

$$P^2 \psi_{p_1 \sigma_1 n_1; p_2 \sigma_2 n_2; \dots} = \eta_{n_1}^2 \eta_{n_2}^2 \dots \psi_{p_1 \sigma_1 n_1; p_2 \sigma_2 n_2}$$

$$I_p^2 P^2 = 1$$

تعلق داخلی تازه؟  
الریضی از تقارن پیوسته سیستم باشد.

$$e^{i(\alpha B + \beta L + \gamma Q)}$$

$$P' \equiv P I_p \quad P'^2 = 1$$

سؤال ۵

$$\Delta \rightarrow e^{i\beta} \Delta$$

تحت U(1)

$$\Delta \rightarrow e^{i\vec{2} \cdot \vec{T}} \Delta$$

تحت SU(2)

$$\left[ \frac{T^i}{2}, \frac{T^j}{2} \right] = i \epsilon^{ijk} \frac{T^k}{2}$$

$$T^1 = -i \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$T^2 = -i \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$T^3 = -i \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[T^1, T^2] = i T^3$$

امتحان

$$D_\mu = \partial_\mu - i g \vec{A}_\mu \cdot \vec{T} - i g' \frac{Y}{2} B_\mu$$

$$\begin{cases} A_\mu^3 = \cos \theta_w Z_\mu + \sin \theta_w A_\mu \\ B_\mu = -\sin \theta_w Z_\mu + \cos \theta_w A_\mu \end{cases}$$

$$i \vec{\Delta} \cdot \vec{D}_\mu \Delta = \dots + A_\mu^3 \bar{\Delta} \gamma^\mu \begin{bmatrix} g' \frac{Y}{2} \cos \theta_w & -i g \sin \theta_w & 0 \\ i g \sin \theta_w & g' \frac{Y}{2} \cos \theta_w & 0 \\ \dots & \dots & \dots \end{bmatrix} \Delta$$

$$L \quad 0 \quad 0 \quad \left. \begin{matrix} g \gamma \cos \theta_w \\ \end{matrix} \right\}$$

$$g \cos \theta_w = g \sin \theta_w = e$$

$$\Delta \rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \Delta \rightarrow \begin{matrix} \text{بار } \Delta^3 = \frac{Y}{2} \\ \text{بار } \frac{\Delta^1 + i\Delta^2}{\sqrt{2}} = 1 + \frac{Y}{2} \\ \text{بار } \frac{i\Delta^1 + \Delta^2}{\sqrt{2}} = -1 + \frac{Y}{2} \end{matrix}$$

$$\Delta' = \begin{bmatrix} \Delta^3 & \frac{\Delta^1 - i\Delta^2}{\sqrt{2}} \\ \frac{\Delta^1 + i\Delta^2}{\sqrt{2}} & -\Delta^3 \end{bmatrix}$$

$$A = B = \Delta^3 \quad n_1 = 1 = -n_2 \quad n_3 = \sqrt{2} \quad n_4 = \sqrt{2}$$

$$\text{بار } C \rightarrow \frac{Y}{2} - 1$$

$$\text{بار } D \rightarrow \frac{Y}{2} + 1$$

$$\Delta^i \rightarrow (e^{i T^k_{\alpha k}} \Delta)^i = (1 + i T^k_{\alpha k})^j_{ij} \Delta^j$$

$$= \Delta^i + \varepsilon^{kij} \Delta^j \alpha^k =$$

$$\Delta^i \tau^i \rightarrow \Delta^i \tau^i + \frac{\varepsilon^{kij} \tau^i \Delta^j \alpha^k}{-\varepsilon^{kji} \tau^i}$$

$$i \left[ \frac{\vec{\tau} \cdot \vec{\alpha}}{2}, \Delta \right]$$

$$\Delta^i \tau^i \rightarrow \Delta^i \tau^i + i \left[ \frac{\vec{\tau} \cdot \vec{\alpha}}{2}, \Delta \right]$$

\* جرم‌های سولف‌های مختلف  $\Delta$  را بدست آورید. برای این کار

باید عمل جنبشی را با نرمالیزاسیون صحیح بنویسید. ضرب جمدی به عمل است

با  $\Phi$  رابطه درست زیر در نظر بگیرید.

