

U(1) gauge symmetry

$$L = \bar{\Psi}(x) (i \gamma^\mu \partial_\mu - m) \Psi(x)$$

$$\Psi(x) \rightarrow e^{-i\theta} \Psi(x) \quad (1 - i\theta) \Psi$$

$$\bar{\Psi}(x) \rightarrow \bar{\Psi}(x) e^{+i\theta}$$

$$j_\mu = \frac{\partial L}{\partial \partial^\mu \Psi} \overbrace{i\Psi}^{\delta\Psi} \quad j_\mu = \bar{\Psi} \gamma_\mu \Psi$$

$$\theta \rightarrow \theta(x) \quad \text{متغیر}$$

$$L \rightarrow L + j^\mu \partial_\mu \theta$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ie A_\mu \quad \text{مشتق عمود}$$

$$\Psi(x) \rightarrow e^{-i\theta} \Psi(x)$$

$$A_\mu \rightarrow A_\mu - \frac{1}{e} \partial_\mu \theta$$

$$\bar{\Psi} i \gamma^\mu D_\mu \Psi - \bar{\Psi} m \Psi$$

$$D_\mu \Psi \rightarrow e^{-i\theta} D_\mu \Psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\text{بنا} \quad [D_\mu, D_\nu] \Psi = -ie F_{\mu\nu} \Psi$$

~~$$m^2 A_\mu A^\mu$$~~

kinetic term

عقد همتراباينى بنيم

$$\mathcal{L}_{QED} = \bar{\Psi}(x) (i\gamma^\mu D_\mu - m) \Psi + \left(-\frac{1}{4}\right) F_{\mu\nu} F^{\mu\nu}$$

* معادلات ادير-لازارت ← معادلات السول

حلمه ي اخذ ۳.۲.۱

بگلتن يو كما ان نوع بيانى ست .

بگلتن اسكار

$$(\partial_\mu \varphi)^\dagger \partial^\mu \varphi$$

$$\varphi \rightarrow e^{-i\theta} \varphi$$

$$\partial^\mu \rightarrow D^\mu = \partial^\mu - ie A^\mu$$



$$(D_\mu \varphi)^\dagger D^\mu \varphi = (\partial_\mu \varphi^\dagger + ie A_\mu \varphi^\dagger) (\partial^\mu \varphi - ie A^\mu \varphi) =$$

$$(\partial_\mu \varphi)^\dagger \partial^\mu \varphi + eA^\mu i (\varphi^\dagger \partial_\mu \varphi - (\partial_\mu \varphi)^\dagger \varphi) + e^2 A^\mu A_\mu \varphi^\dagger \varphi$$

حد غير نسبتي

$$\psi(p) = \frac{1}{\sqrt{2E_p}} \sum_s (a_p^s u^s e^{-ip \cdot x} + a_p^{s\dagger} v^s e^{ip \cdot x})$$

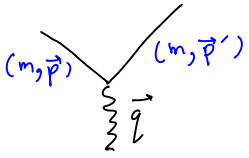
$$u = \begin{bmatrix} \sqrt{p \cdot \sigma} \chi \\ \sqrt{p \cdot \bar{\sigma}} \chi \end{bmatrix}$$

$$E \approx m \quad |\vec{p}| \ll m$$

حد غير نسبتي

$$\sqrt{p \cdot \sigma} = \sqrt{E - \vec{p} \cdot \vec{\sigma}} = \sqrt{m} \left(1 - \frac{\vec{\sigma} \cdot \vec{p}}{2m}\right)$$

$$\sqrt{p \cdot \bar{\sigma}} = \sqrt{E + \vec{p} \cdot \vec{\sigma}} = \sqrt{m} \left(1 + \frac{\vec{\sigma} \cdot \vec{p}}{2m}\right)$$



$$\bar{u}' \gamma^\mu u = 2m \chi^{\dagger} \cdot \chi$$

$$\begin{aligned} \bar{u}' \gamma^\mu u &= \left[\chi^{\dagger} \sqrt{p \cdot \sigma} \quad \chi^{\dagger} \sqrt{p' \cdot \sigma} \right] \begin{bmatrix} -\omega^i & 0 \\ 0 & \omega^i \end{bmatrix} \begin{bmatrix} \sqrt{p \cdot \sigma} \chi \\ \sqrt{p' \cdot \sigma} \chi \end{bmatrix} \\ &= m \left[\chi^{\dagger} \left(1 - \frac{\vec{\sigma} \cdot \vec{p}}{2m} \right) \omega^i \left(1 - \frac{\vec{\sigma} \cdot \vec{p}'}{2m} \right) \chi \right. \\ &\quad \left. - \chi^{\dagger} \left(1 + \frac{\vec{\sigma} \cdot \vec{p}}{2m} \right) \omega^i \left(1 + \frac{\vec{\sigma} \cdot \vec{p}'}{2m} \right) \chi \right] \end{aligned}$$

