۲۰۱۰/۱۰/۳۰
۰۱:۲۷ ب.ظ

$$J_{\mu\nu} = L_{\mu\nu} + S_{\mu\nu}$$

$$\vec{K}^i = J^{0i} = L^{0i} + S^{0i} \qquad [S_{\mu\nu}, P_\alpha] = 0$$

$$\vec{J}^i = \varepsilon^{ijk} J_{jk} \qquad [S_{\mu\nu}, L_{\alpha\beta}] = 0$$

$$[S_{\mu\nu}, S_{\alpha\beta}] \neq 0$$

$$\left.\begin{aligned} [S^{0i}, \vec{S}^k] &\neq 0 \\ [k^i, \vec{S}^k] &\neq 0 \end{aligned}\right\} \text{در حالت کلی}$$

اما اگر جهت خیز اجبت $\vec{K}$ کلی باشد $\vec{L}$ تغییر می‌کند.

$$[Q, \varphi] = i\delta\varphi \qquad \left([Q, \varphi] = i\delta\varphi\right)^\dagger$$

$$Q\varphi - \varphi Q = i\delta\varphi \xrightarrow{\text{Conjugate}} \varphi^\dagger Q^\dagger - Q^\dagger \varphi^\dagger = +i\delta\varphi^\dagger$$

$$[Q^\dagger, \varphi^\dagger] = +i\delta\varphi^\dagger$$

در مورد بارهای معمولی که سراغ داریم. اما باز هم این $\leftarrow$ $Q = Q^\dagger$

ابهام در ترتیب را حل نمی‌کند.

$$\underset{\overline{AB}}{\text{کلاسیک}} \xrightarrow{\text{کوانتمی}} \begin{aligned} &\frac{AB + B^\dagger A^\dagger}{2} \\ &\frac{BA + A^\dagger B^\dagger}{2} \end{aligned}$$

$$\cos\alpha\, AB + \sin\alpha\, BA + H.C.$$

مدل استاندارد

$$SU(3) \times SU(2)_L \times U(1)_Y$$

Glashow - Weinberg - Salam

(GWS)

Glashow, 1961; Weinberg, 1967; Salam 1968

برهمکنش های ضعیف

$$\left\{\begin{aligned} &\mu^- \to e^- + \bar{\nu}_e + \nu_\mu \\ &\pi^- \to \mu^- \bar{\nu}_\mu \\ &n \to p + e^- + \bar{\nu}_e \end{aligned}\right.$$

left-handed leptons

right-handed anti leptons

$$J_\mu(x) \equiv J_\mu^+(x) = \bar{\nu}_{eL}(x)\gamma_\mu e_L(x) = \bar{\nu}_e(x)\gamma_\mu \frac{1-\gamma_5}{2} e(x)$$

$$J_\mu^\dagger(x) \equiv J_\mu^-(x) = \bar{e}_L \gamma_\mu \nu_{eL} = \frac{1}{2}\bar{e}\gamma_\mu(1-\gamma_5)\nu_e$$

$$L = \frac{1-\gamma_5}{2}\begin{pmatrix}\nu_e \\ e^-\end{pmatrix} = \begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L$$

$$\tau^+ = \frac{\tau^1 + i\tau^2}{2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\tau^- = \frac{\tau^1 - i\tau^2}{2} \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$J_\mu^+ = \bar{L}\gamma_\mu \tau^+ L$$

$$J_\mu^- = \bar{L}\gamma_\mu \tau^- L$$

مشابه موارد بالا و با به کارگیری ماتریس τ^3 می‌توان نوشت

$$J_\mu^3 = \bar{L}\gamma_\mu \frac{\tau^3}{2} L = \frac{1}{2}\bar{\nu}_e\gamma_\mu\nu_e - \frac{1}{2}\bar{e}_L\gamma_\mu e_L$$

$W_\mu^{\pm}$ $\quad W_\mu^3$

$$J_\mu^i(x) = \bar{L}\gamma_\mu \frac{\tau^i}{2} L$$

$$T^i = \int J_0^i(x)\, d^3x$$

$$[T^i, T^j] = i\,\varepsilon^{ijk}\, T^k$$

Levi – Civita تانسور

$$\varepsilon^{123} = 1$$

$$R = \frac{1+\gamma_5}{2} e = e_R$$

$$J_\mu^3 \neq J_\mu^{em}$$ وسوسه

$$J_\mu^{em} = -\bar{e}\gamma^\mu e = -\bar{e}_L\gamma^\mu e_L - \bar{e}_R\gamma^\mu e_R$$

$$J_\mu^{em} = q\,\bar{\psi}\gamma_\mu\psi$$

$$Q = \int J_0^{em} d^3x = -\int \bar{e}\gamma_0 e\, d^3x =$$
$$-\int (e_L^\dagger e_L + e_R^\dagger e_R)\, d^3x$$

$[Q, T^3] = ?$ $\quad [Q, T^1] = ?$ $\quad [Q, T^2] = ?$

$SU(2)_L$ و $U(1)_{em}$ جریان نمی‌توانند تقارن های مدل باشند.

$SU(2)_L : \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ $\qquad U(1)_{em} : \begin{pmatrix} \nu_e \\ e \end{pmatrix}$

$U(1)_Y$

$$\frac{Y}{2} = Q - T^3 = \int d^3x \left(-\frac{1}{2}\, \nu_{eL}^\dagger \nu_{eL} - \frac{1}{2} e_L^\dagger e_L - e_R^\dagger e_R \right)$$

$$[Q - T^3, T^i] = 0$$

$U(1)_Y$ و $SU(2)$ تقارن های جریان مدل می‌توانند باشند.

$$Q = T^3 + \frac{Y}{2}$$

ذرات	Q	(T, T^3)	Y
ν_e, ν_μ, ν_τ	0	$(\frac{1}{2}, +\frac{1}{2})$	-1
e_L, μ_L, τ_L	-1	$(\frac{1}{2}, -\frac{1}{2})$	-1
e_R, μ_R, τ_R	-1	0	-2

نکته ی تاریخی (دقت زیاد تا اشتباه نشود) !!!

رابطه ی Nakano – Nishijima – Gell-Mann

{Nakano & Nishijima , 1953

(Gell-Mann 1953

$Y = B + S$

p درمورد $= ?$

K^+ درمورد $= ?$

$K^+ = |u\bar{s}\rangle$

$SU(2)$ ایزواسپین با $SU(2)$ پیمانه‌ای فرق دارد.

به دیدگاه مدرن بازگردیم.

$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \qquad R = e_R$

$SU(2)_L : \quad L \to L' = e^{-i\frac{\alpha^i \tau^i}{2}} L \ , \quad R \to R' = R$

$U(1)_Y : \quad L \to L' = e^{i\beta/2} L \ , \quad R \to R' = e^{i\beta} R$

$\alpha^i = \alpha^i(x) \qquad \beta = \beta(x)$

$SU(2)_L \times U(1)_Y$ لاگرانژی ناوردا تحت

$$\mathcal{L}_F = \bar{L}\, i\gamma^\mu \left(\partial_\mu - ig\,\frac{\vec{\tau}}{2}\cdot\vec{A}_\mu + \frac{i}{2} g' B_\mu\right) L$$
$$+ \bar{R}\, i\gamma^\mu \left(\partial_\mu + ig' B_\mu\right) R$$

$$D_\mu = \partial_\mu - ig\,\frac{\vec{\tau}}{2}\cdot\vec{A}_\mu - ig'\,\frac{Y}{2}\,B_\mu$$

$$m\,\bar{e}e = \cancel{m\,\bar{e}_R e_L + m\,\bar{e}_L e_R}$$

matter field L, R
gauge field $\vec{A}_\mu, B_\mu$

$$\mathcal{L}_G = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\,\varepsilon_{ijk} A^j_\mu A^k_\nu$$
$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

No mass term for B_μ or $\vec{A}_\mu$

در دنیای واقعی این میدان‌ها جرم دارهستند.

