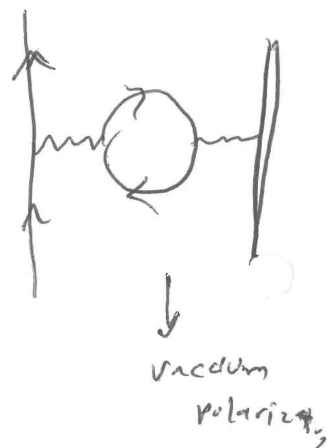
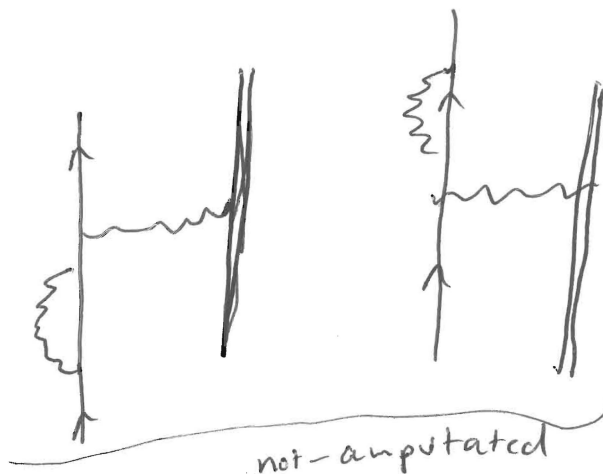


Radiative Corrections: Introduction

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- loop - corrections
- brem sstrahlung

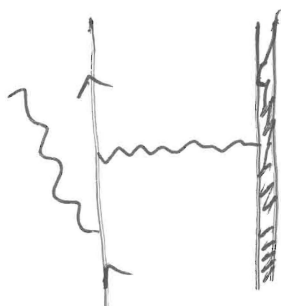


ultraviolet divergence
infrared divergence

حالت ذره سنگین نچانیم

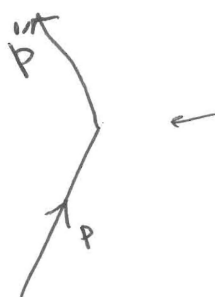
7.5

بذایم برای جفت



Soft Bremsstrahlung

classical



sudden kick at time $t=0$
when $x=0$

2

$$j^\mu(x) = (1, 0)^\mu e \delta^3(x) \leftarrow \text{charged particle at rest at } x=0$$

$$= \int dt (1, 0)^\mu e \delta^4(x - y(t)) \quad y^\mu(t) = (t, 0)^\mu$$

$$j^\mu(x) = e \int d\tau \frac{dy^\mu(\tau)}{d\tau} \delta^4(x - y(\tau))$$

تابع دلتا که در جایی غیر از صفر نیست

$$\int d^4x f(x) \partial_\mu j^\mu(x) = \int d^4x f(x) e \int d\tau \frac{dy^\mu(\tau)}{d\tau} \delta_{,\mu} \delta^4(x - y(\tau))$$

$$= -e \int d\tau \frac{dy^\mu(\tau)}{d\tau} \frac{\partial f(x)}{\partial x^\mu} \Big|_{x=y(\tau)}$$

$$= -e \int d\tau \frac{d f(y(\tau))}{d\tau}$$

$$= -e f(y(\tau)) \Big|_{-\infty}^{\infty} = 0$$

$$y^\mu(\tau) = \begin{cases} \frac{p^\mu}{m} \tau & \text{for } \tau < 0 \\ \frac{p'^\mu}{m} \tau & \text{for } \tau > 0 \end{cases}$$

$$j^\mu(x) = e \int_0^{\infty} d\tau \frac{p'^\mu}{m} \delta^4(x - \frac{p'\tau}{m}) + e \int_{-\infty}^0 d\tau \frac{p^\mu}{m} \delta^4(x - \frac{p\tau}{m})$$

$$\tilde{j}^\mu(k) = \int d^4x e^{ik \cdot x} j^\mu(x) = e \int_0^{\infty} d\tau \frac{p'^\mu}{m} e^{i(\frac{k \cdot p'}{m} + i\varepsilon)\tau} + e \int_{-\infty}^0 d\tau \frac{p^\mu}{m} e^{i(\frac{k \cdot p}{m} - i\varepsilon)\tau}$$

$$= ie \left(\frac{p'^\mu}{k \cdot p' + i\varepsilon} - \frac{p^\mu}{k \cdot p - i\varepsilon} \right)$$