$$\vec{p}' = \langle \vec{p} \rangle + \frac{\vec{q}}{2}$$

$$\vec{p} = \langle \vec{p} \rangle - \frac{\vec{q}}{2}$$

$$\bar{u}' \gamma^\mu u = -2m \chi^{\dagger} \left[\frac{2 \langle \vec{p} \rangle^i}{2m} + \frac{2i \epsilon^{jik} \frac{q_j \sigma_k}{2}}{2m} \right] \chi$$

$$\Rightarrow A_i \bar{u}' \gamma^\mu u = -2\vec{A} \cdot \langle \vec{p} \rangle \chi^{\dagger} \chi - \vec{B} \cdot \chi^{\dagger} \vec{\sigma} \chi$$

$$A_\mu \bar{\Psi} \gamma^\mu \Psi = A_0 \chi^{\dagger} \chi + \frac{\vec{A} \cdot \langle \vec{p} \rangle}{m} \chi^{\dagger} \chi + \frac{\vec{B} \cdot \vec{\sigma}}{2m} \chi^{\dagger} \chi$$

$\chi' = \chi$

تجدید خاطرت

(الترنجان)

فصل ۵.۳ ساکوری

$$A = -\frac{1}{2} (B_y \hat{x} - B_x \hat{y})$$

از صیغ تحت نابداستفاد کنیم

$$\vec{A} \cdot \vec{p} = |\vec{B}| \left(-\frac{y}{2} p_x + \frac{x}{2} p_y \right)$$

$$\vec{A} \cdot \vec{p} = \frac{|\vec{B}| L_z}{2}$$

از صیغ صوح با بایستفاد کنیم

$$V = -e \bar{\Psi} \gamma^\mu \Psi A_\mu = \frac{-e B}{2m} (L + 2S)$$

$$g = 2 \leftarrow \text{tree level}$$

$$\mu = \frac{e g B}{2m} \int \quad : \text{ذره ی باردار با اسپین } \frac{1}{2}$$

$$\frac{e\hbar}{2m_e} = g_B \quad \text{مگنتون بور}$$

$$\frac{e\hbar}{2m_p} = g_N \quad \text{Nuclear magneton}$$

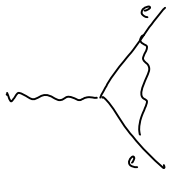
PDG 2006

در مورد الکترون $\mu_e = 1.0011596521859 \pm 0.0000000038 \mu_B$

PDG 2008

Magnetic moment anomaly

$$\frac{g-2}{2} = (1159.6521811 \pm 0.0000007) \times 10^{-6}$$



از جمله سبون

$$\frac{g-2}{2} = (11659208 \pm 6) \times 10^{-10} \quad \text{مشاهده پزیرگی ذرات$$

OLD SM $\rightarrow \mu_\nu = 0$ برای

$$\mu_\nu < 0.79 \times 10^{-10} \mu_B$$

$$\mu_{\pi^+} \quad \mu_{\pi^0}$$

Magnetic dipole

Proton

$$\left. \begin{aligned} \mu &= 2.792847351 \pm 0.00000028 \mu_N \\ \text{neutron} \\ \mu &= -1.9130427 \pm 0.0000005 \mu_N \end{aligned} \right\}$$

anomalous

$d \rightarrow 0$ در مورد همه ی این ذرات

$$\vec{S} \xrightarrow{T} -\vec{S}$$

$$\vec{S} \xrightarrow{T} -\vec{S}$$

$$\vec{B} \xrightarrow{T} -\vec{B}$$

$$\vec{E} \xrightarrow{T} +\vec{E}$$

$$d S \cdot \vec{E} + \mu \vec{S} \cdot \vec{B}$$



$$-d \vec{E} \cdot \vec{E} + \mu \vec{S} \cdot \vec{B}$$

$$d_e < 1.4 \times 10^{-27} \text{ e cm} \quad \leftarrow \text{PDG}$$

$$d_\mu < 1.9 \times 10^{-16} \text{ e cm} \quad \leftarrow \text{arXiv: 0811.1207}$$