$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}$

$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \qquad \leftarrow Q = T_3 + \frac{Y}{2} \to Y_\varphi = ?$

$$\mathcal{L}_S = (D_\mu \phi)^\dagger D^\mu \phi - V(\phi^\dagger \phi)$$

کردآن

$$D_\mu \phi = \left(\partial_\mu - ig\,\frac{\vec{\tau}\cdot\vec{A}_\mu}{2} - ig' B_\mu\right)\phi$$

$$V(\phi^\dagger\phi) = m^2 \phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$\lambda > 0$

کتاب یک اشتباه تایپی دارد:

$$\mathcal{L}_Y = -G_e\left(\bar{L}\Phi R + \bar{R}\Phi^{\dagger}L\right) + h.c.$$

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_G + \mathcal{L}_S + \mathcal{L}_Y$$

$SU(2)_L \times U(1)_Y$ شکست خودبه‌خود تقارن

$$m^2 = -\mu^2 \qquad \mu^2 > 0$$

$$\varphi_0 = \langle 0|\varphi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\frac{\tau^3}{2}\varphi_0 = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix} = -\frac{\varphi_0}{2}$$

$$Y\varphi_0 = \varphi_0$$

بنابراین

$$e^{-i\alpha^3\frac{\tau^3}{2}}\varphi_0 \neq \varphi_0 \qquad e^{-i\beta\frac{Y}{2}}\varphi_0 \neq \varphi_0$$

اما

$$Q = T^3 + \frac{Y}{2}$$

$$Q\varphi_0 = \left(T^3 + \frac{Y}{2}\right)\varphi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix} = 0$$

$$e^{-i\varepsilon Q}\varphi_0 = \varphi_0$$

نتیجه: تقارن متناظر با T^3 و Y شکسته اما تقارن متناظر با Q خیر.

به زبان دیگر

$$\varphi \longrightarrow \varphi' = e^{iT^3\alpha}\varphi\, e^{-iT^3\alpha}$$

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \to \begin{bmatrix} e^{i\alpha}\varphi_1 \\ e^{-i\alpha}\varphi_2 \end{bmatrix}$$

$$\langle 0|\varphi|0\rangle \neq \langle 0|\varphi'|0\rangle$$

$$T^3|0\rangle \neq 0$$

اما

$$Q|0\rangle = 0$$

$$\Phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} = e^{i\frac{\vec{\tau}\cdot\vec{\xi}}{2v}}\begin{bmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{bmatrix}$$

$$U(\xi) = e^{-i\frac{\vec{\tau}\cdot\vec{\xi}}{2v}}$$

پیمانه‌ی یکانی

$$\varphi' = U(\xi)\varphi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

$$L' = U(\xi)L$$

$$\vec{A}'_\mu = U(\xi)\vec{A}_\mu U(\xi)^{-1} - \frac{i}{g}\left(\partial_\mu U(\xi)\right)U^{\dagger}(\xi)$$

$$R' = R$$

$$B'_\mu = B_\mu$$

$$\mathcal{L}_F = \bar{L}' i\gamma^\mu\left(\partial_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{A}'_\mu + \frac{i}{2}g'B'_\mu\right)L'$$

$$\mathcal{L}_F = \bar{L}' i\gamma^\mu (\partial_\mu - ig \tfrac{\vec{\tau}}{2} \cdot \vec{A}'_\mu + \tfrac{i}{2} g' B'_\mu) L'$$

$+ \cdots$

$$\mathcal{L}_s = (D_\mu \varphi)' (D^\mu \varphi)'$$

$$(D_\mu \varphi)' = \left(\partial_\mu - ig \tfrac{\vec{\tau}}{2} \cdot \vec{A}'_\mu - \frac{ig' B'_\mu}{2}\right) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

جملات درجه‌ی ۲

$$\mathcal{L}_{mass} = \frac{v^2}{2} (0 \quad 1) \left(g \tfrac{\vec{\tau}}{2} \cdot \vec{A}'_\mu + \tfrac{g'}{2} B'_\mu\right) \left(g \tfrac{\vec{\tau}}{2} \cdot \vec{A}'_\mu + \tfrac{g'}{2} B'_\mu\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$\frac{v^2}{8} \left(g^2 A'^1_\mu A'^{1\mu} + g^2 A'^2_\mu A'^{2\mu} + (g A'^3_\mu - g' B'_\mu)^2\right)$$

$$W^\pm_\mu = \frac{A^1_\mu \mp i A^2_\mu}{\sqrt{2}}$$

$$\frac{v^2}{8} g^2 (A'^1_\mu A'^{1\mu} + A'^2_\mu A'^{2\mu}) = \frac{m_W^2}{2} W^+_\mu W^{-\mu}$$

$$m_W = \frac{gv}{2} \qquad (W^+_\mu)^\dagger = W^-_\mu$$

$$\frac{v^2}{8} (A'^3_\mu \quad B'_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} A'^{3\mu} \\ B'^\mu \end{pmatrix}$$

$\downarrow$ قطری کردن

$$\frac{v^2}{8} (Z_\mu \quad A_\mu) \begin{bmatrix} g^2 + g'^2 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$= \frac{v^2}{8} (g^2 + g'^2) \; Z_\mu Z^\mu + 0 \, A_\mu A^\mu$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A'^3_\mu \\ B'_\mu \end{pmatrix}$$

$\theta_W =$ زاویه‌ی واینبرگ یا زاویه‌ی اختلاط ضعیف

$$\operatorname{tg}\theta_W = \frac{g'}{g}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \qquad \cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

$$m_Z = \frac{v}{2} \sqrt{g^2 + g'^2}$$

$$m_Z = \frac{m_W}{\cos\theta_W} \quad \leftarrow \quad \text{پیش‌بینی !}$$

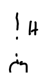
در پیمانه یکانی

$$V(\varphi^\dagger \varphi) = -\frac{\mu^2 v^2}{4} + \frac{1}{2}(2\mu^2) H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4 \qquad m_H = \sqrt{2\mu^2}$$

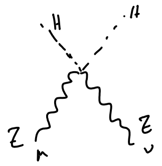
$$\mathcal{L}_s = (D_\mu \varphi)(D^\mu \varphi) - V(\varphi^\dagger \varphi) =$$

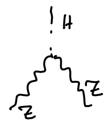
$$\frac{1}{2} \partial_\mu H \partial^\mu H - \frac{M_H^2}{2} H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4$$

$$+ \frac{g^2}{8} (H^2 + 2Hv) \left[\frac{Z_\mu Z^\mu}{\cos^2\theta_W} + 2 W^+_\mu W^{-\mu}\right]$$

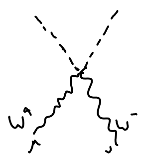


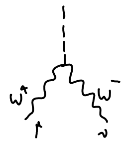
$+ \frac{g}{8}(H^2 + 2Hv)\left[\frac{Z_\mu Z^\mu}{\cos^2\theta_W} + 2W_\mu^+ W^{-\mu}\right]$





$i\frac{g^2}{2}\frac{\eta_{\mu\nu}}{\cos^2\theta_W}$ $\qquad$ $\frac{ig^2\eta_{\mu\nu}v}{2\cos^2\theta_W}$



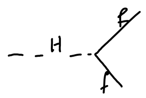


$\frac{ig^2}{2}\eta_{\mu\nu}$ $\qquad$ $\frac{ig^2 v}{2}$

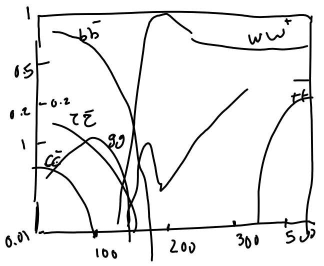
$$\mathcal{L}_Y = -G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L)$$

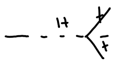
$$= -\frac{G_e v}{\sqrt{2}}\bar{e}e - \frac{G_e}{\sqrt{2}}H\bar{e}e$$

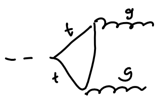
$$m_e = \frac{G_e v}{\sqrt{2}}$$

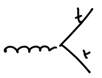


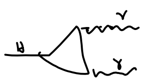
$-i\frac{m_f}{v}$











نکته اول شکست خودبه‌خودی تقارن

تقدم و تأخر زمانی

β-decay و $U(1)_{em}$ در انرژی پایین

weak interactions

$E \ll m_{EW} \simeq m_W$

$E > m_{EW}$ $\qquad W^+$, W^-, $U(1)_{em}$

در کیهان‌شناسی اول و آخر فرق دارد.

