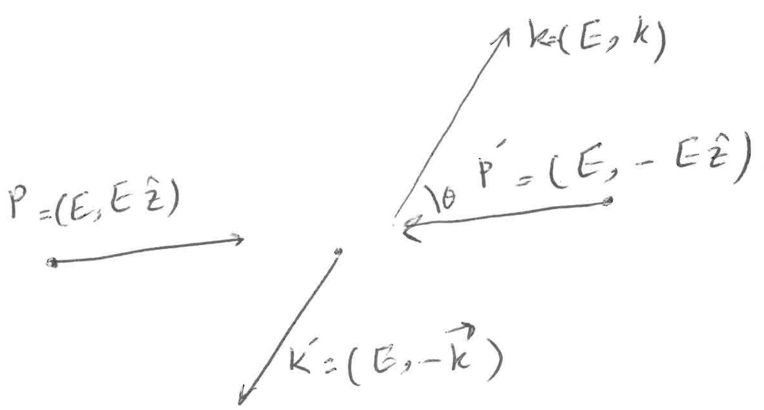
Non-relativistic limit : $e^+e^- \longrightarrow \mu^+\mu^-$

$E \simeq m_\mu$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_{cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left[\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right.$$

$$\longrightarrow \frac{\alpha^2}{2E_{cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}} = \frac{\alpha^2}{2E_{cm}^2}\frac{|k|}{E}$$



$$\mathcal{M} = \frac{e^2}{q^4}\left(\bar{v}(p')\gamma^\mu u(p)\right)\Big(\underbrace{\bar{u}(k)\gamma_\mu v(k')}_{}\Big)$$

non-relativistic

relativistic

relativistic $\quad e^-_L e^+_R$

$$\bar{u}(p')\gamma^\mu u(p) = -2E\,(0, 1, i, 0)$$

$$u(k) = \sqrt{m}\begin{pmatrix}\xi\\ \xi\end{pmatrix} \qquad v(k') = \sqrt{m}\begin{pmatrix}\xi'\\ -\xi'\end{pmatrix}$$

$$\bar{u}(k)\,\gamma^\mu\, v(k^-) = m\,(\xi^{+\dagger}\ \xi^{+\dagger}) \begin{pmatrix} \bar\sigma^\mu & 0 \\ 0 & \sigma^\mu \end{pmatrix} \begin{pmatrix} \xi^- \\ -\xi^- \end{pmatrix}$$

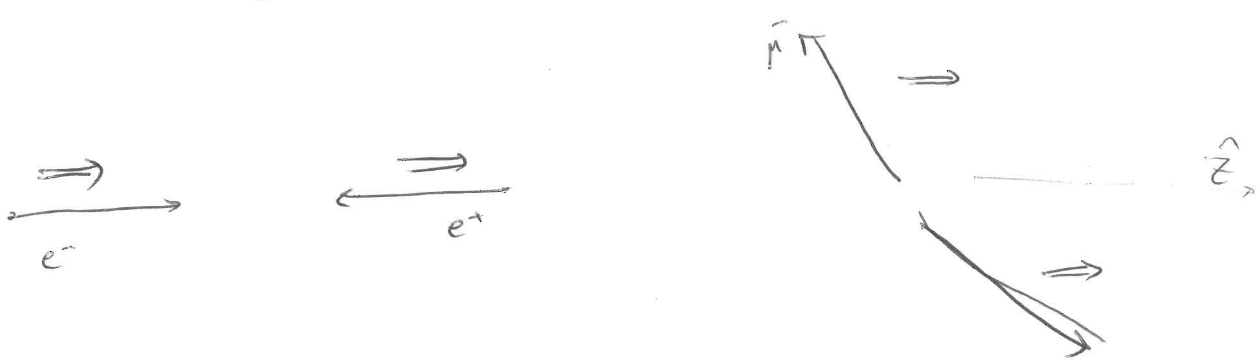
$$= \begin{cases} 0 & \mu = 0 \\ -2m\,\xi^{\dagger}\sigma^i\xi^- & \mu = i \end{cases}$$

$$\boxed{\frac{e^2}{q^2} \simeq \frac{e^2}{4m^2}}$$

$$\mathcal{M}(e^-_R e^+_L \to \mu^+\mu^-) = -2e^2\,\xi^{\dagger}\underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{2\times 2}\xi^-$$

no angular dependence

S-wave
$L=0$



$$\frac{d\sigma}{d\Omega}(e^-_R e^+_L \to \mu^+\mu^-) = \frac{\alpha^2}{E_{cm}^2}\frac{|k|}{E} = \frac{d\sigma}{d\Omega}(e^+_L e^-_R \to \mu^+\mu^-)$$

