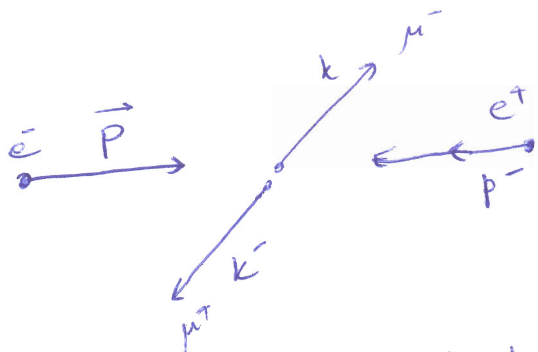
منبع فصل اول پسکین

هدف کتاب: روش‌های اصلی و متعارف نظریه‌ی میدان کوانتمی و فرمالیزم دیاگرام‌های فاینمن

این فرمالیزم ← الکترودینامیک کوانتمی

اول فرمالیزم   فعلاً هدف ما



$e^+e^- \to \mu^+\mu^-$

$$|p| = |p'| = |k| = |k'| = E = \frac{E_{cm}}{2}$$

$\vec{p}' = -\vec{p}$

$\vec{k}' = -\vec{k}$

$\begin{cases} \text{unpolarized} \\ \text{polarization-blind} \end{cases}$ → average over initial polarizations

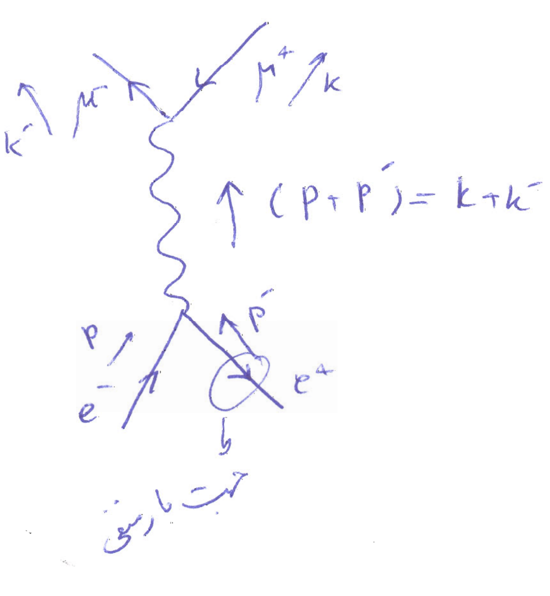
sum 〃 〃 final 〃

quantum mechanical amplitude

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{c.m}^2} |M|^2$$

بعدش؟

M دقیق را نمی‌دانیم.



Feynman rule ←

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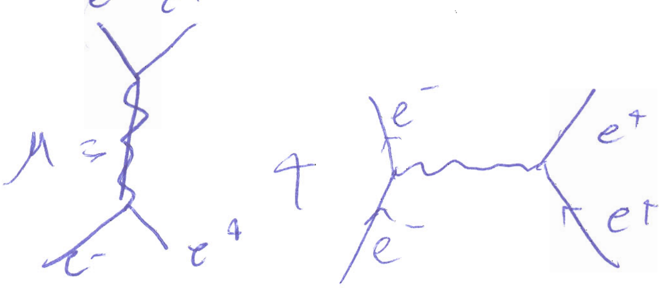
$$M \sim \langle \mu^+\mu^- | H_I | \gamma\rangle^\mu \langle \gamma | H_I | e^+e^-\rangle_\mu$$

polarization قطبش

جمع در جهت دامنه → A_μ

Bhabha scattering



حالا

$$\varepsilon^\mu = (0,1,i,0)$$

$\cancel{\langle \mu^+\mu^- | H_I}$ $\quad \langle\gamma|H_I|e^+e^-\rangle^\mu \propto e(0,1,i,0)$