$\mu^2(T)$

همه‌ی بحث‌های ما در دمای صفر است.

دقت کنید که دمای صفر و انرژی صفر یکی نیستند.

در LHC دما ؟

انرژی ؟

نکته‌ی دوم

$SU(2) \times U(1)$ یا $U(2)$

$e^{i\beta} \quad e^{i\frac{\tau^i}{2}\alpha^i} \begin{bmatrix} - \\ - \end{bmatrix}$

دو جور ضریب جفت‌شدگی داریم

$g \,,\, g'$

نه چهار جور!! و نه یک جور!!

$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \longrightarrow$ یک Hypercharge دارند

اما e_R و e_L هایپرچارج متفاوت دارند.

نکته‌ی سوم

بالای ص ۶۱

Ghost در پیمانه‌ی یکانی لزومی نیست!

CC and NC

CC ← مخفف Charged current

NC ← مخفف Neutral current

kinetic

$\mathcal{L} = \bar{L}\, i\gamma_\mu \partial^\mu L + \bar{R}\, i\gamma_\mu \partial^\mu R =$

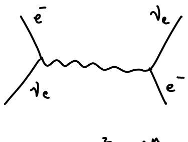
$\bar{e}\, i\gamma^\mu \partial_\mu e + \bar{\nu}_{eL}\, i\gamma^\mu \partial_\mu \nu_{eL}$

$\mathcal{L}_{CC} = g(J_\mu^{\,1} A^{1\mu} + J_\mu^{\,2} A^{2\mu}) =$

$\frac{g}{\sqrt{2}}(J_\mu^{+} W_\mu^{-} + J_\mu^{+} W^{+\mu})$

$J_\mu^{\pm} = \bar{L}\, \gamma_\mu \tau^{\pm} L \qquad \tau^{\pm} = \frac{\tau^1 \pm i\tau^2}{2}$

$J_\mu^{+} = \frac{1}{2} \bar{\nu}_e \gamma_\mu (1-\gamma_5) e$



$M = -\frac{g^2}{2} J^{+\mu} \frac{i\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2 + i\varepsilon} J^{-\nu}$

از کجا آمد؟

$q^2 \ll m_W^2$

$M = -i\frac{g^2}{} \quad J^{+\mu} J^{-}$

$$2 m_W^2$$

$$\mathcal{L}_{cc}^{eff} = -\frac{G_F}{\sqrt{2}}\, 4 J^{+\mu} J_\mu^- = -\frac{G_F}{\sqrt{2}} (\bar{\nu}_e \gamma^\mu (1-\gamma_5) e)$$

$$(\bar{e} \gamma_\mu \underline{(1-\gamma_5)} \nu_e)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2} \qquad G_F = 1.16639 \times 10^{-5}\, GeV^2$$

$$m_W = \frac{g v}{2} \qquad v = \frac{1}{(\sqrt{2} G_F)^{\frac{1}{2}}} = 246\, GeV$$

$$\frac{v}{\sqrt{2}} \simeq 170\ GeV \simeq m_t$$

$$\mathcal{L}_{cc}^{eff} = -\frac{G_F}{\sqrt{2}} (\bar{\nu}_e \gamma^\mu (1-\gamma_5) e)(\bar{\mu} \gamma_\mu (1-\gamma_5) \nu_\mu) ,$$

$$\mathcal{L}_{NC} = g J_\mu^3 A^{3\mu} + \frac{g'}{2} J_\mu^Y B^\mu =$$

$$(g \sin\theta_W\, J_\mu^3 + g' \cos\theta_W\, \frac{J_\mu^Y}{2}) A^\mu$$

$$+ (\underbrace{g \cos\theta_W\, J_\mu^3 - g' \sin\theta_W\, \frac{J_\mu^Y}{2}}_{\frac{g}{\cos\theta_W} J_\mu^Z}) Z^\mu$$

$$J_\mu^{em} = J_\mu^3 + \frac{J_\mu^Y}{2}$$

$$g \sin\theta_W\, J_\mu^3 + g' \cos\theta_W\, \frac{J_\mu^Y}{2} = g' \cos\theta_W\, J_\mu^{em}$$
$$+ (g \sin\theta_W - g' \cos\theta_W)\, J_\mu^3$$

$$e = g' \cos\theta_W = g \sin\theta_W$$

$$\frac{1}{e^2} = \frac{1}{g^2} + \frac{1}{g'^2} \qquad e = \frac{g g'}{\sqrt{g^2 + g'^2}}$$

$$\frac{g}{\cos\theta_W} J_\mu^Z = g \cos\theta_W\, J_\mu^3 - g' \sin\theta_W\, \frac{J_\mu^Y}{2} = \frac{g}{\cos\theta_W} (J_\mu^3 - \sin^2\theta_W J_\mu^{em})$$

$$L = \begin{pmatrix} f \\ f' \end{pmatrix}_L \qquad R^f = f_R \qquad R^{f'} = f'_R$$

$$J_\mu^Z = J_\mu^3 - \sin^2\theta_W\ J_\mu^{em} =$$
$$\bar{L} \gamma_\mu \frac{\tau^3}{2} L - \sin^2\theta_W (Q_f \bar{f} \gamma_\mu f + Q_f' \bar{f}' \gamma_\mu f')$$
$$= a_L^f\ \bar{f}_L \gamma_\mu f_L + a_R^f\ \bar{f}_R \gamma_\mu f_R + a_L^{f'}\ \bar{f}'_L \gamma_\mu f'_L + a_R^{f'}\ \bar{f}'_R \gamma_\mu f'_R$$

$$a_L^f = \frac{1}{2} - q_f \sin^2\theta_W \qquad a_L^{f'} = -\frac{1}{2} - q_f' \sin^2\theta_W$$

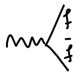
$$a_R^f = -q_f\ \sin^2\theta_W \qquad a_R^{f'} = -q_f'\ \sin^2\theta_W$$

$$J_\mu^Z = \bar{f} \gamma_\mu (C_V^f - C_A^f \gamma_5) f + \bar{f}' \gamma_\mu (C_V^{f'} - C_A^{f'} \gamma_5) f'$$

$$C_V^f = \frac{1}{2} (a_L^f + a_R^f) = \frac{1}{4} - Q_f \sin^2\theta_W$$
$$C_A^f = \frac{1}{2} (a_L^f - a_R^f) = \frac{1}{4}$$
$$C_V^{f'} = \frac{1}{2} (a_L^{f'} + a_R^{f'}) = -\frac{1}{4} - Q_f' \sin^2\theta_W$$
$$C_A^{f'} = \frac{1}{2} (a_L^{f'} - a_R^{f'}) = -\frac{1}{4}$$