$$\mathcal{L}_{int} = -\frac{\lambda}{\Lambda} \Phi^\dagger \Delta_i \Delta_j \Phi \varepsilon_{ij}$$

# Quantum chromodynamics

## SU(3) color symmetry

Yukawa ← 1930

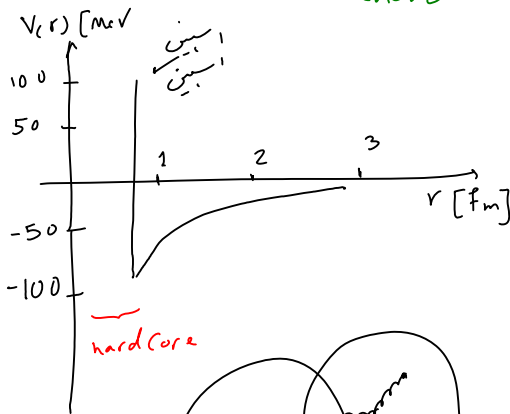
idea of strong interaction

$\pi$  - exchange

Particles & Nuclei

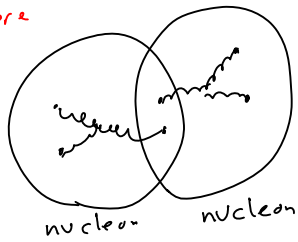
فضل ۱۶

Povh · Rith · Scholz · Zetsche



بروتون  
نیوترون  
interaction

$E < \text{few } \times 100 \text{ MeV}$



شبه نیروی  
کووالانسی  
H<sub>2</sub>

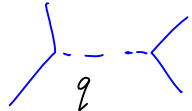
Chromomagnetism  
→ 350  $\frac{\text{MeV}}{c^2}$

نیروی دافعه:  
↑↑↑  
uud ↑ Δ<sup>+</sup>

Yukawa → تبادل بايون

$$\frac{e^{-mr}}{r}$$

پيون بي جرم π : m را با π عوضی دهند.

$\Delta E \ll 100 \text{ MeV}$    $g \ll 100 \text{ MeV}$

تبادل گلفون P, g

Dynamics of the SM

۹

J. Donoghue, Golowich & B. Holstein

دسته‌تارن ایزواسپین

$$\mathcal{L} = \bar{\Psi} (i \not{\partial} - m) \Psi + \frac{1}{2} [\partial_\mu \pi \cdot \partial^\mu \pi - m_\pi^2 \pi \cdot \pi] + i g \bar{\Psi} \tau \cdot \pi \gamma_5 \Psi - \frac{\lambda}{4} (\pi \cdot \pi)^2$$

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad \pi = \{ \pi^i \}$$

$$m = \begin{bmatrix} m & \\ & m \end{bmatrix}$$

نکات  
نکته‌ی اول:  $\lambda, g$  در اینجا  $g$  و  $\lambda$  قبل از  $g$  می‌باشد.

نکته‌ی دوم:  $\gamma_5$  انجام می‌دهد.

نکته‌ی سوم:

$$\Psi \xrightarrow{SU(2) \text{ of isospin}} U \Psi$$

$$\vec{\pi} \cdot \pi \xrightarrow{} U \vec{\pi} \cdot \pi U^\dagger$$

$$\tau \cdot \pi = \begin{bmatrix} \pi^0 & \pi^+ \sqrt{2} \\ \sqrt{2} \pi^- & -\pi^0 \end{bmatrix} \quad \pi^+ = \frac{\pi^1 - i \pi^2}{\sqrt{2}}$$

$$\pi^- = \frac{\pi^1 + i \pi^2}{\sqrt{2}}$$

$$\bar{\Psi} \vec{\pi} \cdot \vec{\tau} \gamma_5 \Psi = \pi^0 (\bar{p} \gamma_5 p - \bar{n} \gamma_5 n)$$