RL

RL $\quad \langle\gamma|H_I|\mu^+\mu^-\rangle^\mu \propto e(0,\cos\theta, i, -\sin\theta)$

$$\mathcal{M}(RL\to RL) = -e^2(1+\cos\theta) \leftarrow \cos\theta = 180$$

$\cancel{\mathcal{M}(RR \to RR \ldots) = \mathcal{M}(LL \to RL \ldots) = \mathcal{M}(R \to LL)}$

$$\langle\gamma|H_I|\gamma e^+_L e^-_L\rangle = 0 = \langle\gamma|H_I|\mu^+_L\mu^-_L\rangle = \langle\gamma|H_I|e^+_R e^-_R\rangle$$

$$= \langle\gamma|H_I|\mu^+_R\mu^-_R\rangle$$

تمرین نشان دهید

$$\mathcal{M}(RL\to LR) = -e^2(1-\cos\theta)$$

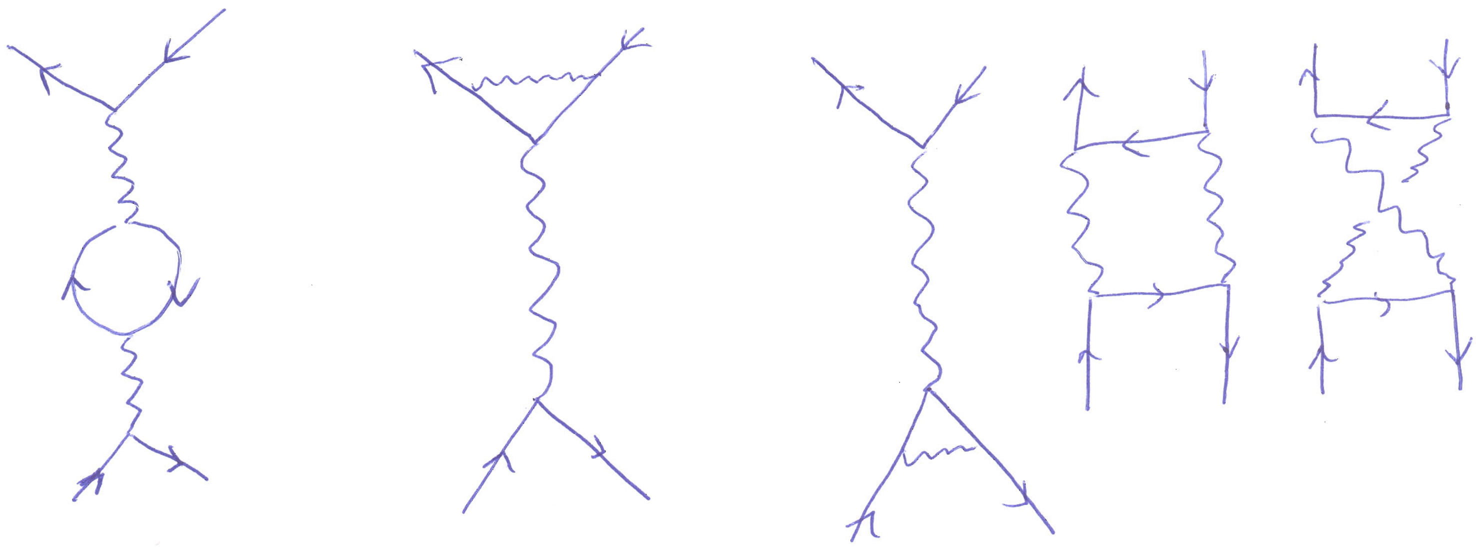
$$\mathcal{M}(LR\to RL) = -e^2(1-\cos\theta)$$