واپاشی بوزون Z^0

$$\mathcal{L}^{Z}_{NC} = \frac{g}{\cos\theta_W} \; J^{Z}_{\mu} Z^{\mu}$$



$$\frac{ig}{\cos\theta_W} \; \gamma_\mu \left(C_V^f - C_A^f \gamma_5 \right)$$

دستگاه مرکز جرم $\quad P_Z = (m_Z, 0, 0, 0)$

$$P_f = \left(\frac{m_Z}{2}, \vec{k} \right) \qquad m_f \ll m_Z$$

$$P_{\bar{f}'} = \left(\frac{m_Z}{2}, -\vec{k} \right)$$

در روش قطبش خاص برای Z درنظر بگیریم و حل کنیم

$$\epsilon^{\mu}(0,0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \epsilon^{\mu}(0,1) = -\frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ i \\ 0 \end{bmatrix} \quad \epsilon^{\mu}(0,-1) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -i \\ 0 \end{bmatrix}$$

$$\Pi^{\mu\nu}(p) = \sum_{\sigma} e^{\mu}(p,\sigma) \, e^{\nu *}(p,\sigma)$$

روی همه اسپین ها جمع ببندیم و تقسیم بر ۳ کنیم.

$$\Pi^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \underbrace{\eta^{\mu\nu} + \frac{p^\mu p^\nu}{m^2}}$$

نوتاسیون وایبرگ

$$\frac{p^\mu p^\nu}{m} - \eta^{\mu\nu}$$

مال ما باید این گونه باشد

نوتاسیون وایبرگ $\quad \eta^{\mu\nu} = \mathrm{diag}(1,1,1,-1)$

مال ما $\quad \mathrm{diag}(1,-1,-1,-1)$

(t, x, y, z) نوتاسیون ما

(x, y, z, t) " وایبرگ

$$\mathcal{M} = \frac{ig}{\cos\theta_W} \; \bar{f}' \gamma_\mu \left(C_V^f - C_A^f \gamma_5 \right) f \; Z^\mu$$

$$|\mathcal{M}|^2 = \left| \frac{g}{\cos\theta_W} \right|^2 Z^\mu Z^\nu \; \bar{f}' \gamma_\mu \left(C_V^f - C_A^f \gamma_5 \right) f f^\dagger$$

$$\left(C_V^{f*} - C_A^f \gamma_5^* \right) \gamma_\nu^\dagger \gamma^{0\dagger} f'$$

$$\gamma_\nu^\dagger = \begin{cases} \gamma_\nu & \nu = 0 \\ -\gamma_\nu & \nu = 1,2,3 \end{cases} \Rightarrow \gamma_\nu^\dagger \gamma^{0\dagger} = \gamma^0 \gamma_\nu$$

$$|\mathcal{M}|^2 = \left| \frac{g}{\cos\theta_W} \right|^2 Z^\mu Z^\nu \; \bar{f}' \gamma_\mu \left(C_V^f - C_A^f \gamma_5 \right) f \bar{f} \left(C_V^{f*} + C_A^{f*} \gamma_5 \right)$$

$$\gamma_\nu f' = \left| \frac{g}{\cos\theta_W} \right|^2 Z^\mu Z^\nu$$

$$\mathrm{Tr}\left\{ \underbrace{f' \bar{f}'}_{\not{p}_{f'}} \gamma_\mu \left(C_V^f - C_A^f \gamma_5 \right) \underbrace{f \bar{f}}_{\not{p}_f} \left(C_V^{f*} + C_A^{f*} \gamma_5 \right) \gamma_\nu \right\}$$

$$= \left| \frac{g}{\cos\theta_W} \right|^2 \left(\frac{P_Z^\mu P_Z^\nu}{m_Z^2} - \eta^{\mu\nu} \right) P_{f'}^\alpha P_f^\beta$$

$$\Big\{ \left(|C_V^f|^2 + |C_A^f|^2 \right) \Big(\mathrm{Tr}\{ \gamma_\alpha \gamma_\mu \gamma_\beta \gamma_\nu \} + 2\mathrm{Re}\left[C_V^f C_A^{f*} \right]$$

$$\mathrm{Tr}\{ \gamma_\nu \gamma_\alpha \gamma_\mu \gamma_\beta \gamma_5 \} \Big)$$

$$P_Z = P_f + P_{\bar{f}} \qquad X_\mu \gamma^\mu X_\nu \gamma^\nu = X_\mu X_\nu \left[\frac{\{\gamma^\mu,\gamma^\nu\}}{2} + \frac{[\gamma^\mu,\gamma^\nu]}{2}\right]$$

$$= X \cdot X$$

$$\Downarrow$$

$$\cancel{P}_{\bar{f}} \; \cancel{P}_Z \; P_f = 0 + O(m_f^2)$$

$$\underbrace{Tr\{\gamma_\nu \gamma_\alpha \gamma_\mu \gamma_\beta \gamma_5\}}_{\eta^{\mu\nu} \times} = 4i\, \varepsilon_{\nu\alpha\mu\beta} \qquad = 0$$

$$Tr\{\gamma_\alpha \gamma_\beta\} = 4\eta_{\alpha\beta}$$

$$Tr\{\underbrace{\gamma_\mu \gamma_\nu}_{2\eta_{\mu\nu} - \gamma_\nu\gamma_\mu} \gamma_\rho \gamma_\sigma\} = 8\eta_{\mu\nu}\eta_{\rho\sigma} - Tr\{\gamma_\nu \underbrace{\gamma_\mu \gamma_\rho}_{2\eta_{\mu\rho} - \gamma_\rho\gamma_\mu} \gamma_\sigma\} =$$

$$= 8\eta_{\mu\nu}\eta_{\rho\sigma} - 8\eta_{\mu\rho}\eta_{\nu\sigma} + Tr\{\gamma_\nu \gamma_\rho \underbrace{\gamma_\mu \gamma_\sigma}_{2\eta_{\mu\sigma} - \gamma_\sigma\gamma_\mu}\}$$

$$= 8\eta_{\mu\nu}\eta_{\rho\sigma} - 8\eta_{\mu\rho}\eta_{\nu\sigma} + 8\eta_{\mu\sigma}\eta_{\nu\rho} - Tr\{\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\mu\}$$

$$Tr\{\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma\} = 4(\eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho})$$

$$\overline{|M|^2} = -\frac{1}{3}\left|\frac{g}{\cos\theta_W}\right|^2 \eta^{\mu\nu} \; Tr\{\cancel{P}_{\bar{f}}\, \gamma_\mu \cancel{P}_f \gamma_\nu\}$$

$$(|C_V^f|^2 + |C_A^f|^2) = -\left|\frac{g}{\cos\theta_W}\right|^2 \frac{4}{3}\times\left(2 P_{\bar{f}} \cdot P_f\right.$$

$$\left. - 2P_f \cdot P_{\bar{f}} \underbrace{\eta^{\mu\nu}\eta_{\mu\nu}}_{4}\right)(\;) = \frac{8}{3}\left|\frac{g}{\cos\theta_W}\right|^2 \underbrace{P_f \cdot P_{\bar{f}}}_{\frac{m_Z^2}{2}} (\;)$$

$$= \frac{4}{3}\left|\frac{g}{\cos\theta_W}\right|^2 m_Z^2 \; (|C_V^f|^2 + |C_A^f|^2)$$

$$d\Gamma = \frac{1}{2m_Z}\int \frac{d^3P_f}{(2\pi)^3} \frac{d^3 P_{\bar{f}}}{(2\pi)^3} \frac{1}{2E_f} \frac{1}{2E_{\bar{f}}}$$

$$\overline{|M|^2} \; (2\pi)^4 \, \delta(m_Z - |P_f| - |P_{\bar{f}}|) \; \delta^3(\vec{P}_f + \vec{P}_{\bar{f}})$$

$$= \frac{1}{2m_Z} \frac{1}{(2\pi)^2} \int \frac{d\Omega \, |P_f|^2 \, dP_f}{4|P_f|^2} \delta(m_Z - 2|P_f|) \, |\bar{M}|^2$$

$$= \frac{\overline{|M|^2}}{m_Z \;\; 16\pi} = \frac{2}{3}\frac{G_F}{\sqrt{2}} m_Z^3 \left(|C_V^f|^2 + |C_A^f|^2\right)$$