$$+ \sqrt{2} \pi^+ \bar{p} \gamma_5 n + \sqrt{2} \pi^- \bar{n} \gamma_5 p$$

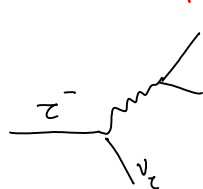
Quantum Chromodynamics

$$g_s \frac{\lambda^i}{2} \quad \lambda^i \leftarrow \text{Gell-Mann}$$

$$SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)_{em}$$

شاهد برای رزنگ

$$\Delta^{++} (1232)$$



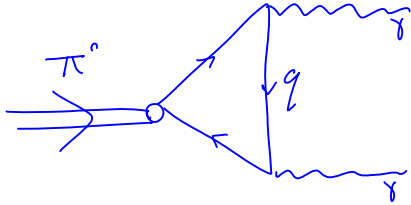
$$\bar{\nu}_e, \quad \nu_\tau, \quad d, \quad s$$

$$B = B_r (\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$$

$$R = \sigma(e^+ e^- \rightarrow \text{hadrons})$$

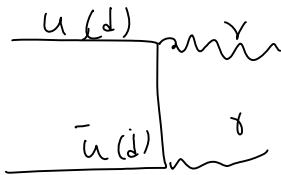
$$\Delta^{++} (1232)$$

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



$$\Gamma(\pi^0 \rightarrow \gamma\gamma) = N_c^2 (e_u^2 - e_d^2)^2 \left(\frac{\alpha}{\pi}\right)^2 \frac{m_\pi^3}{32\pi f_\pi^2}$$

$$f_\pi = 130 \text{ MeV}$$



???

$$\pi \not\sim F^{\mu\nu}$$

$$\pi^0 \sim \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$$

$$\langle 0 | \bar{u} \gamma^\mu u \bar{u} \gamma^\nu u | \pi^0 \rangle A_\mu A_\nu = 0$$

$$\langle 0 | \bar{u} \gamma^{\mu 5} u | \pi^0 \rangle \neq 0$$

لا لائزگی سوئی

$$\pi^0 \sim \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta}$$

## QCD Lagrangian and strength of color forces

$$\mathcal{L} = \bar{q} (i\not{D} - m)q - \frac{1}{4} F_{\mu\nu}^i F^{i\mu\nu}$$

$$q = \begin{pmatrix} q^R \\ q^G \\ q^B \end{pmatrix} \quad D_\mu = \partial_\mu - ig_s \frac{\lambda^i A_\mu^i}{2}$$

$$F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g_s f_{ijk} A_\mu^j A_\nu^k$$

$$L_{int} = g_s \bar{q} \gamma_r \lambda^i q A^{ir} =$$

$$\frac{g_s}{\sqrt{2}} \left\{ \left( \bar{q}^R \gamma^r q^G G_r^1 + \bar{q}^G \gamma^r q^R G_r^2 \right) \right. \\ \left. + \frac{1}{\sqrt{2}} \left( \bar{q}^R \gamma^r q^R - \bar{q}^G \gamma^r q^G \right) G_r^3 \right. \\ \left. + \bar{q}^R \gamma^r q^B G_r^4 + \bar{q}^B \gamma^r q^R G_r^5 + \right. \\ \left. \bar{q}^G \gamma^r q^B G_r^6 + \bar{q}^B \gamma^r q^G G_r^7 \right. \\ \left. + \frac{1}{\sqrt{6}} \left( \bar{q}^R \gamma^r q^R + \bar{q}^G \gamma^r q^G - 2 \bar{q}^B \gamma^r q^B \right) G_r^8 \right\}$$

$$G_r^1 = \frac{A_r^1 - i A_r^2}{\sqrt{2}}$$

$$G_r^2 = \frac{A_r^1 + i A_r^2}{\sqrt{2}} \quad G_r^3 = A_r^3$$

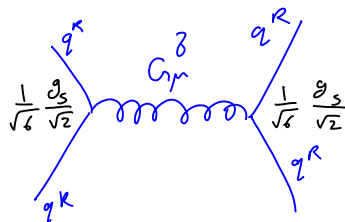
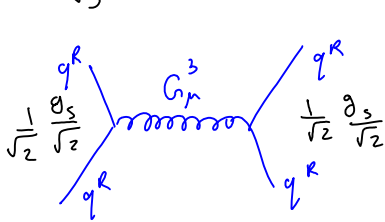
$$G_r^4 = \frac{A_r^4 - i A_r^5}{\sqrt{2}}$$