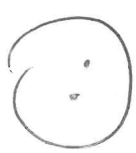
$$\frac{d\sigma}{d\Omega} = (e^-e^+ \to \mu^-\mu^+) = \frac{\alpha^2}{2E_{cm}^2}\frac{|k|}{E}$$

$2 \times \frac{1}{4}$

Bound states

$e^+e^- \longrightarrow \underbrace{\mu^+\mu^-}_{\text{bound}}$ even below threshold

Reviews $\begin{cases} \text{Bodwin, Yennie, Gregorio} & \text{RMP 57 (1985) 723} \\ \text{Sapirstein Yennie} & \text{1990} \end{cases}$



bound state

non-relativistic

~~Quantum mechanics~~

Quantum Field Theory

$$\mathcal{M}(\uparrow\uparrow \longrightarrow k_1\uparrow, k_2\uparrow) = -2e^2$$

$$R = \frac{\vec{r}_1 + \vec{r}_2}{2} \qquad r = \vec{r}_1 - \vec{r}_2$$

$$K = \vec{k}_1 + \vec{k}_2 \qquad \vec{k} = \frac{\vec{k}_1 - \vec{k}_2}{2} \qquad \underset{\text{COM}}{\vec{K} = 0}$$

$\mu^+\mu^-$ نیروی کلمب

حل معادله شرودینگر

$$\tilde{\psi}(k) = \int d^3x \, e^{i\vec{k}\cdot\vec{r}} \, \psi(\vec{r}) \qquad \int \frac{d^3k}{(2\pi)^3} |\tilde{\psi}(k)|^2 = 1$$

$$|B\rangle = \sqrt{2M} \int \frac{d^3k}{(2\pi)^3} \tilde{\psi}(k) \frac{1}{\sqrt{2m}\sqrt{2m}} \, |k\uparrow, -k\uparrow\rangle$$

Bound state جرم

$M \simeq 2m$

normalization

$$|p, s\rangle \triangleq \sqrt{2E_p}\, a_p^{s\dagger}|0\rangle \quad \Big\{ \quad \underbrace{\langle p, r | q, s\rangle = 2E_p (2\pi)^3 \delta^3(p-q)\, \delta^{rs}}_{\text{relativistically normalized}}$$

$$\mathcal{M}(\uparrow\uparrow \to B) = \sqrt{2M} \int \frac{d^3k}{(2\pi)^3}\, \tilde{\Psi}^*(k) \frac{1}{\sqrt{2m}} \frac{1}{\sqrt{2m}}$$

$$\times\, \mathcal{M}(\uparrow\uparrow \to k\uparrow, -k\uparrow)$$

فرض کنید $\mathcal{M}(\uparrow\uparrow \to k\uparrow, -k\downarrow)$ مستقل از k است (S-wave) (مثل حالتی ک

(در نظر نگیرم)

$$\int d^3k \frac{\tilde{\Psi}^*(k)}{(2\pi)^3} = \Psi^*(0)$$

راس نقطه‌ای

در مورد ما $(e^-e^+ \to \mu^+\mu^-)$

$$\mathcal{M}(\uparrow\uparrow \to B) = \sqrt{\frac{2}{M}}\,(-2e^2)\,\Psi^*(0)$$

حالا کلی

$$i\mathcal{M}(\text{something} \to k, k^-) = \xi^\dagger \left[\Gamma(k)\right] \xi^-$$

$$\Gamma(k) = \begin{pmatrix} - & - \\ - & - \end{pmatrix}$$

حالت اسپین 1

$$\xi \bar{\xi}^{\dagger} = \frac{1}{\sqrt{2}} \hat{n}^{*} \cdot \sigma \qquad \hat{n} = \text{بردار یکه}$$

$$\hat{n} = \frac{\hat{x} + i\hat{y}}{\sqrt{2}} \qquad \uparrow\uparrow$$

$$\hat{n} = \frac{x - i\hat{y}}{\sqrt{2}} \qquad \downarrow\downarrow$$

$$\hat{n} = \hat{z} \qquad \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}$$