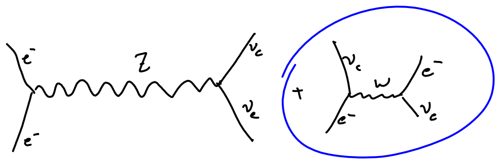
$$\frac{g^2}{8 m_Z^2 \cos\theta_W} = \frac{G_F}{\sqrt{2}}$$

$$e^{i(A+B)} = 1 + i(A+B) - \frac{(A+B)^2}{2!} + \dots$$

$$= 1 + i(A+B) - \frac{A^2 + B^2 + 2AB}{2!}$$

$$\int \mathcal{L}\, d^4x = A + B$$

$$A = \int d^4x \frac{g}{\cos\theta_W} J^Z_\mu Z^\mu$$



$$M = -\frac{i}{2!}\,\frac{g^2}{M_Z^2\cos^2\theta_W}\,J^Z_\mu J^{Z\mu}$$

$$J^Z_\mu = a^e_L \bar{e}_L\gamma_\mu e_L + a^e_R\,\bar{e}_R\gamma_\mu e_R + a^\nu_L\,\bar{\nu}_L\gamma_\mu\nu_L$$

$$a^f = T^3 - q_f\sin^2\theta_W$$

$\downarrow$

$$a^e_L = -\frac{1}{2} + \sin^2\theta_W \simeq -0.27$$

$$a^e_R = -\sin^2\theta_W = -0.23$$

$$a^\nu_L = \frac{1}{2} = 0.5$$

$$\mathcal{L}_{eff} = \frac{G_N}{\sqrt{2}}\,4\,J^Z_\mu J^{Z\mu}$$

$$\frac{G_N}{\sqrt{2}} = \frac{g^2}{8m_Z^2\cos^2\theta_W}$$

$$\mathcal{L}^{eff} = -\frac{G_F}{\sqrt{2}}\,4\left(J^+_\mu J^{-\mu} + \rho\,J^Z_\mu J^{Z\mu}\right)$$

$$\rho = \frac{G_N}{G_F} = \frac{m_W^2}{M_Z^2\cos^2\theta_W} = 1$$

مشاهده. $\rho = 1 \Longrightarrow$ Higgs doublet

$\varphi \to$ n-plet $\quad \langle\varphi\rangle = v_h\,\chi_h \quad$ (در مورد مدل استاندارد $\chi = \begin{bmatrix}0\\1\end{bmatrix}$)

$$(D_\mu\varphi)^\dagger D_\mu\varphi = \cdots +$$

$$\frac{v_h^2}{2}\chi_h^\dagger\,(g\underbrace{\vec{T}_h}\cdot\vec{A}_\mu + g'\frac{Y_h}{2}B_\mu)(g\vec{T}_h\cdot\vec{A}_\mu + g'\frac{Y_h}{2}B_\mu)\chi_h$$

مولدهای جبر در نمایشی که φ عضو آن هست

$$Q_h = T_{h3} + \frac{Y_h}{2} \leftarrow$$ این رابطه باز هم برقرار است.

$$Q_h = 0 \longrightarrow T_{h3} = -\frac{Y_h}{2}$$

$$\mathcal{L}_{mass} = \frac{v_h^2 g^2}{2}\chi_h^\dagger\left(A^1_\mu A^{1\mu}\,T_{h1}^2 + A^2_\mu A^{2\mu}\,T_{h2}^2\right)\chi_h$$

$$+\frac{g^2 v_h^2}{2}\underbrace{\left(A^3_\mu A^{3\mu} + \frac{g'^2}{g^2}B_\mu B^\mu - 2\frac{g'}{g}A_{3\mu}B^\mu\right)}_{\left(A^3_\mu + \frac{g'}{g}B_\mu\right)^2 = \frac{Z_\mu^2}{\cos^2\theta_W}}\chi_h^\dagger T_{h3}^2\chi_h$$

نکته‌ی مهم

$$\chi_h^\dagger T_{h1}^2\chi_h = \chi_h^\dagger T_{h2}^2\chi_h$$

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\theta_W} = \frac{\sum_h v_h^2\,T_{h\perp}^2}{\sum_h v_h^2\,T_{h3}^2}$$

بیش از یک هیگز

$$T_{h\perp}^2 = \frac{1}{2}\left(T_{h1}^2 + T_{h2}^2\right) = \frac{1}{2}\left(\vec{T}_h^2 - T_{h3}^2\right) = \frac{1}{2}\left[I_h(I_h+1) - T_{h3}^2\right]$$

اگر همه هیگزها در یک نمایش باشند

$$\rho = \frac{1}{2}\,\frac{I_h(I_h+1) - T_{h3}^2}{T_{h3}^2}$$

$$I_h = \frac{1}{2}\ , \quad T_{h3} = \pm\frac{1}{2} \longrightarrow \rho = 1$$

$$I_h = 1\ , \quad \left\{\begin{matrix} -1 \\ +1 \\ 0 \end{matrix}\right\} \begin{matrix} \longrightarrow \frac{1}{2} \\ \longrightarrow \frac{1}{2} \\ \longrightarrow \infty \quad (m_Z = 0) \end{matrix}$$

و تصحیح کوانتی می‌گیرد.

سه نسل لپتونی

$$\begin{array}{cc|cc|cc} L_\alpha & R_\alpha & & & & \\ \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L & e_R & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L & \mu_R & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L & \tau_R \end{array}$$

$$\mathcal{L}_K = \sum_\alpha \bar{L}'_\alpha\, i\gamma^\mu \partial_\mu L'_\alpha + \sum_\alpha \bar{R}'_\alpha\, i\gamma^\mu \partial_\mu R'_\alpha$$

$$\mathcal{L}^{CC}_{gauge} = \sum_\alpha \frac{g}{\sqrt{2}} \left(\bar{L}'_\alpha \gamma_\mu \tau^- L'_\alpha W^{-\mu} + \bar{L}'_\alpha \gamma_\mu \tau^+ L'_\alpha W^{+\mu} \right)$$

$$\mathcal{L}^{NC+em}_{gauge} = \left(g\sin\theta_W \sum_\alpha \bar{L}'_\alpha \gamma_\mu \frac{\tau^3}{2} L'_\alpha + g'\cos\theta_W \frac{Y_L}{2} \sum_\alpha \bar{L}'_\alpha \gamma_\mu L'_\alpha \right) A^\mu$$

$$+ \left(g\cos\theta_W \sum_\alpha \bar{L}'_\alpha \gamma_\mu \frac{\tau^3}{2} L'_\alpha - g'\sin\theta_W \frac{Y_L}{2} \sum_\alpha \bar{L}'_\alpha \gamma_\mu L'_\alpha - g'\sin\theta_W \frac{Y_R}{2} \sum_\alpha \bar{R}'_\alpha \gamma_\mu R'_\alpha \right.$$

$$- \sum_\alpha e \bar{R}'_\alpha \gamma^\mu R'_\alpha A_\mu$$

$$\mathcal{L}_Y = -\sum_{\alpha\beta} G_{\alpha\beta}\, \bar{L}'_\alpha \phi R'_\beta + H.c.$$

$$G = U^\dagger G^{diag} V \qquad G^{diag}_{\alpha\beta} = G_\alpha \delta_{\alpha\beta}$$

$$R = VR' \qquad L = UL'$$

$$\mathcal{L}_Y = -\sum_\alpha G_\alpha \bar{L}_\alpha \phi R_\alpha + H.c.$$

$$\sum_\alpha \bar{R}_\alpha \gamma^\alpha R_\alpha = \sum_\alpha \bar{R}'_\alpha \gamma^\alpha R'_\alpha$$