$$e + \gamma \rightarrow e \gamma$$

برعکس تابش

$$e^- e^- \rightarrow e^- e^-$$

Moller scattering

$$e^- e^+ \rightarrow \gamma \gamma$$

pair annihilation

$$e^- e^+ \rightarrow e^- e^+$$

BHABHA scattering

$$\mathcal{L}(\varphi, \partial_\mu \varphi)$$

$$\varphi_a \rightarrow \varphi_a + \delta \varphi_a$$

$$\delta \mathcal{L} = \partial_\mu K^\mu = \frac{\partial \mathcal{L}}{\partial \varphi_a} \delta \varphi_a + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_a)} \delta \partial_\mu \varphi_a$$

$$= \left(\frac{\partial \mathcal{L}}{\partial \varphi_a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi_a} \right) \delta \varphi_a + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_a)} \delta \varphi_a \right) =$$

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi_a} \delta \varphi_a \right)$$

$$\text{چنانچه } \mathcal{J}^\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi_a} \delta \varphi_a - K^\mu$$

در بیشتر مواردی که در فیزیک ذرات ماسه بار داریم $K=0$ یعنی

تحت تبدیل نسبتاً کنش $(\int d^4x \mathcal{L})$ بلکه جهانی لادارشی

(یعنی \mathcal{L}) ناورداست .

مانسور انرژي - تکانه گسترده است و این سوکار دارد

یک استادت.

تبدیل
انتقال
فضا-زمان

$$x^\mu \rightarrow x^\mu + a^\mu \quad \varphi_a(x) \rightarrow \varphi_a(x) + a^\mu \partial_\mu \varphi_a(x)$$

$$L \rightarrow L + a^\mu \delta_\mu L \quad T^{\mu\nu} = \frac{\delta L}{\delta \partial_\mu \varphi} \partial_\nu \varphi - L \delta^\mu_\nu$$

SU(n)

$$\varphi_a \rightarrow \varphi_a + \delta \varphi_a$$

$$\delta \varphi_a = -i \theta^i T_{ab}^i \varphi_b \quad \varphi_a \rightarrow \varphi'_a = e^{-i \theta^i T_{ab}^i} \varphi_b$$

$$a, b = 1 \dots n$$

$$j^i(x) = -i \frac{\delta L}{\delta \partial_r \varphi_a} T_{ab}^i \varphi_b$$

$$Q^i(t) = \int d^3x j^{i0}(\vec{x}, t)$$

Canonical quantization

$$\pi_a = \frac{\delta L}{\delta (\partial_0 \varphi_a)}$$

$$[\varphi_a(\vec{x}, t), \varphi_b(\vec{y}, t)] = [\pi_a(\vec{x}, t), \pi_b(\vec{y}, t)] = 0$$

$$[\varphi_a(\vec{x}, t), \pi_b(\vec{y}, t)] = i \delta_{ab} \delta^3(\vec{x} - \vec{y})$$

استادانی کتاب رابط ۳۱۶

$$\partial_\mu j^{i0} = i [P_\mu, j^{i0}]$$

له عملرهار - تکار

$$\frac{dQ^i}{dt} = i [H, Q^i] = 0$$

$$[Q^i, Q^j] = i f_{ijk} Q^k$$

*

$$[Q^i, \varphi] = -T^i \varphi$$

*

$$\varphi \rightarrow \varphi' = e^{i\theta^i Q^i} \varphi e^{-i\theta^i Q^i} = e^{-i\theta^i T^i} \varphi$$

تئوری بیان‌های Yang-Mills

SU(2)

Yang Mills 1954

$$U(1) \rightarrow SU(2)$$

ایزاسپین ستاری است.

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}$$

$$\psi \rightarrow \psi'(x) = U \psi(x)$$

$$U = e^{-ig \frac{\tau^i}{2} \theta^i(x)}$$

$$\tau^i = \sigma^i$$

$$\tau^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \tau^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \tau^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

ایده‌ی اولیه Yang-Mills

کند و ملیز برای طرح کردن ایده‌ی اولیه‌ی خود از توانایی (P_n) استفاده کرد، بردند

$$D_\mu = \partial_\mu - ig \vec{A}_\mu$$

$$\vec{A}_\mu = \sum_{i=1}^3 \frac{\tau^i}{2} A_\mu^i = \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu$$

مشابه U(1) می‌خواهیم داشته باشیم

$$D_\mu \psi \rightarrow (D_\mu \psi)' = D_\mu' \psi' = U (D_\mu \psi)$$

در این صورت $\psi D_\mu \psi \rightarrow \psi' D_\mu' \psi'$ ندارد خواهد ماند.