$$G_r^5 = \frac{A_r^4 + i A_r^5}{\sqrt{2}}$$

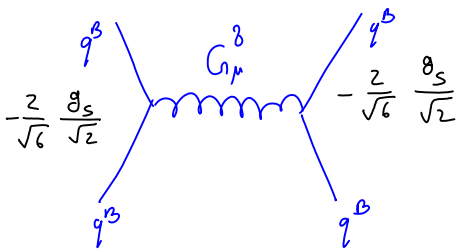
$$G_r^6 = \frac{A_r^6 - i A_r^7}{\sqrt{2}}$$

$$G_r^7 = \frac{A_r^6 + i A_r^7}{\sqrt{2}} \quad G_r^8 = A_r^8$$

$$\frac{1}{\sqrt{3}} (\bar{R}R + \bar{G}G + \bar{B}B)$$

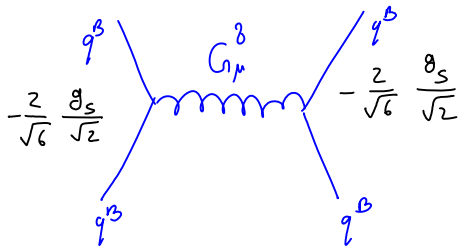


$$\left( \frac{1}{\sqrt{2}} \frac{g_s}{\sqrt{2}} \right)^2 + \left( \frac{1}{\sqrt{6}} \frac{g_s}{\sqrt{2}} \right)^2 = \frac{2}{3} \left( \frac{g_s}{\sqrt{2}} \right)^2$$

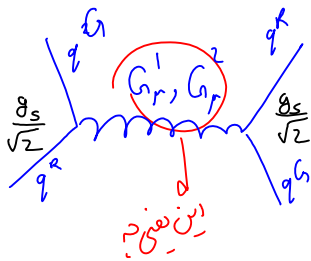
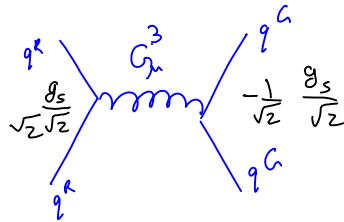
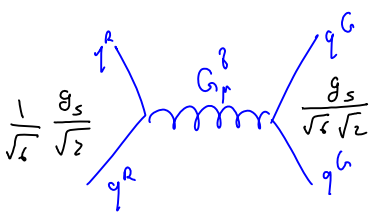


$$\left( -\frac{2}{\sqrt{6}} \frac{g_s}{\sqrt{2}} \right)^2 = \frac{2}{3} \left( \frac{g_s}{\sqrt{2}} \right)^2$$

$$\left(\frac{1}{\sqrt{2}} \frac{g_s}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{6}} \frac{g_s}{\sqrt{2}}\right)^2 = \frac{2}{3} \left(\frac{g_s}{\sqrt{2}}\right)^2$$



$$\left(-\frac{2}{\sqrt{6}} \frac{g_s}{\sqrt{2}}\right)^2 = \frac{2}{3} \left(\frac{g_s}{\sqrt{2}}\right)^2$$



$$\begin{cases} G_\mu^1 = \frac{A_\mu^1 - iA_\mu^2}{\sqrt{2}} \\ G_\mu^2 = \frac{A_\mu^1 + iA_\mu^2}{\sqrt{2}} \end{cases}$$

$$L_{int} = \frac{g_s}{\sqrt{2}} \left\{ \bar{q}^R \gamma^\mu q^G G_\mu^1 + \bar{q}^G \gamma^\mu q^K G_\mu^2 + \dots \right\}$$

$$\left(\frac{1}{\sqrt{6}} \frac{g_s}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}} \frac{g_s}{\sqrt{2}}\right) \left(-\frac{1}{\sqrt{2}} \frac{g_s}{\sqrt{2}}\right) + \left(\frac{g_s}{\sqrt{2}}\right)^2 = \frac{2}{3} \left(\frac{g_s}{\sqrt{2}}\right)^2$$