No lepton flavor violation
in the SM

$$\sum_{\alpha\beta} F_{\alpha\beta}\, \bar{L}_\alpha i\sigma_2 \varphi^* \nu_{R\beta} + H.c. \longleftarrow \text{flavor violation}$$

$$\sum_\alpha Z'_\mu f_\alpha \bar{L}_\alpha \gamma^\mu \frac{\tau^3}{2} L_\alpha \qquad f_\alpha \neq f_\beta \leftarrow \alpha \neq \beta$$

تعمیم به کوارک ها

quark family	Q	(T, T^3)	Y
u_L, c_L, t_L	$+\frac{2}{3}$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$
d_L, s_L, b_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$
u_R, c_R, t_R	$+\frac{2}{3}$	0	$\frac{4}{3}$
d_s, s_R, b_R	$-\frac{1}{3}$	0	$-\frac{2}{3}$

quark family	Q	(T, T^3)	Y
u_L, c_L, t_L	$+\frac{2}{3}$	$(\frac{1}{2}, \frac{1}{2})$	$+\frac{1}{3}$
d_L, s_L, b_L	$-\frac{1}{3}$	$(\frac{1}{2}, -\frac{1}{2})$	$+\frac{1}{3}$
u_R, c_R, t_R	$+\frac{2}{3}$	0	$\frac{4}{3}$
d_R, s_R, b_R	$-\frac{1}{3}$	0	$-\frac{2}{3}$

$$SU(3)_c \times \underbrace{SU(2) \times U(1)}_{\text{Color-blind}}$$

کوررنگی دارد

$$\underset{\hookrightarrow (r\,b\,g)}{q_i} \Longrightarrow q$$

$$Q_{Li} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \qquad U_{Ri} \qquad D_{Ri} \qquad (i = 1, 2, 3)$$

$$\mathcal{L}_F = \sum_{i=1}^{3} \bar{Q}_{Li}\, i\gamma^\mu \left(\partial_\mu - ig \frac{\vec{\tau}\cdot\vec{A}}{2} - \frac{i}{6} g' B_\mu\right) Q_{Li}$$
$$+ \sum_{i=1}^{3} \bar{U}_{Ri}\, i\gamma^\mu \left(\partial_\mu - i\frac{2}{3} g' B_\mu\right) U_{Ri}$$
$$+ \sum_{i=1}^{3} \bar{D}_{Ri}\, i\gamma^\mu \left(\partial_\mu + i\frac{g'}{3} B_\mu\right) D_{Ri}$$

$$\mathcal{L}_Y = -\sum_{i,j} \left(\Gamma^{(D)}_{i,j}\, \bar{Q}_{Li}\, \phi\, D_{Rj} + \Gamma^{(U)}_{ij}\, \bar{Q}_{Li}\, \tilde{\phi}\, U_{Rj} + H.c.\right)$$

$$\tilde{\phi} = i\tau_2 \varphi^* = \begin{bmatrix} \varphi^{0*} \\ -\varphi^- \end{bmatrix} \qquad Y_\varphi = 1 \qquad Y_{\tilde{\varphi}} = -1$$

$$\begin{cases} \varphi \xrightarrow{SU(2)} U\varphi \\ \varphi^* \longrightarrow U^*\varphi^* \\ \tilde{\varphi} \longrightarrow U\tilde{\varphi} \end{cases}$$

$$\tau_1 = \tau_1^* \quad -\tau_2 = \tau_2^* \quad \tau_3 = \tau_3^*$$
$$\tau_{1,3} \cdot \tau_2 = -\tau_2 \tau_{1,3}$$
$$\tau_2 \tau_2 = \tau_2 \tau_2$$

$SU(N)$

$$\tilde{q} = \varepsilon^{i_1 \cdots i_n}\, q^{i_1 *}_{j_1} \cdots q^{i_n *}_{j_n} \qquad q_i = \begin{bmatrix} - \\ - \\ - \\ - \\ - \end{bmatrix}$$

$$q_i \xrightarrow{SU(N)} U q_i$$

$$\tilde{q} \xrightarrow{SU(N)} U\tilde{q}$$

$$q_i^j \longrightarrow U_{jk}\, q_i^k \qquad Det[U] = Det[U^*] = 1$$

$$SU(2) \qquad \varepsilon^{ij} = i\sigma^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\varphi \to \frac{v+H}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \bar{Q}_{Li}\, \phi\, D_{Rj} = \frac{v}{\sqrt{2}} \bar{D}_{Li} D_{Rj} + \frac{H}{\sqrt{2}} \bar{D}_{Li} D_{Rj}$$

$$\tilde{\varphi} \to \frac{v+H}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \bar{Q}_{Li}\, \tilde{\phi}\, U_{Rj} = \frac{v}{\sqrt{2}} \bar{U}_{Li} U_{Rj} + \frac{H}{\sqrt{2}} \bar{U}_{Li} U_{Rj}$$

ماتریس جرمی کوارک‌ها

$$\mathcal{L}^{(m_q)} = -\sum \bar{D}\, M^D_{ij}\, D - \sum \bar{U}\, M^U_{ij}\, U + H.c.$$

$$M^{(D)}_{ij} = \Gamma^{D}_{ij} \frac{v}{\sqrt{2}} \qquad M^{(U)}_{ij} = \Gamma^{U}_{ij} \frac{v}{\sqrt{2}}$$

$$\left.\begin{aligned} \mathcal{U}_D^{\dagger} M^{D} \mathcal{V}_D &= M^{D} \\ \mathcal{U}_U^{\dagger} M^{U} \mathcal{V}_U &= M^{(U)} \end{aligned}\right\} \text{قطری}$$

$$\begin{cases} D'_L = \mathcal{U}_D^{\dagger} D_L \\ D'_R = \mathcal{V}_D^{\dagger} D_R \end{cases} \qquad \begin{cases} U'_L = \mathcal{U}_U^{\dagger} U_L \\ U'_R = \mathcal{V}_U^{\dagger} U_R \end{cases}$$

$$\mathcal{L}^{(m_a)} = -\bar{D}'_L M^{(D)} D'_R - \bar{D}'_R M^{(D)\dagger} D'_L - \bar{U}'_L M^{(U)} U'_R - \bar{U}'_R M^{(U)} U'_L$$

$$M^{(D)} = \begin{bmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{bmatrix} \qquad M^{(U)} = \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{bmatrix}$$

$$D' = \begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} \qquad U' = \begin{bmatrix} u' \\ c' \\ t' \end{bmatrix}$$

$$Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix}$$

$Q \longrightarrow ?$

$$\sum_{\alpha} \bar{Q}_{\alpha} \gamma_{\mu} \frac{\tau^{3}}{2} Q_{\alpha} = ?$$

$$\bar{u}_L \gamma^{\mu} u_L \qquad \bar{u}_R \gamma^{\mu} u_R$$

$$\bar{D}_L \gamma^{\mu} D_L \qquad \bar{D}_R \gamma^{\mu} D_R$$

$$\bar{Q} \gamma_{\mu} \frac{\tau^{\pm}}{2} Q \longrightarrow ?$$

$$\mathcal{L}^{(a)}_{cc} = \frac{g}{\sqrt{2}} \sum_i \bar{U}_{Li} \gamma^{\mu} D_{Li} W^{+}_{\mu} + \frac{g}{\sqrt{2}} \sum_i \bar{D}_{Li} \gamma^{\mu} U_{Li} W^{-}_{\mu}$$

$$\Downarrow$$

$$\mathcal{L}^{a}_{cc} = \frac{g}{\sqrt{2}} \bar{U}'_L \mathcal{U}_U^{\dagger} \gamma^{\mu} \mathcal{U}_D D'_L W^{+}_{\mu} + \frac{g}{\sqrt{2}} \bar{D}'_L \mathcal{U}_D^{\dagger} \gamma^{\mu} \mathcal{U}_U U'_L W^{-}_{\mu}$$

