$$P^2 \psi_{p_1\sigma_1 n_1;\, p_2\sigma_2 n_2;\ldots} = \eta_{n_1}^2\, \eta_{n_2}^2 \cdots \psi_{p_1\sigma_1 n_1;\, p_2\sigma_2 n_2}$$

$$I_p^2 P^2 = 1$$

تقارن داخلی تازه ؟!   اگر بخشی از تقارن پیوسته‌ی سیستم باشد.

$$e^{i(\alpha B + \beta L + \gamma Q)}$$

$$P' \equiv P I_p \qquad\qquad P'^2 = 1$$

سؤال ۵)

$$\Delta \to e^{i\beta} \Delta \qquad\qquad \text{تحت } U(1)$$

$$\Delta \to e^{i\vec{\alpha}\cdot\vec{T}} \Delta \qquad\qquad \text{تحت } SU(2)$$

$$\left[\frac{\tau^i}{2}, \frac{\tau^j}{2}\right] = i\,\varepsilon^{ijk}\,\frac{\tau^k}{2}$$

$$T^1 = -i\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \qquad T^2 = -i\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$T^3 = -i\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[T^1, T^2] = i\,T^3 \qquad\qquad \text{امتحان}$$

$$D_\mu = \partial_\mu - i g \vec{A}_\mu \cdot \vec{T} - i g' \frac{Y}{2} B_\mu$$

$$\begin{cases} A_\mu^3 = \cos\theta_w Z_\mu + \sin\theta_w A_\mu \\ B_\mu = -\sin\theta_w Z_\mu + \cos\theta_w A_\mu \end{cases}$$

$$i\bar{\Delta}\, \not{D}_\mu \Delta = \text{جمله جنبشی} + \cdots + A_\mu \bar{\Delta}\gamma^\mu \begin{bmatrix} g'\frac{Y}{2}\cos\theta_w & -ig\sin\theta_w & 0 \\ ig\sin\theta_w & g'\frac{Y}{2}\cos\theta_w & 0 \\ & & \end{bmatrix} \Delta$$

$$L \quad 0 \quad 0 \quad g\frac{Y}{2}\cos\theta_w$$

$$g'\cos\theta_w = g\sin\theta_w = e$$

$$\Delta \to \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \Delta \longrightarrow$$

بار $\Delta^3 = \frac{Y}{2}$

بار $\frac{\Delta^1 + i\Delta^2}{\sqrt{2}} = 1 + \frac{Y}{2}$

بار $\frac{i\Delta^1 + \Delta^2}{\sqrt{2}} = -1 + \frac{Y}{2}$

$$\Delta' = \begin{bmatrix} \Delta^3 & \frac{\Delta^1 - i\Delta^2}{\sqrt{2}}\sqrt{2} \\ \frac{\Delta^1 + i\Delta^2}{\sqrt{2}} \propto \sqrt{2} & -\Delta^3 \end{bmatrix}$$

$$A = B = \Delta^3_{\downarrow} \qquad n_1 = 1 = -n_2 \qquad n_3 = \sqrt{2} \qquad n_4 = \sqrt{2}$$

بار $C \longrightarrow \frac{Y}{2} - 1$

بار $D \longrightarrow \frac{Y}{2} + 1$

$$\Delta^i \longrightarrow \left(e^{iT^k\alpha^k}\Delta\right)^i = \left(1 + iT^k\alpha^k\right)_{ij}\Delta^j$$

$$= \Delta^i + \varepsilon^{kij}\Delta^j\alpha^k =$$

$$\Delta^i\tau^i \longrightarrow \Delta^i\tau^i + \underbrace{\underbrace{\varepsilon^{kij}\tau^i}_{-\varepsilon^{kji}\tau^i}\Delta^j\alpha^k}_{i\left[\frac{\tau^k}{2},\, \tau^j\right]\Delta^j\alpha^k}$$

$$\Delta^i\tau^i \longrightarrow \Delta^i\tau^i + i\left[\frac{\vec{\tau}\cdot\vec{\alpha}}{2}, \Delta\right]$$

* جرم‌های مولفه‌های مختلف Δ را بدست آورید. برای این کار باید جمله جنبشی را با نرمالیزاسیون صحیح بنویسید. ضریب جدی برهمکنش با Φ را به صورت زیر در نظر بگیرید.

