Dec. 17

Top Quark

The last of the six quarks is the top quark. This quark is exceptionally heavy compared to the others, and, just by virtue of this, it has some unusual properties. In this lecture, I will present the picture that the Standard Model gives for the top quark and present some experimental tests of that picture.

As we will see later, the mass of the top quark is about 170 GeV. I have already noted that, in the Standard Model, the top quark obtains mass from a Yukawa coupling

$$\lambda_t \approx 1$$

This seems large, but actually it gives only

$$\frac{\lambda_t^2}{4\pi} = \frac{1}{13}$$

which is weaker than the asymptotically free QCD coupling. So it is reasonable to treat the top quark as being weakly coupled to the Higgs boson, as we do for the other quarks. From this point of view, the mystery is not why the top quark is so heavy but why all other quarks and leptons are so light.

The large mass of the quark implies, in particular, that

$$m_t > m_W + m_b$$

So, the top quark does not decay through the effective $(V-A)$ coupling like the other heavy quarks. Instead, it decays to an on-shell W^+ boson

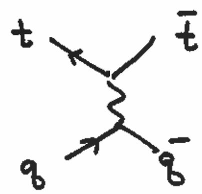
$$t \rightarrow b + W^+$$

This gives a very clear picture of how the top quark should appear in experiments. Depending on whether the W boson has a hadronic or leptonic decay,

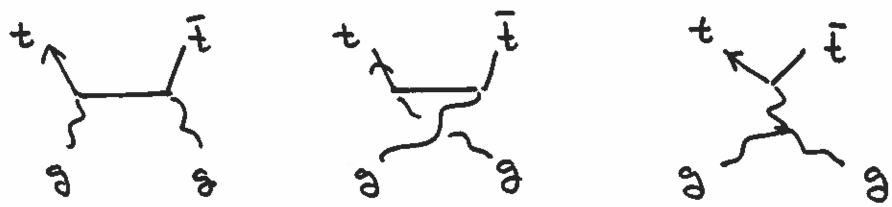
$$t \rightarrow b\, q\bar{q} \qquad\qquad t \rightarrow b\, \ell^+ \nu$$

the top quark appears as a 3-jet system or as a jet, a lepton, and an unseen neutrino.

Top quarks are too heavy to have been produced at any e^+e^- collider built so far. At hadron colliders, top quarks are pair-produced through the reactions $q\overline{q} \rightarrow t\overline{t}$



and $gg \rightarrow t\overline{t}$



At the Fermilab Tevatron, with 2 TeV $p\overline{p}$ collisions, $q\overline{q}$ annihilation dominates. At the CERN LHC, with 14 TeV pp collisions, there should be enough high energy gluons available from the protons that gg collisions become the dominant production process. In fact, the LHC should produce huge samples of top quarks, more than 1 million per year even in its early stages.

From the above discussion of top decays, $t\overline{t}$ events will appear as 6-jet events, 4-jet events with a lepton plus missing momentum, and events with 2 leptons and missing momentum. Figs p. 2, 3, 4 show some examples of events of this type that have been observed at the Tevatron. Figs p. 2 shows a CDF event with 4 jets, an e^+, and unbalanced momentum consistent with the neutrino in the decay $W^+ \to e^+\nu$. The bottom of the figure shows the pattern of high-momentum tracks at three levels of magnificiation. At the highest level, two pairs of tracks are seen to emerge from secondary vertices some mm away from the primary vertex, indicating the presence of two b quarks. Figs p. 3 shows a more recent event from CDF. The green track is a muon, and, again, two b vertices are clearly identified. Figs p. 4 is a 6-jet event from DO. Two of the jets have a muon almost collinear with the jet. This is evidence for the presence of a b quark decaying via