$\cancel{\mathcal{V}^{\dagger}_{D}}$ کتاب اشتباه املایی دارد

$$V_{CKM} = \mathcal{U}_U^{\dagger} \mathcal{U}_D$$

نتیجه برهمکنش های Z_{μ} و فوتون F را نقض نمی‌کنند

اما CC می‌کنند

FCNC در حد درخت

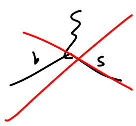
نداریم

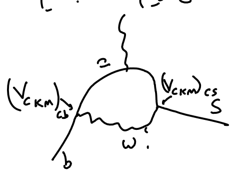
ولی

FCCC داریم.

اما در حد حلقه می‌توانیم داشته باشیم

$b \to s \gamma$





+

J_μ^Z

$$f = \begin{cases} u \\ c \\ t \end{cases} \qquad f' = \begin{cases} d \\ s \\ b \end{cases}$$

برهمکنش Z_μ

$$a^f = T^3 - q \sin^2\theta_W$$

$$C_V^u = \frac{1}{2}(a_L^u + a_R^u) = \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W - \frac{2}{3}\sin^2\theta_W\right) = \frac{1}{4} - \frac{2}{3}\sin^2\theta_W$$

$$= C_V^c = C_V^t$$

$$C_A^u = \frac{1}{2}(a_L^u - a_R^u) = \frac{1}{2}\left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W + \frac{2}{3}\sin^2\theta_W\right) = \frac{1}{4}$$

$$= C_A^t = C_A^c$$

$$C_V^d = \frac{1}{2}(a_L^d + a_R^d) = \frac{1}{2}\left(\frac{-1}{2} + \frac{1}{3}\sin^2\theta_W + \frac{1}{3}\sin^2\theta_W\right) = -\frac{1}{4} + \frac{\sin^2\theta_W}{3}$$

$$= C_V^b = C_V^s$$

$$C_A^d = \frac{1}{2}(a_L^d - a_L^d) = -\frac{1}{4} = C_A^b = C_A^s$$

$Z \to e\bar{e}$? $\quad Z \to \mu\bar{\mu}$? $\quad Z \to \tau\bar{\tau}$?

$Z \to c\bar{c}$? $\quad Z \to u\bar{u}$? $\quad Z \to d\bar{d}$?

$Z \to t\bar{t}$? $\quad Z \to b\bar{b}$?

$Z \to$ invisible

CC & FC

$$\mathcal{L}_{CC}^{(a)} = \frac{g}{2\sqrt{2}} \bar{U}' \gamma^\mu (1-\gamma_5) V_{CKM} D' W_\mu^+ + h.c.$$

$n = 3$ ← نسل

$\frac{n(n-1)}{2}$ ← زاویه

فاز غیرفیزیکی

$n(n+1)$ ← فاز $\quad - (2n-1) = \frac{(n-1)(n-2)}{2}$

آیا می‌توان میدان‌ها را rephase کرد؟

$n=2 \qquad n=3$

تنها دو نسل $n=2$

$$\mathcal{L}_{CC} = \frac{g}{\sqrt{2}} (\bar{u} \;\; \bar{c}) \gamma^\mu \frac{1-\gamma_5}{2} V_{CKM} \begin{pmatrix} d \\ s \end{pmatrix} W_\mu^+ + H.c.$$

$$V_{CKM} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

واپاشی K^+ را چه گونه حساب کنیم؟

$$K^+ \to \mu^+ \nu_\mu$$

مکانیزم GIM

1960

$$Q_L^{(1)} = \begin{pmatrix} u \\ d_c \end{pmatrix}_L \qquad Q_L^{(2)} = \begin{pmatrix} c \\ s_c \end{pmatrix}_L$$

$(d_c \quad (\cos\theta_c \quad \sin\theta_c \quad (d$

$$Q_L^{(1)} = \begin{pmatrix} u \\ d_c \end{pmatrix}_L \qquad Q_L^{(2)} = \begin{pmatrix} c \\ s_c \end{pmatrix}_L$$

$$\begin{pmatrix} d_c \\ s_c \end{pmatrix}_L = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_L$$

$$\bar{Q}_L^{(1)} \gamma_\mu \frac{\tau^3}{2} Q_L^{(1)} = \frac{1}{2} \bar{u}_L \gamma_\mu u_L - \cos^2\theta_c \bar{d}_L \gamma_\mu d_L$$

$$- \sin^2\theta_c \bar{s}_L \gamma_\mu s_L - \sin\theta_c \cos\theta_c (\bar{d}_L \gamma_\mu s_L + \bar{s}_L \gamma_\mu d_L)$$

$$\mathcal{L}^{em} = -\frac{1}{3} e (\bar{d} \gamma_\mu d + \bar{s} \gamma_\mu s) A^\mu$$

$$\left.\begin{array}{l} d \to d\cos\theta_c + s\sin\theta_c \\ s \to -d\sin\theta_c + d\cos\theta_c \end{array}\right\} \mathcal{L}_{em} \text{ فرقی نمی‌کند.}$$

اما $Z^\mu \quad \bar{d}_L \gamma_\mu s_L$

اگر $Q_L^{(2)}$ را کنار بگذاریم

$\uparrow$

سهم Q^2 این را حذف می‌کند.

$K_L^0 \to \mu^+\mu^- \qquad K^+ \to \pi^+ \nu\bar{\nu}$

اما:

$$\begin{cases} Br(K_L^0 \to \mu^+\mu^-) = (7.25 \pm 0.16) \times 10^{-9} \\ Br(K^+ \to \pi^+ \bar{\nu}\nu) = (1.6^{+1.8}_{-0.8}) \times 10^{-10} \end{cases}$$

Glashow, Iliopoulos & Maiani (GIM) 1970

$$Q^{(2)} = \begin{pmatrix} c \\ s_c \end{pmatrix}_L = \begin{pmatrix} c \\ s\cos\theta_c - d\sin\theta_c \end{pmatrix}_L$$

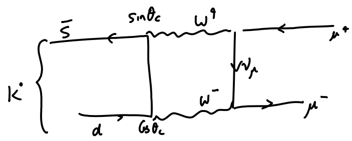
$$\bar{Q}_L^{(2)} \gamma_\mu \frac{\tau^3}{2} Q_L^{(2)} = \frac{1}{2} \Big(\bar{c}_L \gamma_\mu c_L - \cos^2\theta_c \bar{s}_L \gamma_\mu s_L$$

$$+ \sin^2\theta_c \bar{d}_L \gamma_\mu d_L + \sin\theta_c \cos\theta_c (\bar{d}_L \gamma_\mu s_L + \bar{s}_L \gamma_\mu d_L) \Big)$$

