$$



SUSY IS BROKEN → Goldstino

BUT R-SYMMETRY prohibits GAUGINO mass.

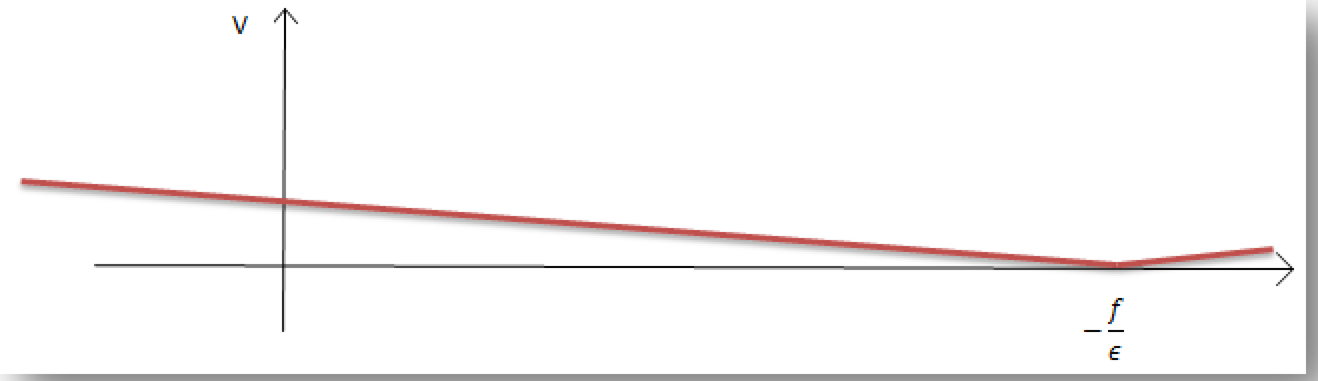
Break R-symmetry!

$$W = fX + \frac{1}{2}\epsilon X^2 \qquad R(X) = 2$$

$$V = |f + \epsilon X|^2$$

$$X = -\frac{f}{\epsilon}$$ supersymmetric vacuum

$$m_0{}^2 = \begin{pmatrix} |\epsilon|^2 & 0 \\ 0 & |\epsilon|^2 \end{pmatrix} = m_{\frac{1}{2}}{}^2$$



Noncanonical Kahler potential

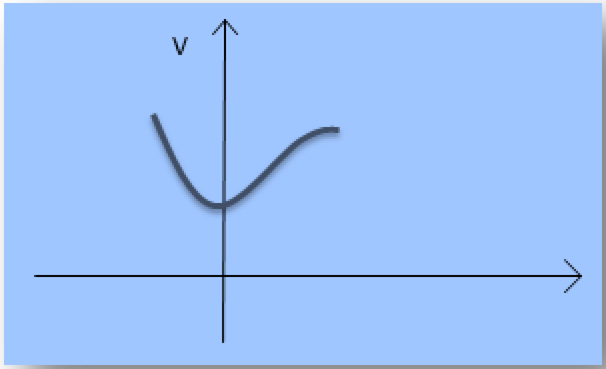
$$W = fX$$

$$K = X\bar{X} - \frac{c}{|\Lambda|^2}(X\bar{X})^2 + \cdots \qquad \text{about } X = 0$$

$$V = K_{X\bar{X}}{}^{-1}|f|^2 = \frac{|f|^2}{1 - \frac{4c}{|\Lambda|^2}(X\bar{X})^2 + \cdots}$$

$$= \left(1 + \frac{4c}{|\Lambda|^2}(X\bar{X})^2 + \cdots\right)|f|^2$$

A local minimum at $X = 0$



Mass spectrum

$$m_X{}^2 = \frac{4|f|^2 c}{|\Lambda|^2}$$

Massless spinor field is the Goldstino.