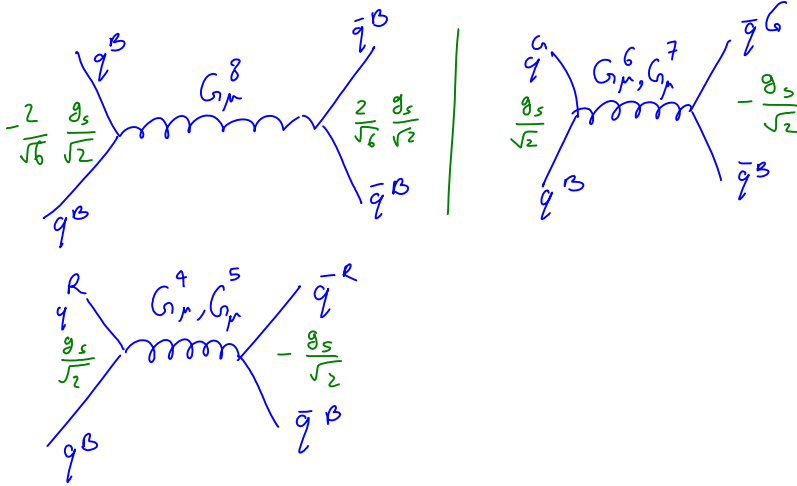
جہاں سے ضرب آتا ہے؟

$$\langle G_\mu^1 G_\mu^2 \rangle = ?$$

$$\langle G_\mu^1 G_\mu^4 \rangle = ?$$

Color singlet

$$\frac{1}{\sqrt{3}} (\bar{R}R + \bar{G}G + \bar{B}B)$$



$$\left(\frac{2}{\sqrt{6}} \frac{g_s}{\sqrt{2}}\right) \cdot \left(-\frac{2}{\sqrt{6}} \frac{g_s}{\sqrt{2}}\right) + \left(\frac{g_s}{\sqrt{2}}\right) \cdot \left(-\frac{g_s}{\sqrt{2}}\right) + \frac{g_s}{\sqrt{2}} \cdot \left(-\frac{g_s}{\sqrt{2}}\right) = -\frac{4}{3} \left(\frac{g_s}{\sqrt{2}}\right)^2$$

$$\langle q_i \dots | \bar{\psi}_i \gamma^m \psi_j | q_j \dots \rangle$$

$$\langle \bar{q}_j | \underbrace{\bar{\psi}_i \gamma^m \psi_j}_{-1} | q_i \dots \rangle$$

عب

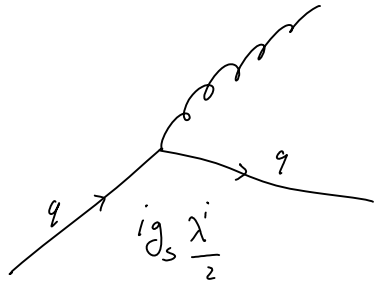
(1) جمع دلہنے کی حالت کی پہلی سفارت؟

(2) s-channel کی سفارت؟

$$\left\langle \frac{\bar{B}B + \bar{G}G + \bar{R}R}{\sqrt{2}} \left| g_s \bar{q} \gamma_\mu \frac{\lambda^i}{2} q A_\mu^i \right| \frac{\bar{B}B + \bar{G}G + \bar{R}R}{\sqrt{3}} \right\rangle \text{ درجہ}$$

$$3 \times \left(\frac{1}{\sqrt{3}}\right)^2 = 1$$





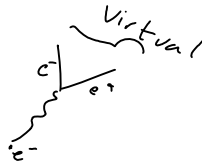
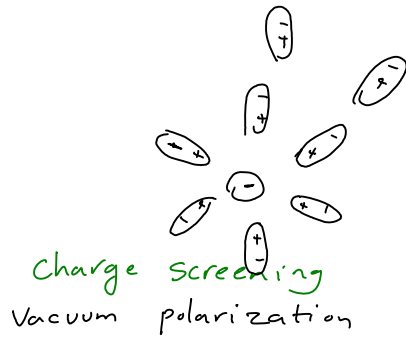
$$M = g_s \bar{u}^b \gamma^\mu \left( \frac{\lambda^i}{2} \right)_{ba} u^a A_\mu^i$$