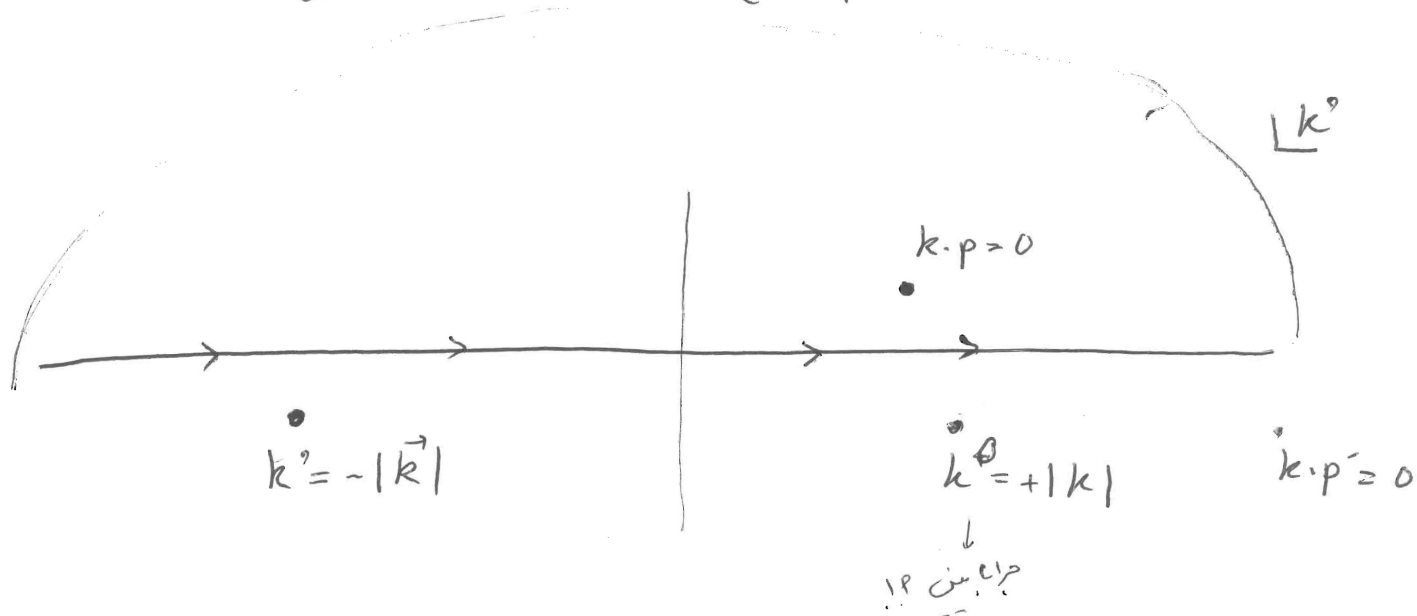
$$\partial^\mu A_\mu = 0$$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

$$\partial^2 A^\mu = j^\mu$$

$$\tilde{A}^\mu(k) = -\frac{1}{k^2} \tilde{j}^\mu(k)$$

$$A^\mu(x) = \int \frac{d^4 k}{(2\pi)^4} e^{-ik \cdot x} \frac{-ie}{k^2} \left(\frac{p'^\mu}{k \cdot p' + i\epsilon} - \frac{p^\mu}{k \cdot p - i\epsilon} \right)$$



for $t < 0$

$$A^\mu(x) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} e^{-\frac{\vec{k} \cdot \vec{p}}{p^0} t} \frac{(2\pi i)(+ie)}{2\pi k^2} \frac{p^\mu}{p^0}$$

$$p^\mu = (p^0, 0)^\mu$$

$$A^\mu(x) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \frac{e}{|\vec{k}|^2} \cdot (1, 0)^\mu$$