آنتی‌تیب می‌دهد؟ $D_\mu \psi \neq \psi D_\mu$

$$(D_\mu \psi)' = D'_\mu U \psi = U (D_\mu \psi)$$

↓

$$D'_\mu = U D_\mu U^{-1}$$

$$\vec{A}'_\mu = U \vec{A}_\mu U^{-1} - \frac{i}{g} (\partial_\mu U) U^{-1}$$

تبدیل‌های بی‌نهایت کوچک

$$U = 1 - i\theta + \mathcal{O}(\theta^2)$$

$$U^{-1} = 1 + i\theta + \mathcal{O}(\theta^2)$$

$$\theta = \sum_{i=1}^3 \frac{\tau^i}{2} \theta^i = \frac{\vec{\tau} \cdot \vec{\theta}}{2}$$

$$\vec{A}'_\mu = \vec{A}_\mu - i [\theta, \vec{A}_\mu] - \frac{\partial_\mu \theta}{g}$$

$$\delta A_\mu^i = \varepsilon_{ijk} \theta^j A_\mu^k - \frac{\partial_\mu \theta^i}{g}$$

$$[D_\mu, D_\nu] \psi = -ig \vec{F}_{\mu\nu} \psi$$

$$\vec{F}_{\mu\nu} = \sum_{i=1}^3 \frac{\tau^i}{2} F_{\mu\nu}^i = \frac{\vec{\tau} \cdot \vec{F}_{\mu\nu}}{2}$$

$$F_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i + g \varepsilon_{ijk} A_\mu^j A_\nu^k$$

$$\vec{F}'_{\mu\nu} = U \vec{F}_{\mu\nu} U^{-1}$$

$$\text{Tr} [\vec{F}'_{\mu\nu} \cdot \vec{F}'^{\mu\nu}] = \text{Tr} [\vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu}]$$

$$-\frac{1}{2} \text{Tr} [\vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu}] = -\frac{1}{2} \sum_{i,j=1}^3 \text{Tr} \left[\frac{\tau^i}{2} F_{\mu\nu}^i \frac{\tau^j}{2} F^{\mu\nu j} \right]$$

$$= -\frac{1}{4} F_{\mu\nu}^i F^{\mu\nu i}$$

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_G$$

$$\mathcal{L}_F = \bar{\Psi} (i \not{D} - m) \Psi$$

$$\mathcal{L}_G = -\frac{1}{2} \text{Tr} [\vec{F}_{\mu\nu} \cdot \vec{F}^{\mu\nu}]$$

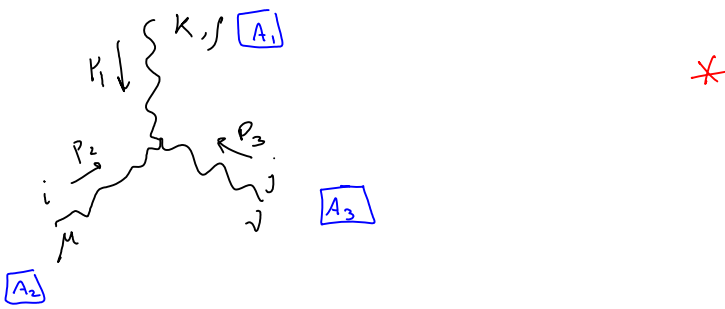
کلاسیکی

$$F_{\cdot i} = -E A^i \quad F^{0i} = E A^i$$

~~$$m A^i A^{i\mu}$$~~

$$\mathcal{L}_G = -\frac{1}{2} \partial_\mu A^i_\nu (\partial^\mu A^{i\nu} - \partial^\nu A^{i\mu})$$

$$-g \varepsilon_{ijk} A^i_\mu A^j_\nu \partial^\mu A^{k\nu} - \frac{g^2}{4} \varepsilon_{ijk} \varepsilon_{ilm} A^j_\mu A^k_\nu A^{l\mu} A^{m\nu}$$



$$f_{i,j,k} A^i_\mu A^j_\nu \partial^\mu A^{k\nu} \mid A_{1j,k} A_{2\mu,i} A_{3\nu,j} \rangle$$