حالت اسپین صفر

$$\xi \xi^{\dagger} \Rightarrow \frac{1}{\sqrt{2}} \mathbb{1}$$

کلی برای اسپین 1

$$i\mathcal{M}(\text{Something} \to B) = \sqrt{\frac{2}{M}} \int \frac{d^3k}{(2\pi)^3} \tilde{\Psi}^{*}(k)\, \mathrm{Tr}\left(\frac{n^{*} \cdot \sigma}{\sqrt{2}} \Gamma(k) \right)$$

کلی برای اسپین صفر

$$i\mathcal{M}(\text{Something} \to B) = \sqrt{\frac{2}{M}} \int \frac{d^3k}{(2\pi)^3} \tilde{\Psi}^{*}(k)\, \mathrm{Tr}\left(\frac{\Gamma(k)}{\sqrt{2}} \right)$$

Vector meson production and decay

$$\mathcal{M}(e^-_R e^+_L \to B) = \sqrt{\frac{2}{M}} (-2e^2)\, n^{*} \cdot \varepsilon_{+}\, \Psi^{*}(0)$$

$$\varepsilon_{+} = \frac{\hat{x} + i\hat{y}}{\sqrt{2}}$$

$$\frac{1}{4}\left(|n^{*}\cdot \varepsilon_{+}|^{2} + |n^{*}\cdot \varepsilon_{-}|^{2} \right) = \frac{1}{4}\left(|n^{x}|^{2} + |n^{y}|^{2} \right)$$

unpolarized $e^{-}e^{+}$ beam

polarized along $\hat{z}$ (یادآوری)

با

مومنتوم

$$\sigma(e^{+}e^{-}\to B) = \frac{1}{2}\,\frac{1}{2m}\,\frac{1}{2m}\int\frac{d^{3}K}{(2\pi)^{3}}\,\frac{1}{2E_{k}}(2\pi)^{4}\delta(p+p'-K)\,\frac{2}{M}(4e^{4})\,\frac{1}{2}\,|\Psi(0)|^{2}$$

اگه کفینه از کجا اومد

اینها از کجا امده؟

$$\delta(P^{0}-K^{0}) = 2K^{0}\,\delta(P^{2}-K^{2})$$

$$\sigma(e^{+}e^{-}\to B) = 64\pi^{3}\alpha^{2}\,\frac{|\Psi(0)|^{2}}{M^{3}}\,\delta(E_{cm}^{2}-M^{2})$$

finite life time

$B \to e^{+}e^{-}$

$$\Gamma(B\to e^{+}e^{-}) = \frac{1}{2M}\int d\Pi_{2}\,|M|^{2}$$

$$M(e^{-}e^{+}\to B) = M^{*}(B\to e^{-}e^{+})$$

ص۷

$$\Gamma = \frac{1}{2M} \int \left(\frac{1}{8\pi} \frac{d\cos\theta}{2} \right) \frac{8e^4}{M} |\psi(0)|^2 \left(|n\cdot\varepsilon|^2 + |n\cdot\varepsilon^*|^2 \right)$$

$$\Downarrow$$

$$\Gamma(B \to e^+e^-) = \frac{16\pi\alpha^2}{3} \frac{|\psi(0)|^2}{M^2}$$

~~کلی~~

$$\sigma(e^+e^- \to B) = 4\pi^2 \frac{3\,\Gamma(B\to e^-e^+)}{M} \delta(E_{cm}^2 - M^2)$$