$$\lambda^1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \sum_{i,j} \text{Tr} \left[ \frac{\lambda^i}{2} \frac{\lambda^j}{2} \right] = \sum_{i,j=1}^8 \frac{\delta^{ij}}{2} = 4$$

دری  
رندگی

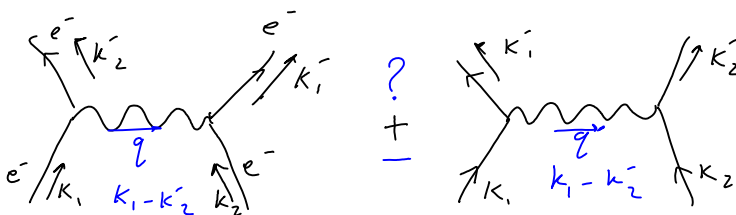
$$\frac{g_s^2}{3} \text{Tr} \left[ \frac{\lambda^i}{2} \frac{\lambda^i}{2} \right] = \frac{4}{3} g_s^2$$

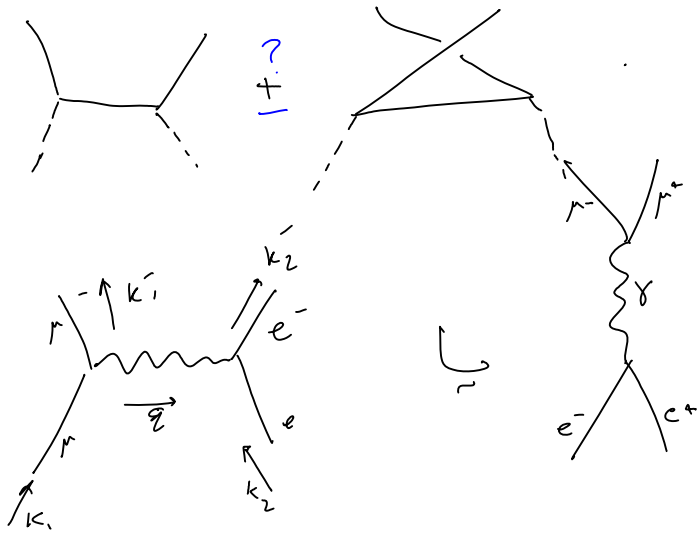
## Running Coupling Constant



$$\alpha = \frac{e^2}{4\pi} \quad \alpha(Q^2)$$

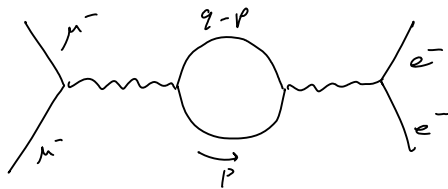
$$\bar{e}(k_1) e(k_2) \rightarrow \bar{e}(k_1') + \bar{e}(k_2')$$





$$q = k_1 - k_1'$$

$$-iM = [ie \bar{u}(k_1') \gamma^\mu u(k_1)] \frac{-i g_{\mu\nu}}{q^2} [ie \bar{u}(k_2) \gamma^\nu u(k_2)]$$

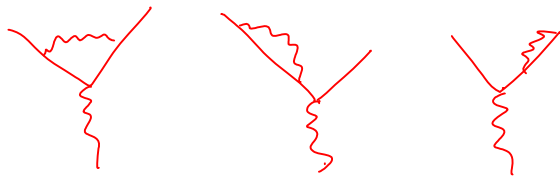


$$-iM = (-1) [ie \bar{u}(k_1) \gamma^\mu u(k_2)] \frac{-i g_{\mu\nu}}{q^2}$$

$$\frac{1}{(2\pi)^4} \int d^4p \text{Tr} [(ie \gamma^\mu) \frac{i(\not{p} + m)}{p^2 - m^2} (ie \gamma^\nu)]$$

$$\left[ \frac{i(\not{q} - \not{p} + m)}{(p-q)^2 - m^2} \right] \times \frac{-ig_{\lambda\nu}}{q^2} [ie \bar{u}(k_2) \gamma^\lambda u(k_2)]$$