$$= f_{i,j,k} A^i_\mu A^j_\nu \partial^\mu A^{k\nu} \mid A_{1j,k} A_{2\mu,i} A_{3\nu,j} \rangle$$

$$= \left(\begin{matrix} A_1 \leftrightarrow A_3 \\ P_3 \leftrightarrow P_1 \\ k \leftrightarrow j \end{matrix} \right) + \left(\begin{matrix} A_2 \leftrightarrow A_3 \\ P_3 \leftrightarrow P_2 \\ j \leftrightarrow i \end{matrix} \right)$$

$$i \left[P_3 \cdot A_1^k A_2^i \cdot A_3^j \quad f_{kij} \right. \\ + P_3 \cdot A_2^i A_1^k \cdot A_3^j \quad f_{ikj} \\ + P_1 \cdot A_3^j A_2^i \cdot A_1^k \quad f_{jik} \\ + P_1 \cdot A_2^i A_1^j \cdot A_3^k \quad f_{ij k} \\ \left. + P_1 \cdot A_1^k A_2^j \cdot A_3^i \quad f_{...} \right]$$

$$+ P_2 \cdot A_1^k \quad A_3^j \cdot A_2^i \quad f_{kji} + \\ P_2 \cdot A_3^j \quad A_1^k \cdot A_2^i \quad f_{jki}] = K$$

$$V = g f_{ijk} A_\mu^i A_\nu^j \delta^\mu A^{k\nu}$$

$$\text{پس} \quad -iV = g K$$

فقط بین چپ دست و راست دست نمی گذارد.

حجم A_μ^i داریم.

$$\mathcal{L} = (\partial_\mu \varphi)^\dagger \partial^\mu \varphi - V(\varphi^* \varphi)$$

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}} \quad \varphi^* = \frac{\varphi_1 - i\varphi_2}{\sqrt{2}}$$

$$V(\varphi^* \varphi) = m^2 \varphi^* \varphi + \lambda (\varphi^* \varphi)^2$$

$\lambda > 0 \rightarrow$ Unbounded from below

$$\langle 0 | \varphi_1 | 0 \rangle = \varphi_{01} \quad \langle 0 | \varphi_2 | 0 \rangle = \varphi_{02}$$

$$V(\varphi_1, \varphi_2) = V(\varphi_{01}, \varphi_{02}) + \sum_{\alpha=1,2} \left(\frac{\partial V}{\partial \varphi_\alpha} \right)_0 (\varphi_\alpha - \varphi_{0\alpha})$$

$$+ \frac{1}{2} \sum_{\substack{\alpha, \beta=1,2 \\ \alpha < \beta}} \left(\frac{\partial^2 V}{\partial \varphi_\alpha \partial \varphi_\beta} \right) (\varphi_\alpha - \varphi_{0\alpha}) (\varphi_\beta - \varphi_{0\beta}) + \dots$$

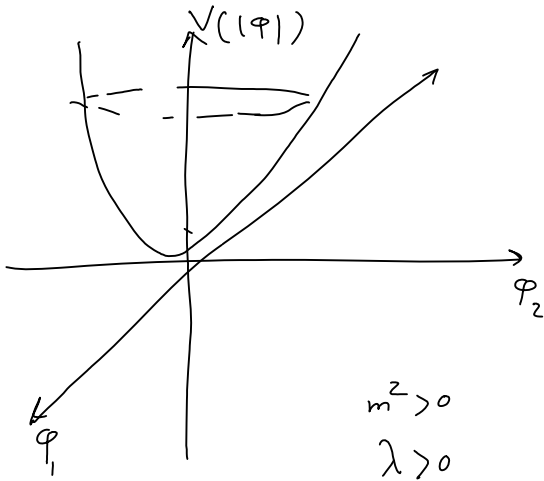
$$\langle \varphi \rangle = (\varphi_{01}, \varphi_{02}) \quad \rightarrow \quad V$$

صفر می شود.

$$m_{\alpha\beta}^2 = \left(\frac{\partial^2 V}{\partial \varphi_\alpha \partial \varphi_\beta} \right)_0$$