$$\mathcal{L}_{int} = -\frac{\lambda}{\Lambda}\Phi^\dagger\Delta_i\Delta_j\Phi\,\varepsilon_{ij}$$

Quantum chromodynamics

SU(3) color symmetry

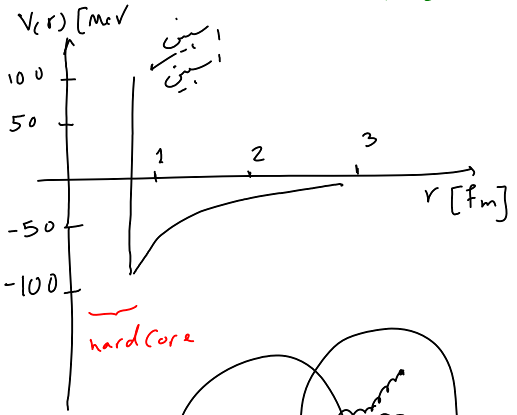
Yukawa ← 1930

idea of strong interaction

π - exchange

Particles & Nuclei — فصل ۱۶

Povh . Rith . Scholz . Zetsche

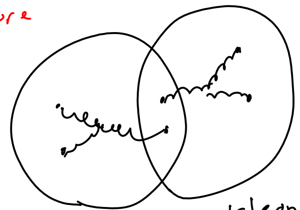


پروتون ← پروتون

پتانسیل نوکلئون نوکلئون

interaction

$E <$ few $\times 100$ MeV



→ شبیه نیروی کووالانسی در H_2

chromomagnetism

نیروی دافعه :

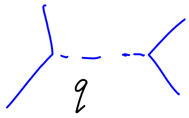
سنگین‌تر → $350 \ \frac{MeV}{c^2}$ $u^{\uparrow} u^{\uparrow} d^{\uparrow}$ Δ^{+}

Yukawa → تبادل پایون

$$\frac{e^{-mr}}{r}$$

پیش‌بینی جرم π : مرا با π عوض کردند.

$\delta E \ll 100$ MeV



$q \ll 100$ MeV

تبادل گلئون چه؟!

Dynamics of the SM — ص ۹

J. Donoghue, Golowich & B. Holstein

در حد تقارن ایزواسپین

$$\mathcal{L} = \bar\psi(i\not\partial - m)\psi + \frac{1}{2}\left[\partial_\mu \pi \cdot \partial^\mu \pi - m_\pi^2\, \pi\cdot\pi\right]$$
$$+ ig\, \bar\psi\, \tau\cdot\pi\, \gamma_5 \psi - \frac{\lambda}{4}(\pi\cdot\pi)^2$$

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \qquad \pi = \{\pi^i\}$$

$$m = \begin{bmatrix} m & \\ & m \end{bmatrix}$$

نکات

نکته‌ی اول: λ و g در اینجا λ و g قبلی نیستند.

نکته‌ی دوم: γ_5 آنجا چه می‌کند؟

نکته‌ی سوم:

$$\psi \xrightarrow{SU(2)\text{ of isospin}} U\psi$$

$$\vec\tau\cdot\vec\pi \longrightarrow U\vec\tau\cdot\vec\pi U^\dagger$$

$$\tau\cdot\pi = \begin{bmatrix} \pi^0 & \pi^+\sqrt2 \\ \sqrt2\pi^- & -\pi^0 \end{bmatrix} \qquad \pi^+ = \left(\frac{\pi^1 - i\pi^2}{\sqrt2}\right)$$
$$\pi^- = \frac{\pi^1 + i\pi^2}{\sqrt2}$$

$$\bar\psi\, \vec\pi\cdot\vec\tau\, \gamma_5 \psi = \pi^0(\bar p\gamma_5 p - \bar n\gamma_5 n)$$
$$+ \sqrt2\, \pi^+\, \bar p\gamma_5 n + \sqrt2\, \pi^-\, \bar n\gamma_5 p$$