از کجا آمد؟ -1



Cancel  $\rightarrow$  all orders of perturbation

Ward-Takahashi

اتحاد

$$\frac{-i g_{\mu\nu}}{q^2} \longrightarrow \frac{-i g_{\mu\nu}}{q^2} + \frac{-i g_{\mu\nu}}{q^2} I^{\mu\nu} \frac{-i g_{\lambda\nu}}{q^2}$$

$$= \frac{-i g_{\mu\nu}}{q^2} + \frac{-i}{q^2} I_{\mu\nu} \frac{-i}{q^2}$$

$$I_{\mu\nu} = \frac{-1}{(2\pi)^4} \int d^4 p \text{Tr} \left[ (-ie\gamma_\mu) \frac{i(\not{p}+m)}{p^2-m^2} (ie\gamma_\nu) \frac{i(\not{q}-\not{p}+m)}{(q-p)^2-m^2} \right]$$

$$I_{\mu\nu} = -i g_{\mu\nu} q^2 I(q^2) + \dots$$

↓  
g<sub>μ</sub>g<sub>ν</sub>

$$g_\mu \bar{u}(k_1) \gamma^\mu u(k_2) = 0$$

$$I(q^2) = \frac{\alpha}{3\pi} \int_0^1 \frac{dx}{x} \log \left( 1 - \frac{q^2 x(1-x)}{m^2} \right)$$

$$\frac{-i g_{\mu\nu}}{q^2} \longrightarrow \frac{-i g_{\mu\nu}}{q^2} (1 - I(q^2))$$

$$I(q^2) = \frac{\alpha}{3\pi} \log \frac{\Lambda^2}{m^2} - \frac{\alpha}{3\pi} \log \left( -\frac{q^2}{m^2} \right) = \frac{\alpha}{3\pi} \log \frac{\Lambda^2}{-q^2}$$

$$(-q^2) \gg m^2$$

-I                      I<sup>2</sup>                      -I<sup>3</sup>

$$\Rightarrow 1 - I + I^2 - I^3 + \dots = \frac{1}{1+I}$$

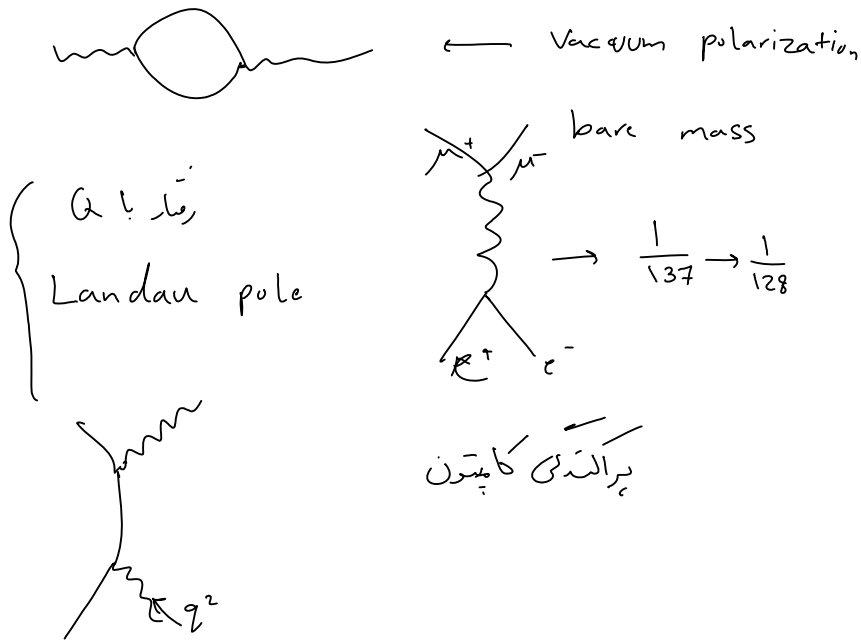
$$-i M = [ie \bar{u}(k_1) \gamma^\mu u(k_1)] \frac{-i g_{\mu\nu}}{q^2} \frac{1}{1 + \frac{\alpha}{3\pi} \log \frac{\Lambda^2}{-q^2}} [ie \bar{u}(k_2) \gamma_\nu u(k_2)]$$