$$\mathcal{M}(LR\to LR) = -e^2(1+\cos\theta)$$

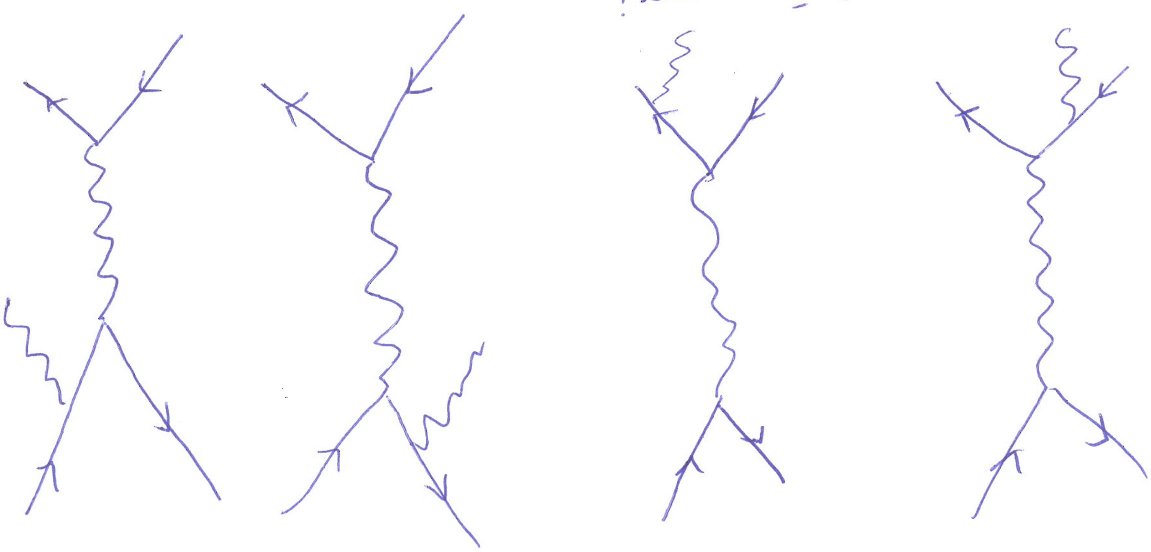
ص ۳

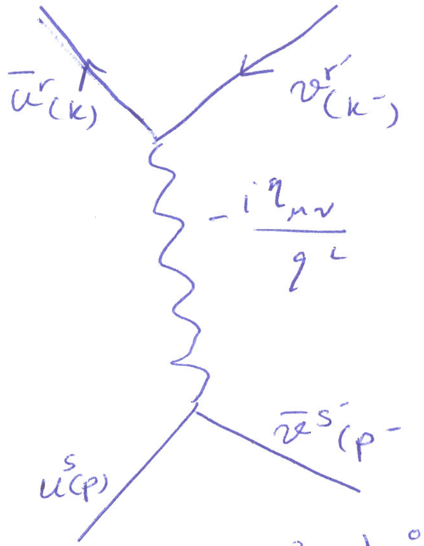
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_{cm}^2}(1+\cos^2\theta) \qquad \alpha = \frac{e^2}{4\pi} \simeq \frac{1}{137}$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3E_{cm}^2} \longrightarrow 10\% \text{ توافق}$$



تصحیحات از چه مرتبه‌ای فکر کنید باشند؟





$$\mathcal{M} = \bar v^{s'}(p')(-ie\gamma^\mu)u^s(p)\frac{-i\eta_{\mu\nu}}{q^2}$$

$$\bar u^{r}(k)(-ie\gamma^\nu)v^{r'}(k') =$$

$$\frac{ie^2}{q^2}\left(\bar v^{s'}(p')\gamma^\mu u^s(p)\right)\left(\bar u^r(k)\gamma_\mu v^{r'}(k')\right)$$

$$\gamma^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad \gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}$$

Compton Scattering

Klein-Gordon Field

فصل دوم پسکین

$$U(t) = \langle x | e^{-iHt} | x_0 \rangle = \langle x | e^{-i\frac{p^2}{2m}t} | x_0 \rangle =$$

$$\int \frac{d^3p}{(2\pi)^3} \langle x | e^{-i\frac{p^2}{2m}t} | p \rangle \langle p | x_0 \rangle = \frac{1}{(2\pi)^3} \int d^3p \, e^{-\frac{ip^2}{2m}t} e^{ip\cdot(x-x_0)}$$

$$= \left(\frac{m}{2\pi i t}\right)^{\frac{3}{2}} e^{im\frac{(x-x_0)^2}{2t}}$$

با سرعت نور

نسبیتی

$$U(t) = \langle x | e^{-i\sqrt{p^2+m^2}\,t} | x_0 \rangle = \frac{1}{(2\pi)^3} \int d^3p \, e^{-it\sqrt{p^2+m^2}} e^{ip\cdot(x-x_0)}$$

$$= \frac{1}{2\pi^2 |x-x_0|} \int_0^{+\infty} dp \, p \sin(p|x-x_0|) \, e^{-it\sqrt{p^2+m^2}}$$