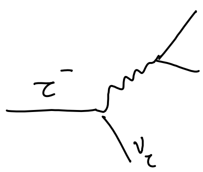
Quantum Chromodynamics

$$g_s \frac{\lambda^i}{2} \qquad \lambda^i \leftarrow \text{Gell-Mann}$$

$$SU(3)\times SU(2)\times U(1) \longrightarrow SU(3)\times U(1)_{em}$$

شاهد برای رنگ

$\Delta^{++}(1232)$



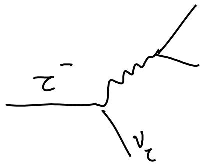
$e^-\,\bar\nu_e\,,\ |\ \mu^-\,\bar\nu_\mu\ |\ d\,\bar u\,,\ |\ s\,\bar u$

$B = Br(\tau^- \to e^-\bar\nu_e\nu_\tau)$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\quad}$$

$\Delta^{++}(1232)$

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

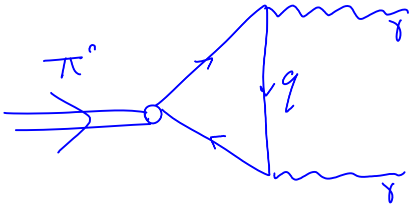


$e^-\ \bar{\nu}_e,\ \Big|\ \mu^-\ \bar{\nu}_\mu\ \Big|\ d\ \bar{u},\ \Big|\ s\ \bar{u}$

$B = Br(\tau^- \to e^- \bar{\nu}_e \nu_\tau)$

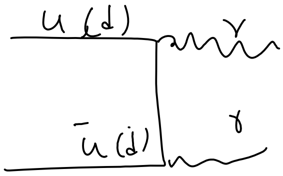
$B = \frac{1}{3}$ ← ب زک

$B = \frac{1}{5}$



$$\Gamma(\pi^0 \to \gamma\gamma) = N_c^2 (e_u^2 - e_d^2)^2 \left(\frac{\alpha}{\pi}\right)^2 \frac{m_\pi^3}{32\pi f_\pi}$$

$$f_\pi \simeq 130 \text{ MeV}$$



???

$\pi\, \cancel{\partial_{\mu\nu}}\, F^{\mu\nu}$ $\quad \pi^0 \varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$

$$\langle 0 | \bar{u}\gamma^\mu u \ \bar{u}\gamma^\nu u | \pi^0 \rangle A_\mu A_\nu = 0$$

$$\langle 0 | \bar{u}\gamma^\mu \gamma^5 u | \pi^0 \rangle \neq 0$$

لاگرانژی موثر $\quad \pi^0 \varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$

QCD Lagrangian and strength of color forces

$$\mathcal{L} = \bar{q}(i\cancel{D} - m)q - \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu}$$

$$q = \begin{pmatrix} q^R \\ q^G \\ q^B \end{pmatrix} \qquad D_\mu = \partial_\mu - i g_s \frac{\lambda^i A^i_\mu}{2}$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_s f_{ijk} A^j_\mu A^k_\nu$$

$$\mathcal{L}_{int} = g_s \, \bar{q} \gamma_\mu \frac{\lambda^i}{2} q \, A^{i\mu} =$$

$$\frac{g_s}{\sqrt{2}} \Big\{ \Big(\bar{q}^R \gamma^\mu q^G G^1_\mu + \bar{q}^G \gamma^\mu q^R \, G^2_\mu$$

$$+ \frac{1}{\sqrt{2}} \left(\bar{q}^R \gamma^\mu q^R - \bar{q}^G \gamma^\mu q^G \right) G^3_\mu$$

$$+ \bar{q}^R \gamma^\mu q^B \, G^4_\mu + \bar{q}^B \gamma^\mu q^R \, G^5_\mu +$$

$$\bar{q}^G \gamma^\mu q^B \, G^6_\mu + \bar{q}^B \gamma^\mu q^G \, G^7_\mu +$$

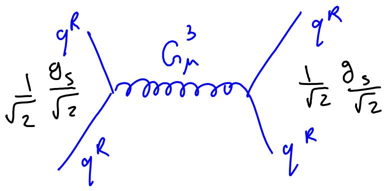
$$\frac{1}{\sqrt{6}} \left(\bar{q}^R \gamma^\mu q^R + \bar{q}^G \gamma^\mu q^G - 2 \bar{q}^B \gamma^\mu q^B \right) G^8_\mu \Big\}$$