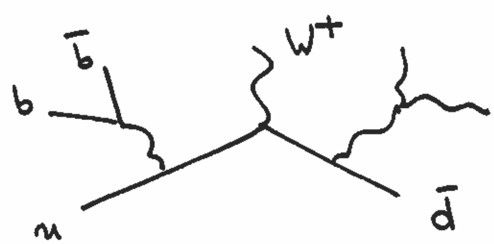
$$b \to c\,\mu^-\,\bar{\nu}_\mu$$

I should say that these events are only *probably* $t\bar{t}$ events. The $t\bar{t}$ cross section at the Tevatron is about 8 pb. This is 10^{-10} times the total $p\bar{p}$ cross section at the Tevatron, leaving a lot of room for more conventional QCD processes to produce events that resemble the top quark signature. A particular dangerous background process is

$$p\bar{p} \to W^+ + g + g + g$$

$$\quad\; \hookrightarrow \ell\nu \qquad\qquad\quad \hookrightarrow b\bar{b} \text{ or } c\bar{c}$$

including a gluon splitting to $b\bar{b}$, or a gluon splitting to $c\bar{c}$, with a c vertex mistaken for a b vertex. These gluons are readily produced from initial state radiation,



To prove that top quark pairs are being produced, the Tevatron experiments made extensive studies of this reaction. For example, data from CDF on the k_T distribution of the 1st, 2nd, 3rd, and 4th jets in $p\bar{p} \to W+$ jets reactions is shown in Figs p. 5. Using this data, it is possible to estimate the expected rate for

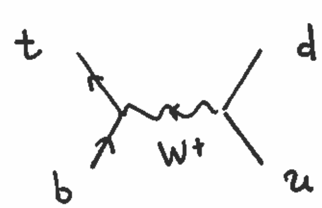
$$p\bar{p} \to \ell \nu + n \text{ jets w. 1 b tag}$$

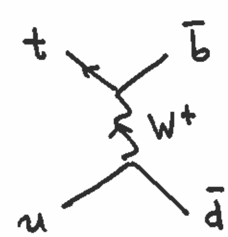
A recent CDF measurement of the rates of these processes is shown in Figs p. 6. The colored regions show the expeced contributions of various physics processes. The dark blue shaded contribution is mainly from misidentified parton-parton scattering events. The green regions are the estimated contributions from $W+$ jets. These contributions can explain the data for $n = 1, 2$, and their precise normalization is checked by this comparison. However, these contributions fall short for $n = 3$and give a very small estimate for $n = 4$. The deficit must be filled in by a new process, $p\bar{p} \to t\bar{t}$.

Figs p. 7 and 8 show some properties of the $t\bar{t}$ events selected in this analysis. Figs p. 7 shows the 2-jet invariant mass distribution for the pair of non-b-tagged jets that give the best fit to be the products of the hadronically decaying W. The distribution does peak at 80 GeV as required. Figs p. 8 show the 3-jet invariant mass distribution for the best combination of this W with one of the other two jets. This distribution should peak at the top quark mass. From this and other measurements of the top quark mass by CDF and DO, one finds

$$m_t = 173.1 \pm 1.2 \text{ GeV}$$

Top quarks can also be produced at the Tevatron singly, through the reactions





and their charge conjugates. The processes are called, respectively, t-channel and s-channel single top production. In the t-channel process, the initial b quark comes splitting of a gluon in the initial proton structure. Evidence from CDF for the presence of this process is shown in Figs p. 9. The orange boxes in the plot show the estimated contribution from single-top production; the other contributions shown are from $W+$ jets, $Z+$ jets, and $t\bar{t}$.

Because the top quark decays directly into $W + b$, its lifetime is extremely short. This means that the top quark does not have enough time to form hadrons. It lives entirely in the weak-coupling regime of QCD from production to decay. In particular, there is not enough time between the production and decay of a top quark for the its spin to be flipped by QCD interactions. Thus, the spin orientation with which a top quark ws produced can be measured. To see how this is done, we should compute the spin structure of top quark decays predicted by the Standard Model.