$x^2 \gg t^2$ $\xrightarrow{\text{غیر صفر}}$

$$U(t) \sim e^{-m\sqrt{x^2-t^2}}$$

- Quantum field theory مسائله حل می‌کند. مسائل علت حل می‌شه
- آمار - اسپین
- طیف مقطع برخورد
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مکانیک کلاسیک $S = \int L\, dt$

$$S = \int L\, dt = \int \mathcal{L}(\varphi, \partial_\mu \varphi)\, d^4x$$

$$0 = \delta S = \int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial \varphi} \delta\varphi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \delta(\partial_\mu \varphi) \right\} =$$

$$\int d^4x \left\{ \frac{\partial \mathcal{L}}{\partial \varphi} \delta\varphi - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \right) \delta\varphi + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \delta\varphi \right) \right.$$

معادله اویلر_لاگرانژ   معادله ی حرکت یک میدان

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \right) - \frac{\partial \mathcal{L}}{\partial \varphi} = 0$$

Hamiltonian Field theory

$$H \triangleq p\dot{q} - L \qquad p(x) \triangleq \frac{\partial L}{\partial \dot{\varphi}(x)} = \frac{\partial}{\partial \dot{\varphi}(x)} \int \mathcal{L}(\varphi(y), \dot{\varphi}(y)) d^3y$$

$$\sim \frac{\partial}{\partial \dot{\varphi}(x)} \sum_x \mathcal{L}(\varphi(y), \dot{\varphi}(y)) d^3y = \pi(x) d^3x$$

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}(x)}$$ ← momentum density conjugate to $\varphi(x)$

$$H = \sum_x p(x) \dot{\varphi}(x) - L$$

↓ پیوستار

$$H = \int d^3x \left[\pi(x) \dot{\varphi}(x) - \mathcal{L} \right] \triangleq \int d^3x \, \mathcal{H}$$

$$\mathcal{L} = \frac{\dot{\varphi}^2}{2} - \frac{(\nabla \varphi)^2}{2} - \frac{m^2 \varphi^2}{2} = \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{m^2 \varphi^2}{2}$$

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2 \right) \varphi = 0$$ یا $$(\partial^\mu \partial_\mu + m^2) \varphi = 0$$

معادله ی Klein_Gordon

کلاسیکی

$$H = \int d^3x \, \mathcal{H} = \int d^3x \left[\frac{\pi^2}{2} + \frac{1}{2} (\nabla \varphi)^2 + \frac{m^2 \varphi^2}{2} \right]$$

$$i\hbar \partial_t \psi = \frac{-\hbar^2}{2m} \nabla^2 \psi + V(x) \psi$$

$\hookrightarrow \quad \partial_t \rho + \nabla_i J^i = 0. \qquad \rho = |\psi|^2 \quad \geq 0$ Propability

$$\vec{J} = \frac{1}{2m}(\psi^\dagger \nabla \psi - \psi \nabla \psi^\dagger)$$

$$(\partial_t^2 - \nabla^2 - m^2)\, \phi = 0$$

$$\partial_\mu J^\mu = 0 \quad , \quad J^\mu = \phi^\dagger \nabla^\mu \phi - \phi \nabla^\mu \phi^\dagger$$

$$\rho = J^0 = \phi^\dagger \dot{\phi} - \dot{\phi}^\dagger \phi$$ is Not +ve def.

$$\phi = \phi_0 e^{ik \cdot x} \qquad k^2 = m^2$$

$$J^\mu = k^\mu |\phi|^2$$