قضیه ی نوتر

$$\varphi \to \varphi' = \varphi + \alpha \Delta \varphi$$

$$\mathcal{L}(x) \longrightarrow \mathcal{L}(x) + \alpha \partial_\mu \mathcal{J}^\mu(x)$$

$$\alpha \Delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \varphi} \alpha \Delta \varphi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \partial_\mu (\alpha \Delta \varphi) =$$

$$= \alpha \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \Delta \varphi \right) + \alpha \left[\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \right) \right] \Delta \varphi$$

$$\partial_\mu j^\mu(x) = 0 \qquad j^\mu(x) = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \Delta \varphi - \mathcal{J}^\mu$$

$$Q \equiv \int_V j^0 d^3 x$$

$V \leftarrow$ فضا

$$\begin{cases} \varphi_1 \to e^{i\alpha} \varphi_1 \\ \varphi_2 \to e^{2i\alpha} \varphi_2 \end{cases} \qquad U(1)$$

$$\mathcal{L} = |\partial_\mu \varphi_1|^2 + |\partial_\mu \varphi_2|^2 + m_1^2 |\varphi_1|^2 + m_2^2 |\varphi_2|^2$$

$$V_1 = \lambda_1 |\varphi_1|^4 + \lambda_2 |\varphi_2|^4 + \lambda_3 |\varphi_1|^2 |\varphi_2|^2$$

$$V_2 = \lambda_4 \varphi_2 (\varphi_1^*)^2$$

$\leftarrow$ درجه ۳

$$\begin{cases} \varphi_1 \to \varphi_1 + i\alpha \varphi_1 \qquad \varphi_2 \to \varphi_2 + 2i\alpha \varphi_2 \\ \varphi_1^* \to \varphi_1^* - i\alpha \varphi_1^* \qquad \varphi_2^* \to \varphi_2^* - 2i\alpha \varphi_2^* \\ j^\mu = i \left[(\partial^\mu \varphi_1^* \varphi_1 - \varphi_1^* \partial^\mu \varphi_1) + 2 (\partial^\mu \varphi_2^* \varphi_2 - \varphi_2^* \partial^\mu \varphi_2) \right] \end{cases}$$

ص ۸

$$x^\mu \to x^\mu - a^\mu$$

$$\varphi(x) \longrightarrow \varphi(x+a) = \varphi(x) + a^\mu \partial_\mu \varphi(x)$$

$$\mathcal{L} \longrightarrow \mathcal{L} + a^\mu \partial_\mu \mathcal{L} = \mathcal{L} + a^\nu \partial_\mu (\delta^\mu{}_\nu \mathcal{L})$$

$$T^\mu{}_\nu \triangleq \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi} \partial_\nu \varphi - \mathcal{L} \delta^\mu{}_\nu$$ ← stress-energy tensor

energy-momentum tensor

تمرین جریان مربوط را همین را برای تبدیل لورنتس بدست بیاورید.

$$H = \int T^{00} d^3x = \int \mathcal{H} d^3x$$

$$P^i = \int T^{0i} d^3x = -\int \pi \partial_i \varphi \, d^3x$$

تکانه‌ی فیزیکی

میدان کلاین-گوردن به عنوان نوسانگر هماهنگ

real field φ

$$\mathcal{L} = \frac{(\partial_\mu \varphi)^2}{2} - \frac{m^2 \varphi^2}{2}$$ ← معادله کلاین گوردن

$\underset{\text{کلاسیکی}}{\varphi} \xrightarrow{\text{quantize}}$ متغیرهای دینامیکی ≡ عملگرها

دارای روابط جابه‌جایی کانونیک (سیداری)

$$\left(\begin{array}{l} \text{Second quantization } \varphi \\ \\ \underset{\text{wave function}}{\varphi} \longrightarrow \underset{\text{operator}}{\varphi} \end{array} \right)$$