Breaking R-symmetry

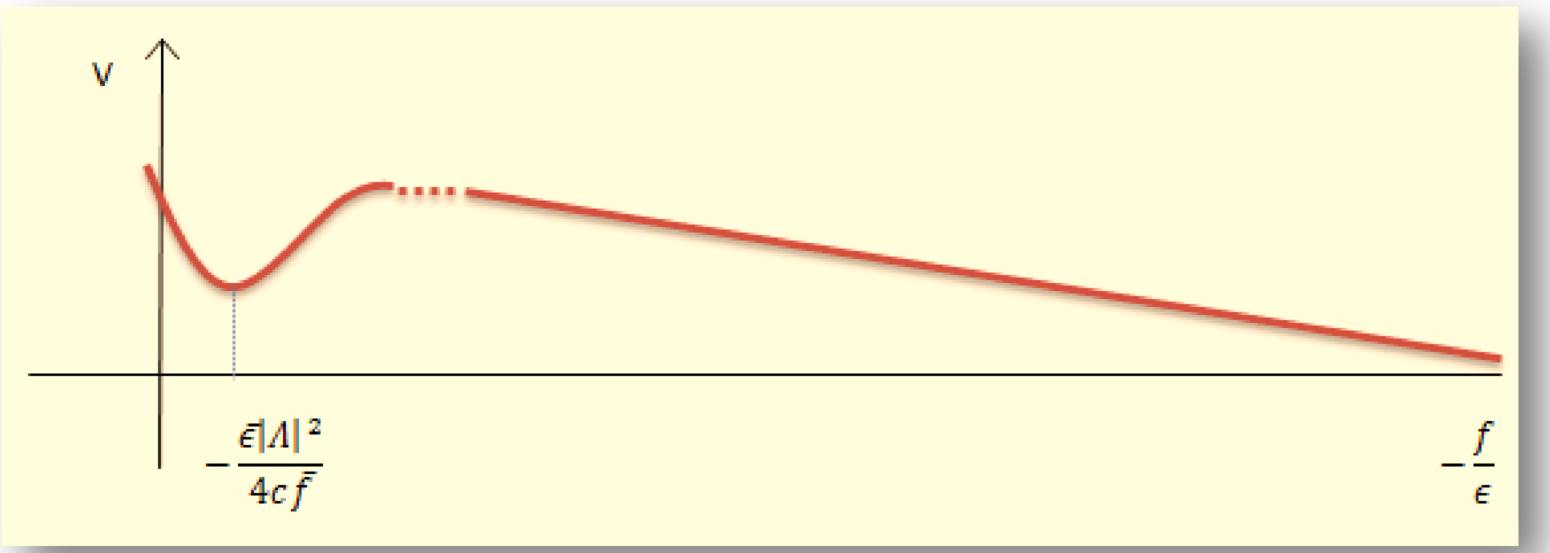
$$W = fX + \frac{1}{2}\epsilon X^2, \quad X = -\frac{f}{\epsilon}$$: supersymmetric vacuum

$$V = K_{X\bar{X}}{}^{-1}|f + \epsilon X|^2 = \left(1 + \frac{4c}{|\Lambda|^2}(X\bar{X})^2 + \cdots\right)|f + \epsilon X|^2$$ •

$$= |f|^2 + \epsilon x\bar{f} + \bar{\epsilon}\bar{x}f + \frac{4c|f|^2}{|\Lambda|^2}X\bar{X} + \cdots$$

$$\rightarrow \langle X\rangle_{meta} = -\frac{\bar{\epsilon}|\Lambda|^2}{4c\bar{f}}$$

$\epsilon \ll 1 \;\rightarrow a\ large\ distance\ between\ two\ minima.$
The possibility of the Long lived metastable state.



Mass terms

- $m_X{}^2 = \frac{4|f|^2 c}{|\Lambda|^2}$

Massless spinor field is the Goldstino.

SUMMARY

Problems of SM → SUSY

Superpartners not observed → SUSY breaking

F-term SUSY breaking → R-symmetry

Explicit R-symmetry breaking → Metastable state

References

- [1] P. Argyres, An Introduction to Global Supersymmetry, http://www.physics.uc.edu/argyres/661/susy2001.pdf
- [2] M.E.Peskin, D.V.Schroeder, An introduction to quantum field theory, Addison-Wesley(1995)
- [3] F.Quevedo, Cambridge Lectures on Supersymmetry and Extra Dimensions , arXiv:1011.1491v1 [hep-th] 5 Nov 2010
- [4] P.B.Pal, Dirac, Majorana and Weyl fermions, arXiv: 1006.1718v2 [hep-ph]
- [5] M. E. Peskin, Duality in Supersymmetric Yang-Mills Theory, arXiv:hep-th/9702094v1
- [6] A.Bilal, Introduction to supersymmetry, arXiv:hep-th/0101055 v1
- [7] J. Lykken, Introduction to supersymmetry, arXiv:hep-th/9612114 v1
- [8] D.Griffiths, Introduction to elementary particles.
- [9] K.Intriligator and N. Seiberg,Lectures on supersymmetry Breaking arXiv:hep-ph/0702069v3
- [10] J. Terning, Modern supersymmetry: Dynamics and duality (Oxford University Press, 2006).
- [11] A.Stergiou, New ways of supersymmetry breaking, (2007)
- [12] D.Bailin, A.Love, Supersymmetric gauge field theory and string theory , lOP Publishing Ltd (1994)
- [13] K. Intriligator, N. Seiberg and D. Shih, Supersymmetry Breaking, R-Symmetry Breaking and Metastable Vacua, arXiv:hep-th/0703281v1
- [14] T. Morii, C. S. Lim, S. N. Mukherjee, The Physics of the Standard Model and Beyond, World Scientific Publishing Co. Pte. Ltd. (2004)