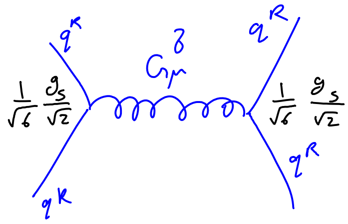
$$G^1_\mu = \frac{A^1_\mu - i A^2_\mu}{\sqrt{2}} \qquad G^2_\mu = \frac{A^1_\mu + i A^2_\mu}{\sqrt{2}} \qquad G^3_\mu = A^3_\mu$$

$$G^4_\mu = \frac{A^4_\mu - i A^5_\mu}{\sqrt{2}} \qquad G^5_\mu = \frac{\left(A^4_\mu + i A^5_\mu\right)}{\sqrt{2}}$$

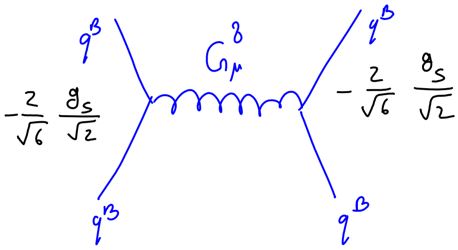
$$G^6_\mu = \frac{A^6_\mu - i A^7_\mu}{\sqrt{2}} \qquad G^7_\mu = \frac{A^6_\mu + i A^7_\mu}{\sqrt{2}} \qquad G^8_\mu = A^8_\mu$$

$$\frac{1}{\sqrt{3}} \left(\bar{R}R + \bar{G}G + \bar{B}B \right)$$



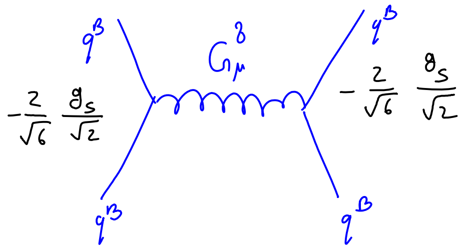


$$\left(\frac{1}{\sqrt{2}} \frac{g_s}{\sqrt{2}} \right)^2 + \left(\frac{1}{\sqrt{6}} \frac{g_s}{\sqrt{2}} \right)^2 = \frac{2}{3} \left(\frac{g_s}{\sqrt{2}} \right)^2$$

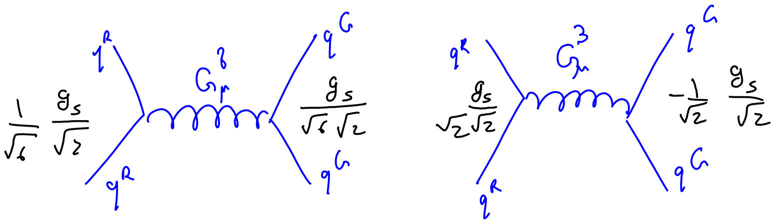


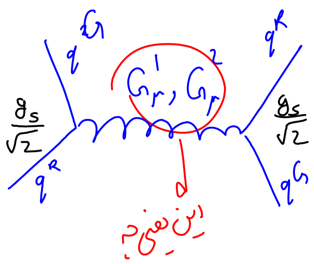
$$\left(-\frac{2}{\sqrt{6}} \frac{g_s}{\sqrt{2}} \right)^2 = \frac{2}{3} \left(\frac{g_s}{\sqrt{2}} \right)^2$$

$$\left(\frac{1}{\sqrt{2}}\frac{g_s}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{6}}\frac{g_s}{\sqrt{2}}\right)^2 = \frac{2}{3}\left(\frac{g_s}{\sqrt{2}}\right)^2$$



$$\left(-\frac{2}{\sqrt{6}}\frac{g_s}{\sqrt{2}}\right)^2 = \frac{2}{3}\left(\frac{g_s}{\sqrt{2}}\right)^2$$





$$\begin{cases} G_\mu^1 = \dfrac{A_\mu^1 - iA_\mu^2}{\sqrt{2}} \\ G_\mu^2 = \dfrac{A_\mu^1 + iA_\mu^2}{\sqrt{2}} \end{cases}$$

$$\mathcal{L}_{int} = \frac{g_s}{\sqrt{2}}\left\{ \bar{q}^R \gamma^\mu q^G G_\mu^1 + \bar{q}^G \gamma^\mu q^R G_\mu^2 + \dots \right\}$$