We can consider first the helicity structure of the decay $t \to W^+b$. The matrix element is

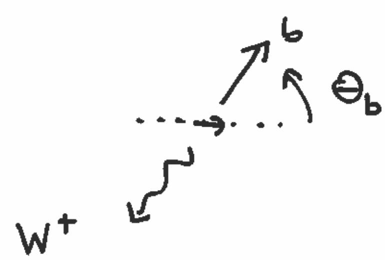
$$\mathcal{M} = i\frac{g}{\sqrt{2}}\, \bar{u}_L(b)\gamma^\mu u_L(t)\, \epsilon^*_\mu(W)$$

For a top in its rest frame with spin along the $+\hat{3}$ axis

$$u(t) = \sqrt{m}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \qquad\qquad u_L = \sqrt{m}\begin{pmatrix}1\\0\end{pmatrix}$$

I will ignore the b quark mass. Then the b is always left-handed, and its spinor is

$$u_L(b) = \sqrt{2E_b}\begin{pmatrix}-\sin\theta_b/2\\ \cos\theta_b/2\end{pmatrix}$$



From this,

$$i\mathcal{M} = \frac{ig}{\sqrt{2}}\sqrt{2E_b m}\left(-\sin\frac{\theta_b}{2}, \cos\frac{\theta_b}{2}\right)(1,-\vec{\sigma})^\mu \begin{pmatrix}1\\0\end{pmatrix}\cdot \varepsilon^*_\mu(W)$$

$$= ig\sqrt{E_b m}\left(-\sin\frac{\theta_b}{2}, -\cos\frac{\theta_b}{2}, -i\cos\frac{\theta_b}{2}, \sin\frac{\theta_b}{2}\right)^\mu \varepsilon^*_\mu(W)$$

The (b,t) vector should be dotted with the possible choices for the W polarization vectors. For transversely polarized W's, the polarization vectors are

$$\varepsilon^*_R(W) = \frac{1}{\sqrt{2}}(0\ 1\ -i\ 0) \qquad \varepsilon^*_L(W) = \frac{1}{\sqrt{2}}(0\ 1\ i\ 0)$$

When we rotate these to a W orientation opposite to the b quark, they become

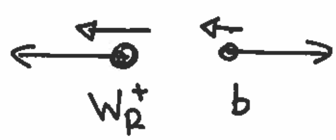
$$\varepsilon^*_R = \frac{1}{\sqrt{2}}(0, -\cos\theta_b, -i, \sin\theta_b)$$

$$\varepsilon^*_L = \frac{1}{\sqrt{2}}(0, -\cos\theta_b, +i, \sin\theta_b)$$

For W^+_R,the scalar product vanishes,

$$i\mathcal{M} = \frac{ig\sqrt{E_b m}}{\sqrt{2}}\left(-\left(\cos\theta_b\cos\frac{\theta_b}{2} + \sin\theta_b\sin\frac{\theta_b}{2}\right) - \left(-\left(\cos\frac{\theta_b}{2}\right)\right)\right)$$
$$= 0$$

This can be understood from angular momentum conservation. The b is left-handed, and a $W^+_R b_L$ final state



would have $J^3 = \frac{3}{2}$ along some axis. This cannot be the product of the decay of a spin $\frac{1}{2}$ top quark. For W_L^+, we find

$$i\mathcal{M} = ig\frac{\sqrt{E_b m}}{\sqrt{2}} \cdot (-2) \cos\frac{\theta_b}{2}$$

For longitudinally polarized W's, a few more remarks are needed. In the W rest frame, the longitudinal polarization vector is

$$\varepsilon_0^{\mu *} = (0\ 0\ 0\ 1)^\mu$$

When the W is boosted along the $\hat{3}$ axis, this becomes

$$\varepsilon_0^{\mu *} = \left(\frac{k_W}{m_W}, 0, 0, \frac{E_W}{m_W}\right)$$

Note that, for a large boost, the components of this vector become large

$$\sim \frac{E_W}{m_W}$$

and ϵ_0^μ becomes approximately equal to k_W^μ/m_W. Rotating this vector into the correct orientation, we find

$$\varepsilon_0^{\mu *} = \left(\frac{k_W}{m_W}, -\sin\theta_b \frac{E_W}{m_W}, 0, -\cos\theta_b \frac{E_W}{m_W}\right)^\mu$$

Then the amplitude for emission of a longitudinally polarized W boson in top decay is

$$i\mathcal{M} = ig\sqrt{E_b m}\left(-\sin\frac{\theta_b}{2}\right)\frac{k_W+E_W}{m_W}$$

$$= -ig\sqrt{E_b m}\,\sin\frac{\theta_b}{2}\,\frac{m_t}{m_W} \qquad (k_W = k_b = E_b)$$

This amplitude is enhanced with respect to the previous one by the factor

$$\frac{m_t}{\sqrt{2}m_W} \sim \frac{\lambda_t}{g}$$

This reflects the fact that the longitudinally polarized W boson arose from the Goldstone boson of the Higgs sector. The coupling of the top quark to that helicity state is that of the size of the Higgs Yukawa coupling rather than the gauge coupling.