$\varphi, \pi \xrightarrow{\text{ارتقا به}}$ عملگر

یادآوری

$$[q_i, p_j] = i\delta_{ij}$$ ⟶ ~~delta~~ Kronecker delta

$$[q_i, q_j] = [p_i, p_j] = 0$$

$$\begin{cases} \varphi(x) \\ \pi(x) \equiv \dfrac{\partial \mathcal{L}}{\partial \dot{\varphi}(x)} \end{cases} \qquad \begin{cases} [\varphi(x), \pi(y)] = i\,\delta^{(3)}(x-y) \\ [\varphi(x), \varphi(y)] = [\pi(x), \pi(y)] = 0 \end{cases}$$

↑
تصویر شرودینگر

$[\varphi(x,t), \pi(y,t)]$ ← تصویر هایزنبرگ

Hamiltonian = $\mathcal{H}(\varphi, \pi)$

$\uparrow$ operator

می‌خواهیم طیف $\mathcal{H}$ را بیابیم.

$$\varphi(x,t) = \int \frac{d^3p}{(2\pi)^3}\, e^{i\vec{p}\cdot\vec{x}}\, \varphi(p,t)$$

بسط فوریه

$$\varphi^*(p) = \varphi(-p)$$

حقیقی φ

$$(\partial^\mu \partial_\mu + m^2)\varphi$$

معادله کلاین-گوردن

$$\left[\frac{\partial^2}{\partial t^2} + (|p|^2 + m^2)\right]\varphi(p,t) = 0$$

$$\omega_p = \sqrt{|p|^2 + m^2}$$

نوسانگر هماهنگ ساده

یادآوری نوسانگر هماهنگ

$$H_{SHO} = \frac{p^2}{2} + \frac{\omega^2\varphi^2}{2}$$

simple Harmonic oscillator

بازنویسی

$$\varphi = \frac{a + a^\dagger}{\sqrt{2\omega}} \qquad p = -i\sqrt{\frac{\omega}{2}}\,(a - a^\dagger)$$

$$[\varphi, p] = i \quad \equiv \quad [a, a^\dagger] = 1$$

$$H_{SHO} = \omega\left(a^\dagger a + \frac{1}{2}\right) \qquad a|0\rangle = 0 \qquad H|0\rangle = \frac{\omega}{2}$$

$$[H_{SHO}, a^\dagger] = \omega a^\dagger \qquad [H_{SHO}, a] = -\omega a$$

$$|n\rangle \triangleq (a^\dagger)^n |0\rangle \quad \longrightarrow \quad H_{SHO}|n\rangle = \left(n + \frac{1}{2}\right)\omega |n\rangle$$

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left(a_p\, e^{i\vec{p}\cdot\vec{x}} + a_p^{\dagger}\, e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\pi(x) = \int \frac{d^3p}{(2\pi)^3} (-i) \sqrt{\frac{\omega_p}{2}} \left(a_p\, e^{i\vec{p}\cdot\vec{x}} - a_p^{\dagger}\, e^{-i\vec{p}\cdot\vec{x}} \right)$$

$$\Big\downarrow \text{ بازنویسی}$$

$$\varphi(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2\omega_p}} \left(a_p + a_{-p}^{\dagger} \right) e^{i\vec{p}\cdot\vec{x}}$$

$$\pi(x) = \int \frac{d^3p}{(2\pi)^3} -i \sqrt{\frac{\omega_p}{2}} \left(a_p - a_{-p}^{\dagger} \right) e^{i\vec{p}\cdot\vec{x}}$$

$$\Downarrow \quad [a_p, a_{p'}^{\dagger}] = (2\pi)^3 \delta^3(p - p')$$

نشان دهید

$$[\varphi(x), \pi(x')] = \int \frac{d^3p\, d^3p'}{(2\pi)^6} \frac{-i}{2} \sqrt{\frac{\omega_{p'}}{\omega_p}} \left([a_{-p}^{\dagger}, a_{p'}] - [a_p, a^{\dagger}] \right) e^{i(p\cdot x + p'\cdot x')}$$