$$\alpha_{\text{eff}} = \frac{\alpha}{1 + \frac{\alpha}{3\pi} \log \frac{\Lambda^2}{-q^2}} = \frac{1}{137} \quad q^2 \gg m^2$$

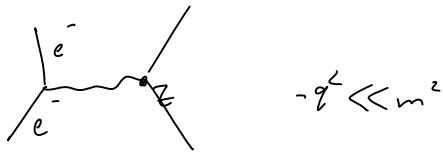
$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \log \frac{Q^2}{\mu^2}}$$

↪ renormalization scale

وجوبت



فصل برای فوتون مجازی



$$\log(1 - q^2 x(1-x)) \approx -\frac{q^2 x(1-x)}{m^2}$$

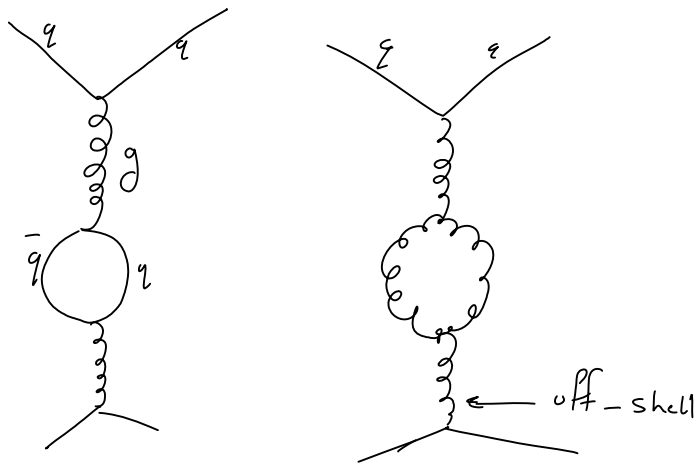
$$I(q^2) = \frac{\alpha}{3\pi} \log \frac{\Lambda^2}{m^2} + \frac{\alpha}{15\pi} \frac{q^2}{m^2}$$

$$-iM = [ie \bar{u}(k_1) \gamma^\mu u(k_1) \frac{-i g_{\mu\nu}}{q^2} \left[ 1 - \frac{\alpha}{3\pi} \log \frac{\Lambda^2}{m^2} - \frac{\alpha}{15\pi} \frac{q^2}{m^2} \right] (-i j^\nu)]$$

$$j^\nu = (j^0 = Ze, \vec{j} = 0)$$

$$-iM = [ie_R \bar{u}(k_1) \gamma_0 u(k_1)] \frac{-i}{q^2} (-iZe_R)$$

$$e_R = e \left[ 1 - \frac{\alpha}{3\pi} \log \frac{\Lambda^2}{m^2} \right]^{\frac{1}{2}}$$



$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{33 - 2n_f}{12\pi} \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2}}$$

Anti-screening

$$\beta \text{ Function} = \mu^2 \frac{d\alpha}{d\mu^2}$$

$$\Lambda_{\text{QCD}} = \mu^2 e^{\frac{-12\pi}{(33-2n_f)\alpha_s(\mu^2)}}$$

$0 = \frac{d}{d\mu^2} \ln \Lambda_{\text{QCD}}$

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2n_f) \log \frac{Q^2}{\Lambda_{\text{QCD}}^2}} \rightarrow$$

u d s c b  $\leftarrow$  
 $Q^2 = 100 \text{ GeV}^2 \quad \alpha_s \approx 0.2$   
 $\Lambda_{\text{QCD}} = 200 \text{ MeV}$

asymptotic freedom  $\xleftrightarrow{\Lambda_{\text{QCD}}}$  Confinement