Wigner phase

فاز ویگنر



$$\left(\frac{\partial V}{\partial \varphi_1}\right)_0 = m^2 \varphi_{01} + \lambda \varphi_{01} (\varphi_{01}^2 + \varphi_{02}^2) = 0$$

$$\left(\frac{\partial V}{\partial \varphi_2}\right)_0 = m^2 \varphi_{02} + \lambda \varphi_{02} (\varphi_{01}^2 + \varphi_{02}^2) = 0$$

$$\varphi_{01} = \varphi_{02} = 0$$

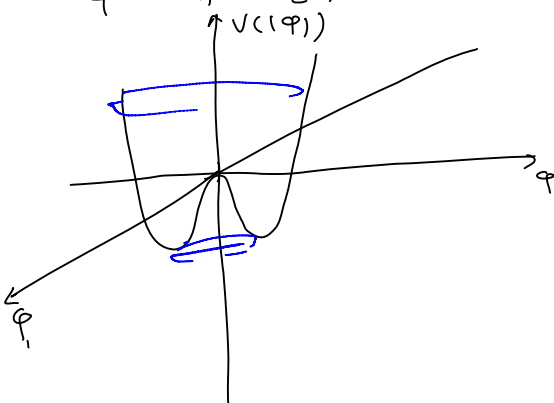
$$m_{ab}^2 = \begin{bmatrix} m^2 & 0 \\ 0 & m^2 \end{bmatrix}$$

فاز نیبو-گلدستون

Nambu-Goldstone phase

$$V(\varphi_1^2 + \varphi_2^2) = -\frac{\mu^2}{2} (\varphi_1^2 + \varphi_2^2) +$$

$$\frac{\lambda}{4} (\varphi_1^2 + \varphi_2^2)^2$$



$$\left(\frac{\partial V}{\partial \varphi_1}\right)_0 = -\mu^2 \varphi_1 + \lambda \varphi_1 (\varphi_{01}^2 + \varphi_{02}^2) = 0$$

$$\left(\frac{\partial V}{\partial \varphi_2}\right)_0 = -\mu^2 \varphi_2 + \lambda \varphi_2 (\varphi_{01}^2 + \varphi_{02}^2) = 0$$

$$\varphi_{01}^2 + \varphi_{02}^2 = v^2 = \frac{\mu^2}{\lambda}$$

$$\frac{\partial^2 V}{\partial \varphi_1^2} = (-\mu^2 + \lambda(\varphi_1^2 + \varphi_2^2)) + 2\lambda \varphi_1^2$$

$$\frac{\partial^2 V}{\partial \varphi_2^2} = (-\mu^2 + \lambda(\varphi_1^2 + \varphi_2^2)) + 2\lambda \varphi_2^2$$

$$\frac{\partial^2 V}{\partial \varphi_1 \partial \varphi_2} = 2\lambda \varphi_1 \varphi_2$$

$$m_{ab}^2 = \begin{bmatrix} 2\lambda v^2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\varphi'_1 = \varphi_1 - v$$

$$\varphi'_2 = \varphi_2$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \varphi'_1)^2 + \frac{1}{2} (\partial_\mu \varphi'_2)^2 - \frac{(2\lambda v^2)}{2} \varphi_1'^2$$

$$+ \lambda v \varphi'_1 (\varphi_1'^2 + \varphi_2'^2) - \frac{\lambda}{4} (\varphi_1'^2 + \varphi_2'^2)^2$$

↑
توازن بینان شده

$$2\lambda v^2 = \varphi_1' = \varphi_1 - v \quad \text{ح.ج}$$

$$0 = \varphi_2' \quad \text{ح.ج}$$

$$\varphi = \frac{f}{\sqrt{2}} e^{i\theta/v}$$

$$\partial_\mu \varphi = \frac{1}{\sqrt{2}} e^{i\theta/v} \left(\partial_\mu f + \frac{i f}{v} \partial_\mu \theta \right)$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu f)^2 + \frac{1}{2v^2} f^2 (\partial_\mu \theta)^2 - V(f^2)$$

$$f(x) = v + \eta(x)$$

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \eta)^2 + \frac{1}{2} (\partial_\mu \theta)^2 + \frac{2}{v} (\partial_\mu \theta)^2 \eta + \frac{\eta^2}{2v^2} (\partial_\mu \theta)^2 - V(\eta^2)$$

$$V(\eta^2) = \frac{1}{2} (\lambda v^2) \eta^2 + \lambda v \eta^3 + \frac{\lambda}{4} \eta^4 - \frac{1}{2} \mu^2 v^2$$

$$m_\eta = \sqrt{2\lambda} v$$

تعويض به مدل $SU(2)$

$$\varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}$$