From the above formulae,

$$\frac{d\Gamma}{d\cos\theta_b}(t\to bW^+_L) = \frac{1}{2m_t}\frac{1}{16\pi}\frac{2k}{m_t}\;g^2\,E_b m\,\cos^2\frac{\theta_b}{2}$$

Using

$$k_b = k_W = E_b = \frac{m_t^2 - m_W^2}{2m_t} \qquad E_W = \frac{m_t^2 + m_W^2}{2m_t}$$

we find

$$\frac{d\Gamma}{d\cos\theta_b}(t \to b W_L^+) = \frac{\alpha_w m_t}{16}(1+\cos\theta_b)\left(1-\frac{m_W^2}{m_t^2}\right)^2$$

$$\frac{d\Gamma}{d\cos\theta_b}(t \to b W_0^+) = \frac{\alpha_w m_t}{16}(1-\cos\theta_b)\,\frac{m_t^2}{2m_W^2}\left(1-\frac{m_W^2}{m_t^2}\right)^2$$

The total width is

$$\Gamma_t = \frac{\alpha_w}{16}\,\frac{m_t^3}{m_W^2}\left(1+2\frac{m_W^2}{m_t^2}\right)\left(1-\frac{m_W^2}{m_t^2}\right)^2$$

which predicts $\Gamma_t = 1.5$ GeV. The W helicities appear with the probabilities

$$W_R^+ \quad 0\% \qquad\qquad W_0^+ \quad 70\% \qquad\qquad W_L^+ \quad 30\%$$

This last relation can be tested, by reconstructing $t\bar{t}$ events, going to the W rest frame, and measuring the angular distribution of the W decays, which reflects the W helicity. The W decay distributions are

$$\frac{d\Gamma}{d\cos\theta_\ell}(W^+ \to \ell^+\nu) \sim \underset{W_R^+}{(1+\cos\theta_\ell)^2} \qquad \underset{W_0^+}{2\sin^2\theta_\ell} \qquad \underset{W_L^+}{(1-\cos\theta_\ell)^2}$$

Figs p. 10 shows these distributions and, in orange, the combination predicted for t decay. Also shown is the CDF measurement of the W decay distribution from leptonic top quark events. This measurement is in excellent agreement with the prediction.

Another way to look at the W spin dynamics is to follow the W decay and consider the orientation of the final particles in $t \to b\ell^+\nu$. The matrix element for the full decay is

$$i\mathcal{M} = \left(\frac{ig}{\sqrt{2}}\right)^2 \frac{-i}{[(k_\ell+k_\nu)^2 - m_W^2 + i m_W \Gamma_W]} u_L^\dagger(b)\bar{\sigma}^\mu u(t)\, u^\dagger(\nu)\bar{\sigma}_\mu v(\ell^+)$$

As we did in our analysis of muon decay, we can use the Fierz identity to rewrite this expression as

$$i\mathcal{M} = ig^2 \frac{1}{[(k_t-k_b)^2 - m_W^2 + i m_W \Gamma_W]} u_L^\dagger(b)_\alpha \epsilon_{\alpha\gamma} u^\dagger_\gamma(\nu) \quad u_\beta(t)\, \epsilon_{\beta\delta} v_\delta(\ell^+)$$

From this, we see that the spinor of the final ℓ^+ is correlated with the spin direction of the t. Since the ℓ^+ is right-handed,

$$\frac{d\Gamma}{d\cos\theta_{\ell^+}} \sim (1+\cos\theta_{\ell^+})$$

where θ_ℓ is the angle between the ℓ^+ direction and the top spin. The ℓ^+ direction is easily measured. We see from this formula that this direction is an excellent—indeed, a maximally effective—polarimeter for top quarks.