کلی

$$\sigma(e^+e^- \to B) \; 4\pi^2 \frac{(2J+1)\,\Gamma(B\to e^+e^-)}{M} \delta(E_{cm}^2 - M^2)$$

quarkonium $\qquad J/\psi \qquad \Upsilon$

$3\times$ درمورد اینها

$$|B\rangle = \frac{|r\bar r\rangle + |g\bar g\rangle + |b\bar b\rangle}{\sqrt{3}}$$

$$|\mathcal{M}(B\to e^+e^-)|^2 = \left| \frac{\langle r\bar r|e e^+\rangle + \langle b\bar b|e^-e^+\rangle + \langle g\bar g|e^-e^+\rangle}{\sqrt{3}} \right|^2 = 3\,|\langle r\bar r|e e^+\rangle|^2$$

can be estimated from a non-relativistic model of the $q\bar{q}$ system with a phenomenological chosen potential

؟ ← $\Psi(0)$

Alternatively

~~$\Gamma(q\bar{q}$~~

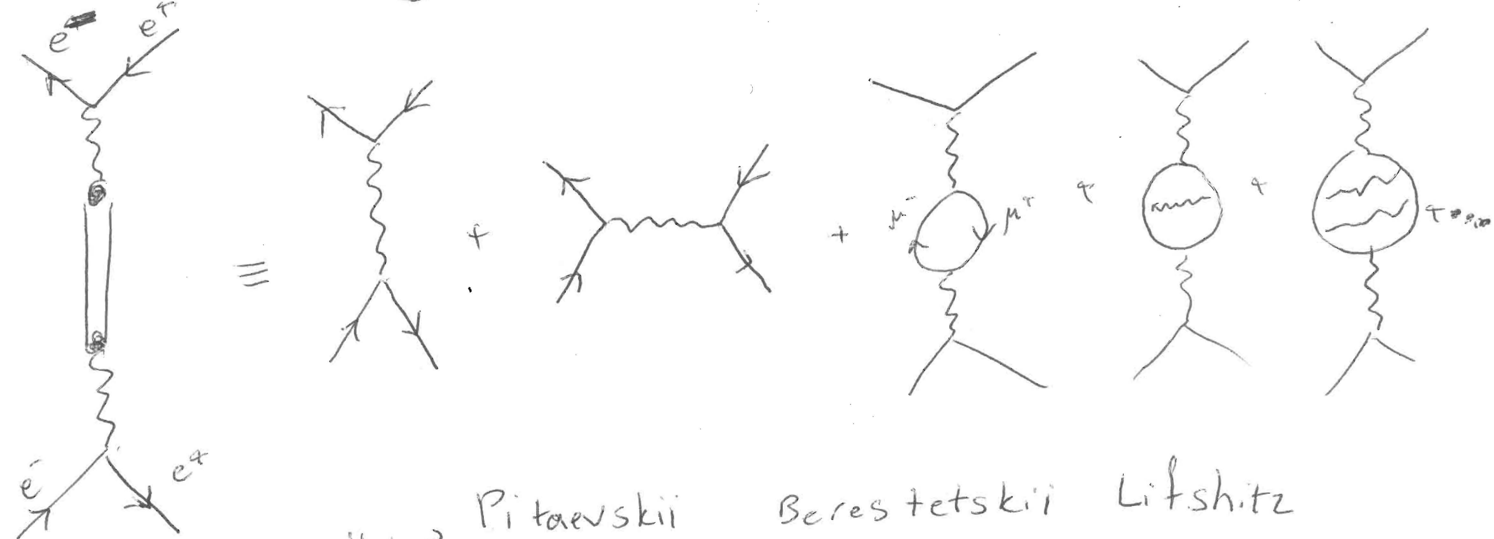
$$\Gamma(B(q\bar{q}) \to e^- e^+) = \frac{16\pi^2 Q^2 |\Psi(0)|^2}{M^2}$$

1 S spin 1 state of $s\bar{s}$ φ meson

$\Gamma(\varphi \to e^+ e^+) = 1.4$ kev $M = 1.02$ GeV

$|\Psi(0)|^2 = (1.2 \text{ fm})^{-3}$ معقول!

نصف مکانیک کوانتمی نصف نظریه میدان



Pitaevskii Berestetskii Lifshitz

در سال ۱۹۸۲ نشان دادند که این

دو رهیافت معادل هستند.