بنابراین کلب (قبل از ضرب به تابعی میست.)

$$A_{\text{rad}}^{\mu}(x) = \int \frac{d^3k}{(2\pi)^3} \frac{-e}{2|k|} \left\{ e^{-ik \cdot x} \left(\frac{p^{\mu}}{k \cdot p'} - \frac{p^{\mu}}{k \cdot p} \right) + c.c. \right\} \Big|_{k^0=|k|} \quad \varepsilon_0$$

$$= \text{Re} \int \frac{d^3k}{(2\pi)^3} A^{\mu}(k) e^{-ik \cdot x}$$

$$A^{\mu}(k) = \frac{-e}{|k|} \left(\frac{p^{\mu}}{k \cdot p'} - \frac{p^{\mu}}{k \cdot p} \right)$$

$$E(x) = \text{Re} \int \frac{d^3k}{(2\pi)^3} E(k) e^{-ik \cdot x}$$

$$B(x) = \text{Re} \int \frac{d^3k}{(2\pi)^3} B(k) e^{-ik \cdot x}$$

$$E(k) = -i \vec{k} \cdot \vec{A}(k) + i k \cdot \vec{A}(k)$$

$$B(k) = i k \times \vec{A}(k) = \hat{k} \times E(k)$$

$$\vec{k} \cdot \vec{E}(k) = 0 \quad \text{شان دهيډ:}$$

$$\text{Energy} \approx \frac{1}{2} \int d^3x (|E(x)|^2 + |B(x)|^2)$$

$$\frac{1}{2} \int d^3x |E(x)|^2 =$$

$$\frac{1}{8} \int d^3x \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3k'}{(2\pi)^3} \left(E(k) e^{-ik \cdot x} + E^*(k) e^{ik \cdot x} \right) \cdot \left(E(k') e^{-ik' \cdot x} + E^*(k') e^{ik' \cdot x} \right)$$

$$= \frac{1}{8} \int \frac{d^3k}{(2\pi)^3} \left(E(k) \cdot E(-k^*) e^{-2ik \cdot t} + 2 E(k) \cdot E^*(k) + E^*(k) \cdot E(-k) e^{2ik \cdot t} \right)$$

$$\text{Energy} = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \mathcal{E}(k) \cdot \mathcal{E}^*(k)$$

$$\mathcal{E}(k) \cdot \mathcal{E}^*(k) = \sum_{\lambda=1,2} \left| \vec{e}_{\lambda} \cdot \vec{\mathcal{E}}(k) \right|^2 = |k|^2 \sum_{\lambda=1,2} \left| \vec{e}_{\lambda}(k) \cdot \vec{A}(k) \right|^2$$

$\uparrow$ transverse
 $\downarrow$ transverse unit polarization vector

$$\text{Energy} = \int \frac{d^3 k}{(2\pi)^3} \sum_{\lambda=1,2} \frac{e^2}{2} \left| \vec{e}_{\lambda}(k) \cdot \left(\frac{\vec{p}'}{k \cdot p'} - \frac{\vec{p}}{k \cdot p} \right) \right|^2$$

می توانیم $\vec{e} \cdot \vec{A} = 0$ را به کار ببریم

$\mathcal{E}_{\lambda=0} = 0$

$$k_{\mu} \left(\frac{p'^{\mu}}{k \cdot p'} - \frac{p^{\mu}}{k \cdot p} \right) = 0$$

توجه کنید $\sum \mathcal{E}_{\mu} \mathcal{E}_{\nu}^* \rightarrow -g_{\mu\nu}$

$$\begin{aligned} \text{Energy} &= \int \frac{d^3 k}{(2\pi)^3} \frac{e^2}{2} (-g_{\mu\nu}) \left(\frac{p'^{\mu}}{k \cdot p'} - \frac{p^{\mu}}{k \cdot p} \right) \left(\frac{p'^{\nu}}{k \cdot p'} - \frac{p^{\nu}}{k \cdot p} \right) \\ &= \int \frac{d^3 k}{(2\pi)^3} \frac{e^2}{2} \left(\frac{2 p \cdot p'}{k \cdot p' k \cdot p} - \frac{m^2}{(k \cdot p')^2} - \frac{m^2}{(k \cdot p)^2} \right) \end{aligned}$$