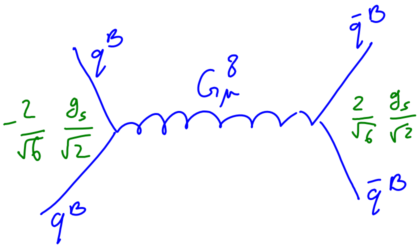
$$\left(\frac{1}{\sqrt{6}}\frac{g_s}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\frac{g_s}{\sqrt{2}}\right)\left(-\frac{1}{\sqrt{2}}\frac{g_s}{\sqrt{2}}\right) + \left(\frac{g_s}{\sqrt{2}}\right)^2 = \frac{2}{3}\left(\frac{g_s}{\sqrt{2}}\right)^2$$

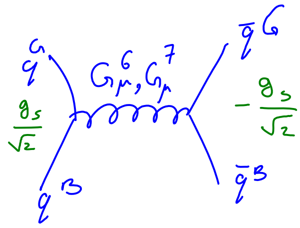
چرا این ضریب ۲ نداره؟

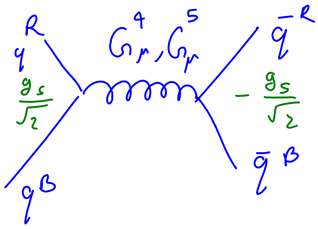
$$\langle G_\mu^1 \; G_\mu^2 \rangle = ? \qquad \langle G_\mu^1 \; G_\mu^4 \rangle = ?$$

Color singlet

$$\frac{1}{\sqrt{3}}\left(\bar{R}R + \bar{G}G + \bar{B}B\right)$$







$$\left(\frac{2}{\sqrt{6}}\frac{g_s}{\sqrt{2}}\right)\cdot\left(-\frac{2}{\sqrt{6}}\frac{g_s}{\sqrt{2}}\right) + \left(\frac{g_s}{\sqrt{2}}\right)\cdot\left(\frac{-g_s}{\sqrt{2}}\right) + \frac{g_s}{\sqrt{2}}\cdot\left(-\frac{g_s}{\sqrt{2}}\right) = -\frac{4}{3}\left(\frac{g_s}{\sqrt{2}}\right)^2$$

$$\langle q_i \cdots | \bar{\psi}_i \gamma^\mu \psi_j | q_j \cdots \rangle$$

$$\langle \bar{q}_j \quad | \underbrace{\bar{\psi}_i \gamma^\mu \psi_j}_{-1} | \bar{q}_i \cdots \rangle$$

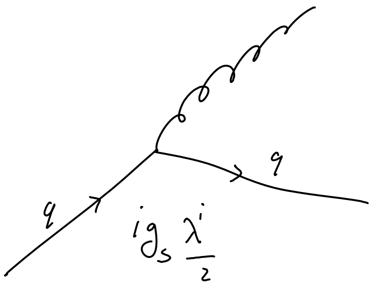
عجب !!

۱) جمع دامنه‌ی حالت‌های نهایی متفاوت؟

۲) s-channel چی شد؟

$$\left\langle \frac{\bar{B}B + \bar{G}G + \bar{R}R}{\sqrt{2}} \right| g_s \bar{q} \gamma_\mu \frac{\lambda^i}{2} q A^i_\mu \left| \frac{\bar{B}B + \bar{G}G + \bar{R}R}{\sqrt{3}} \right\rangle$$ در واقع

$$3 \times \left(\frac{1}{\sqrt{3}}\right)^2 = 1$$



$i g_s \frac{\lambda^i}{2}$

$$M = g_s \; \bar{u}^b \gamma^\mu \left(\frac{\lambda^i}{2}\right)_{ba} u^a A^i_\mu$$

$$\lambda^1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \sum_{i,j} Tr\left[\frac{\lambda^i}{2}\frac{\lambda^j}{2}\right] = \sum_{i,j=1}^{8} \frac{\delta^{ij}}{2} = 4$$

جمع روی
رنگ نهایی

$$\frac{g_s^2}{3} Tr\left[\frac{\lambda^i}{2}\frac{\lambda^i}{2}\right) = \frac{4}{3} g_s^2$$