$$= i\,\delta^3(x - x')$$

$\uparrow$ تمرین

$$H = \int d^3x \int \frac{d^3p\, d^3p'}{(2\pi)^6} e^{i(\vec{p}+\vec{p}')\cdot\vec{x}} \Big\{ -\frac{\sqrt{\omega_p \omega_{p'}}}{4} (a_p - a_{-p}^{\dagger})(a_{p'} - a_{-p'}^{\dagger})$$

$$+ \frac{-\vec{p}\cdot\vec{p}' + m^2}{4\sqrt{\omega_p \omega_{p'}}} (a_p + a_{-p}^{\dagger})(a_{p'} + a_{-p'}^{\dagger}) \Big\} =$$

$$\int \frac{d^3p}{(2\pi)^3} \omega_p \Big(a_p^{\dagger} a_p + \underbrace{\tfrac{1}{2} [a_p, a_p^{\dagger}]}_{\delta(0)} \Big)$$

$\delta(0) \rightarrow$ بی‌نهایت

$[H, a_p^\dagger] = \omega_p a_p^\dagger \qquad [H, a_p] = -\omega_p a_p$

$\forall p$

$a_p|0\rangle = 0$

ground state

خلاء

حالت کلی $\quad a_p^\dagger a_q^\dagger \cdots |0\rangle \xrightarrow{\text{انرژی}} \omega_p + \omega_q + \cdots$

$p^i = \int T^{0i} d^3x = -\int \pi \partial_i \varphi \, d^3x$

عکس ✓

$$P = -\int d^3x \, \pi(x) \nabla\varphi(x) = \int \frac{d^3p}{(2\pi)^3} \vec{p} \, a_p^\dagger a_p$$

$a_p^\dagger a_q^\dagger \cdots |0\rangle \longrightarrow \vec{p} + \vec{q} + \cdots$

It is quite natural to call these excitations particles, since they are discrete entities that have the proper relativistic energy-momentum relation.

$E_p = +\sqrt{|p|^2 + m^2}$

$a_p^\dagger a_q^\dagger |0\rangle = a_q^\dagger a_p^\dagger |0\rangle \rightarrow$ Bose_Einestein Statistics

$\langle 0|0\rangle = 1 \qquad |p\rangle = a_p^\dagger |0\rangle$

$\langle p|q\rangle = (2\pi)^3 \delta^{(3)}(p-q)$

$$E' = \gamma (E + \beta P_3) \qquad P_3' = \gamma (P_3 + \beta E)$$

$$\delta(f(x) - f(x_0)) = \frac{1}{|f'(x_0)|} \delta(x - x_0)$$

$$\delta^3(p-q) = \delta^3(p'-q') \times \frac{dp_3'}{dp_3} = \delta^3(p'-q')\, \gamma \left(1 + \beta \frac{dE}{dp_3}\right)$$

$$= \delta^3(p'-q') \times \frac{\gamma}{E}(E + \beta P_3) = \delta^3(p'-q') \frac{E'}{E}$$

حجم ناوردا نیست.

$$\underset{\text{حجم}}{V} \longrightarrow \underbrace{\frac{V}{\gamma}}$$

Boosted frame

تعریف

$$|p\rangle \triangleq \sqrt{2E_p}\, a_p^\dagger |0\rangle$$

$$\Downarrow$$

$$\langle p | q \rangle = 2 E_p (2\pi)^3 \delta^3(p-q)$$

فاکتور ۲ ضروری نیست

$$U(p)\, |p\rangle = |\Lambda p\rangle$$

$$U(\Lambda)\, a_p^\dagger\, U^{-1}(\Lambda) = \sqrt{\frac{E_{\Lambda p}}{E_p}}\, a_{\Lambda p}^\dagger$$

$$(1)_{1\text{-particle}} = \int \frac{d^3p}{(2\pi)^3} |p\rangle \frac{1}{2E_p} \langle p|$$ حالت تک ذره

$$\int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} = \int \frac{d^4p}{(2\pi)^4} (2\pi)\, \delta(p^2 - m^2) \Big|_{p^0 > 0}$$

$$\varphi(x)|0\rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} e^{-i\vec{p}\cdot\vec{x}} |p\rangle$$