$$\text{Energy} = \frac{e^2}{(2\pi)^2} \int dk I(v, v')$$

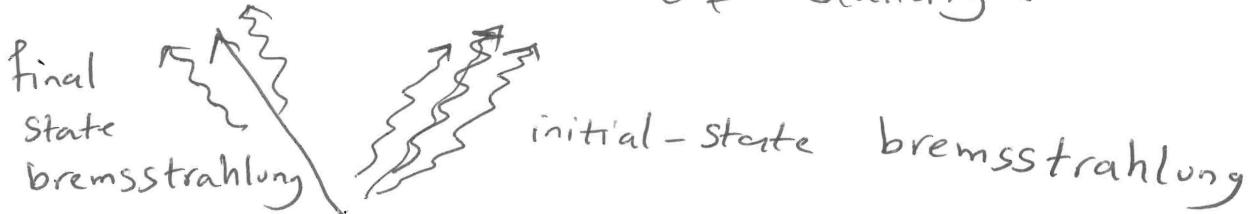
$$I(v, v') = \int \frac{d\Omega_k}{4\pi} \left(\frac{2(1-v \cdot v')}{(1-\hat{k} \cdot v)(1-\hat{k} \cdot v')} - \frac{m^2/E^2}{(1-\hat{k} \cdot v')^2} - \frac{m^2/E^2}{(1-\hat{k} \cdot v)^2} \right)$$

لکه k بی متناهی ندارد!

$$\text{Energy} \rightarrow \infty$$

$$\text{Energy} = \frac{\alpha}{\pi} k_{\max} I(\nu, \nu')$$

$$k_{\max} \sim \frac{1}{\tau} \leftarrow \text{scattering time}$$



peaks

$$\begin{cases} \hat{k} \parallel \nu \\ \hat{k} \parallel \nu' \end{cases}$$

در حد فراسمتی
زنجیر بلک

$$I(\nu, \nu') \approx \int_{\hat{k} \cdot \nu = \nu' \cdot \nu}^{\cos \theta = 1} d\cos \theta \frac{1 - \nu \cdot \nu'}{(1 - \nu \cos \theta)(1 - \bar{\nu} \cdot \nu')}$$

$$+ \int_{\hat{k} \cdot \nu' = \nu \cdot \nu'}^{\cos \theta = 1} d\cos \theta \frac{1 - \nu \cdot \nu'}{(1 - \nu' \cos \theta)(1 - \bar{\nu} \cdot \nu')}$$

$$I(\nu, \nu') \approx \log \frac{1 - \nu \cdot \nu'}{1 - |\nu|} + \log \frac{1 - \nu \cdot \nu'}{1 - |\nu'|} = \log \frac{(E^2 - \vec{p} \cdot \vec{p}')^2}{E^2 (E - |\vec{p}|)^2}$$

$$E - p = \frac{m}{\sqrt{1 - \nu^2}} - \frac{m\nu}{\sqrt{1 - \nu^2}} \approx \frac{m\sqrt{1 - \nu^2}}{\sqrt{2}} \approx \frac{m\sqrt{1 - \nu'^2}}{\sqrt{2}}$$

$E' = E$

log₂
آلفا تقریباً 1/137

✓✓

$$I(\nu, \nu') \approx 2 \log \frac{P \cdot P'}{(E^2 - |P|^2)^{1/2}} = 2 \log \frac{-q^2}{m^2}$$

where $q^2 = (P' - P)^2$

$$\text{Energy} = \frac{\alpha}{\pi} \int_0^{k_{\max}} dk \, I(\nu, \nu') \xrightarrow{E \gg m} \frac{2\alpha}{m} \int_0^{k_{\max}} dk \log \frac{-q^2}{m^2}$$

$$\# \text{ of photons} = \frac{\alpha}{\pi} \int_0^{k_{\max}} dk \, \frac{1}{k} I(\nu, \nu')$$

↑
each photon contributes

محاسبہ کروائیگی ؟