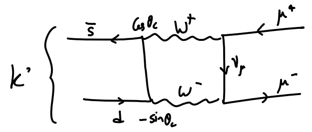
$$\Downarrow$$

$$\bar{Q}_L^{(1)} \gamma_\mu \frac{\tau^3}{2} Q_L^{(1)} + \bar{Q}_L^{(2)} \gamma_\mu \frac{\tau^3}{2} Q_L^{(2)} =$$

$$\frac{1}{2} (\bar{u}_L \gamma_\mu u_L + \bar{c}_L \gamma_\mu c_L - \bar{d}_L \gamma_\mu d_L - \bar{s}_L \gamma_\mu s_L)$$



$$G_F \propto \sin\theta_c \cos\theta_c \left(1 + O\left(\frac{m_u^2}{m_W^2}\right)\right)$$



$$-G_F \propto \sin\theta_c \cos\theta_c \left(1 + O\left(\frac{m_c^2}{m_W^2}\right)\right)$$

$$A(K_L^0 \to \mu^+\mu^-) \propto G_F^2 \sin\theta_c \cos\theta_c \, (m_c^2 - m_u^2)$$

Gaillard & Lee, 1974

$Br(K_L^0 \to \mu\mu) \longrightarrow m_c \simeq 1.5$ GeV

$1974 \longleftarrow J/\psi$

PDG — پارامتریزاسیون استاندارد

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & \cdots & s_{23}c_{13} \\ \cdots & \cdots & c_{23}s_{13} \end{pmatrix}$$

دوران حول محور 3

$$V_{CKM} = R_1(\theta_2)\, R_3(\theta_1)\, C(0,0,\delta)\, R_1(\theta_3)$$

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{bmatrix} \qquad C(0,0,\delta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{bmatrix}$$

$\vdots$

Wolfenstein — پارامتریزاسیون

$$V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$A, \rho, \eta \sim 1 \qquad \lambda = \sin\theta_c \simeq 0.22$

تجربی

$Br(b \to s\gamma) \gg Br(b \to d\gamma)$

مشاهده پذیرهای فیزیکی نباید به پارامتریزاسیون ربط داشته باشند.

پتانسیل هیگز

$$\bar{Q}_{Li}\, \phi\, D_{Rj} = \frac{(v+H)}{\sqrt{2}} \bar{D}_{Li}\, D_{Rj}$$

$$\bar{Q}_{Li}\, \tilde{\Phi}\, U_{Rj} = \frac{(v+H)}{\sqrt{2}} U_{Li} U_{Rj}$$

$$\mathcal{L}_{Hqq} = -\frac{H}{v}\left(m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + m_b \bar{b}b + m_c \bar{c}c + m_t \bar{t}t\right)$$

BSM

$\phi_1 \quad \phi_2 \qquad \langle\phi_1\rangle = \begin{pmatrix}0\\ v_1\end{pmatrix} \quad \langle\phi_2\rangle = \begin{pmatrix}0\\ v_2\end{pmatrix}$

$v^2 = v_1^2 + v_2^2$

$G^{(1)}_{ij}\,\bar{Q}_{Li}\,\phi_1\,D_{Rj} + G^{(2)}_{ij}\,\bar{Q}_{Li}\,\phi_2\,D_{Rj}$

پایه جرم $\qquad v_1 G^{(1)}_{ij} + v_2 G^{(2)}_{ij} \to$ قطری

در این پایه لزوماً $G^{(1)}_{ij}$ و $G^{(2)}_{ij}$ قطری نیستند

$\downarrow$

FC

Weyl   فرمیون وایل

$\psi = \underbrace{\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}}_{\text{اسپینور دیراک}} \qquad \underbrace{\psi_L \quad \psi_R}_{\text{اسپینور وایل}}$

$J_{\mu\nu} \propto [\gamma_\mu, \gamma_\nu]$ تحت تبدیل لورنتس

$J_{0i} \propto \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{bmatrix} - \begin{bmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = 2\begin{bmatrix}-\sigma^i & 0\\ 0 & \sigma^i\end{bmatrix}$

$J_{ij} \propto \begin{bmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{bmatrix}\begin{bmatrix}0 & \sigma^j\\ -\sigma^j & 0\end{bmatrix} - \begin{bmatrix}0 & \sigma^j\\ -\sigma^j & 0\end{bmatrix}\begin{bmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{bmatrix} \propto \epsilon^{ijk}\begin{bmatrix}\sigma^k & \\ & \sigma^k\end{bmatrix}$

تحت تبدیل لورنتس $\psi_L \to \psi_L'$ و $\psi_R \to \psi_R'$ اما یکسان تبدیل نمی‌شوند.

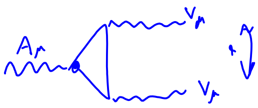
$\psi_L^\dagger \psi_L \quad \longrightarrow$ ناوردای لورنتس نیست

$\psi_R^\dagger \psi_R \quad \longrightarrow$ 〃 〃 〃

$\psi_R^\dagger \psi_L \quad$ 〃 〃 هست.

بی هنجاری

منبع بمنش ۱۹.۲ و ۲۰.۲ پسکین



$\bar\psi \gamma^\mu \gamma^5 \psi$

$\partial_\mu \overbrace{j^{\mu 5}} = \frac{-e^2}{16\pi^2}\,\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$

triangle anomaly = γ_5 anomaly = axial anomaly

= Adler - Bell - Jackiw (ABJ) anomaly

Adler 1969   Bell - Jackiw 1969

Bardeen 1969

.a .b .c

$A_\mu^a \quad A_\nu^b \quad A_\lambda^c$

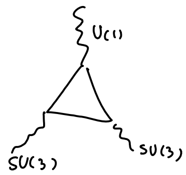
$tr\left[\gamma^5 t^a \{t^b, t^c\}\right]$

over all Fermions

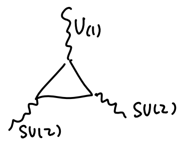
QED QCD → no anomaly

$tr[T^a]_{\text{SU(N) در}} = tr[\lambda]_{\text{SU(3) در}} = 0$

Chiral theory → dangerous

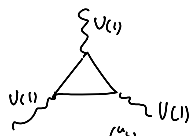


$tr\{\gamma^5 t^a \{t^b, t^c\}\} = \sum_q Y_q = \underset{u_R}{\frac{2}{3}} - \underset{d_R}{\frac{1}{3}} - \underset{\text{left}}{2} \times \frac{1}{6} = 0$ (d_L, u_L)



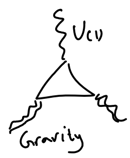
$tr\{\gamma^5 Y_{f_L} \underbrace{\{t^a, t^b\}}_{\delta_{ab}}\}$

$= \left(-(-\frac{1}{2}) - \underbrace{3}_{\text{رنگ}}(\frac{1}{6})\right) \times 2 = 0$



$tr(Y^3) = -2\underset{\binom{\nu_L}{e_L}}{(-\frac{1}{2})^3} + \underset{e_R}{(-1)^3}$

$-3\left[2\underset{\binom{u_L}{d_L}}{(\frac{1}{6})^3} - \underset{u_R}{(\frac{2}{3})^3} - \underset{d_R}{(-\frac{1}{3})^3}\right] = 6$



$tr[Y] = -2(-\frac{1}{2}) + (-1) - 3[2 \times \frac{1}{6} - (\frac{2}{3})$
$- (-\frac{1}{3})] = 0$

فکر کنید. آیا شرط کنسل شدن همه‌ی هایپرچارج

ها را به ما می‌دهد؟

به خاطر داشته باشیم که:

(۱) بی‌هنجاری مربوط به تئوری های کایرال است.

اگر اسکالر اضافه کنیم مشکلی نیست.

اگر زوج فرمیون کایرال (فرمیون دایل) با هایپرچارج

مخالف اضافه کنیم باز هم جای نگرانی نیست.

(۲) عدم وابستگی به جرم

(۳) One-loop exact!

* جریان مربوط به عدد باریونی و ... را بنویسید

و بی‌هنجاری های این جریان‌ها را در حالت مدل استاندارد به دست آورید

عدد باریونی کوارک‌ها ... ذرات مخلوق