اگر $E_p \cong m$

$$\varphi(x)|0\rangle \propto |x\rangle$$

$\varphi(x)|0\rangle \rightarrow$ ذره‌ای در x

$$\langle 0|\varphi(x)|p\rangle = \langle 0| \int \frac{d^3p'}{(2\pi)^3} \frac{1}{\sqrt{2E_{p'}}} \left(a_{p'} e^{i\vec{p}'\cdot\vec{x}} + a^\dagger_{p'} e^{-i\vec{p}'\cdot\vec{x}} \right) \sqrt{2E_p}\, a^\dagger_p |0\rangle$$
$$= e^{i\vec{p}\cdot\vec{x}}$$

میدان کلاین گوردن در تصویر هایزنبرگ

$$\varphi(x) = \varphi(x,t) = e^{iHt} \varphi(x) e^{-iHt}$$

$$i\frac{\partial O}{\partial t} = [O, H]$$

$$i\frac{\partial \varphi(x,t)}{\partial t} = \left[\varphi(x,t), \int d^3x' \left\{ \frac{1}{2}\pi^2(x',t) + \frac{1}{2}(\nabla\varphi(x',t))^2 + m^2\varphi^2(x',t) \right\}\right] = \int d^3x' \left(i\,\delta^{(3)}(x-x')\,\pi(x',t) \right) = i\,\pi(x,t)$$

$$i\frac{\partial}{\partial t}\pi(x,t) = \left[\pi(x,t), \int d^3x' \left\{ \frac{\pi^2(x',t)}{2} + \frac{\varphi(x',t)}{2}(-\nabla^2+m^2)\varphi(x',t) \right\}\right]$$
$$= \int d^3x' \left(-i\,\delta^{(3)}(x-x')(-\nabla^2+m^2)\varphi(x',t) \right) = -i(-\nabla^2+m^2)\varphi(x,t)$$

$$\frac{\partial^2}{\partial t^2}\varphi = (\nabla^2 - m^2)\varphi$$

$$H a_p = a_p (H - E_p)$$

$$\Downarrow$$

$$H^n a_p = a_p (H - E_p)^n$$

$$e^{iHt} a_p e^{-iHt} = a_p e^{-iE_p t} \qquad e^{iHt} a_p^\dagger e^{-iHt} = a_p^\dagger e^{iE_p t}$$

Operator on Hilbert space

$$\varphi(x,t) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_p}} \left(a_p e^{-i\vec{p}\cdot\vec{x}} + a_p^\dagger e^{i\vec{p}\cdot\vec{x}} \right) \Big|_{p^0 = E_p}$$

$$\pi(x,t) = \frac{\partial}{\partial t} \varphi(x,t) \qquad p \cdot x = Ht - \vec{p}\cdot\vec{x}$$

عملگر مشتق $\rightarrow \underline{\underline{P}}$ همین طور

$$e^{-i\vec{P}\cdot\vec{x}} a_p e^{i\vec{P}\cdot\vec{x}} = a_p e^{i\vec{P}\cdot\vec{x}} \qquad e^{-i\vec{P}\cdot\vec{x}} a_p^\dagger e^{i\vec{P}\cdot\vec{x}} = a_p^\dagger e^{-i\vec{P}\cdot\vec{x}}$$

$$\varphi(x) = e^{i(Ht - \vec{P}\cdot\vec{x})} \varphi(0) e^{-i(Ht - \vec{P}\cdot\vec{x})} = e^{iP_\mu x^\mu} \varphi(0) e^{-iP_\mu x^\mu}$$

$$P^\mu = (H, \vec{P})$$

ورکاسن مشت وسنی

ص ۲۶ روخوانی