Gravitational Aharonov-Bohm effect

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Outline of the talk

1-The C~olella~O~werhauser~W~erner~ effect

2-Aharonov-Bohm effect

3-1+3 formulation of spacetime decomposition

4-Gravitoelectromagnetism and Quasi-Maxwell form of EFEs

5-Two different versions of GAB effect

6-GA-B in stationary and static spacetimes

7-Gaussian curvature and global effects

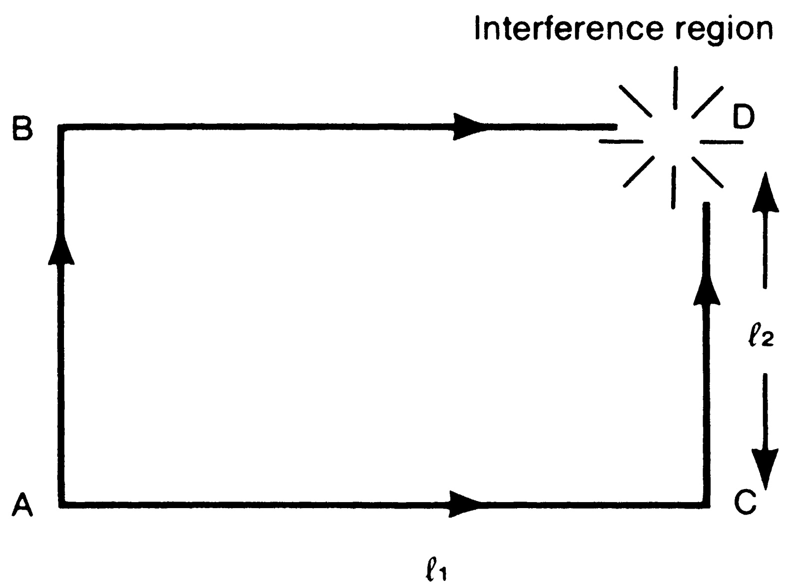
The COW effect (experiment)

Gravity in quantum mechanics?

$$\left[-\left(\frac{\hbar^2}{2m}\right)\nabla^2 + m\Phi_{\text{grav}}\right]\psi = i\hbar\frac{\partial\psi}{\partial t}.$$

$$m\ddot{\mathbf{x}} = -m\nabla\Phi_{\text{grav}} = -mg\hat{\mathbf{z}}.$$

$$\frac{\mathbf{p}^2}{2m} + mgz = E.$$



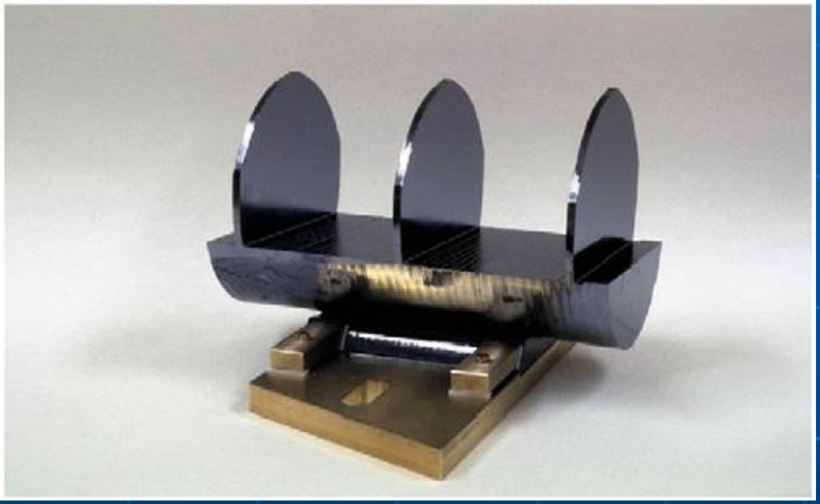
$$\phi_{ABD} - \phi_{ACD} = -\frac{\left(m_n^2 g l_1 l_2 \lambda \sin\delta\right)}{\hbar^2}.$$

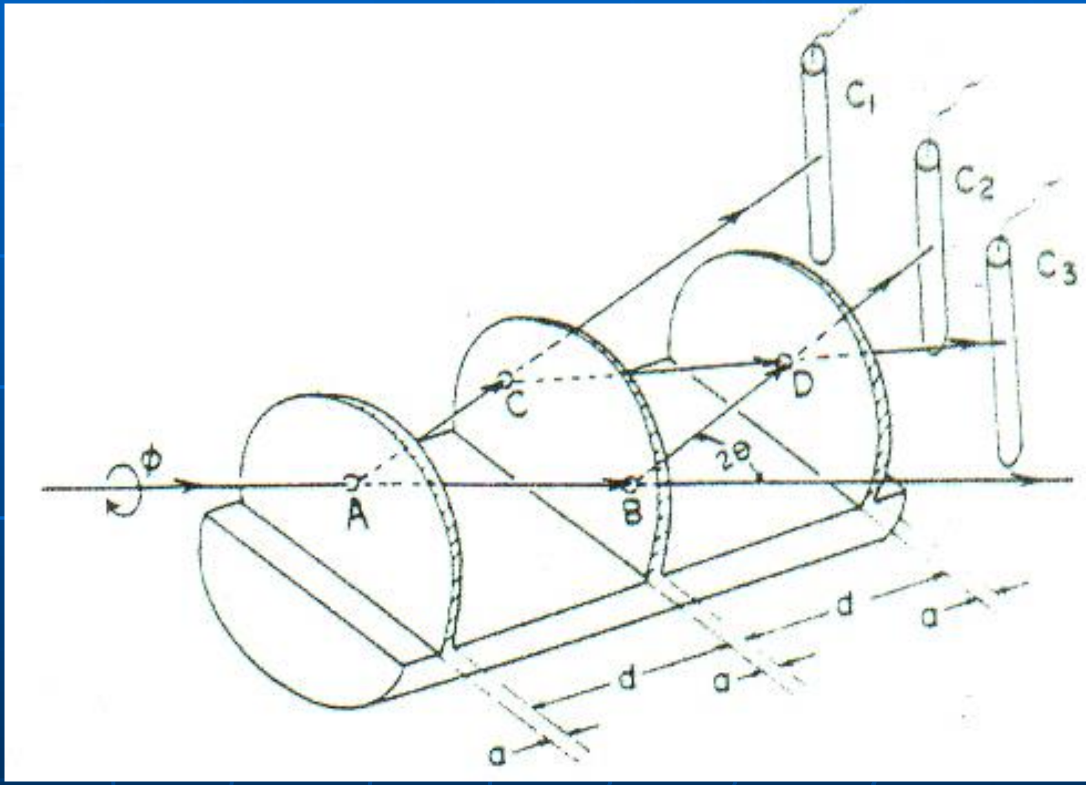
A.W. Overhauser, R. Colella, PRL (1974)

How the COW happened?

Gravitationally-Induced Quantum Interference

Neutron interferometry





$$\phi_{ABD} - \phi_{ACD} = -\frac{\left(m_n^2 g l_1 l_2 \lambda\!\!\!^{-} \sin\delta\right)}{\hbar^2}.$$

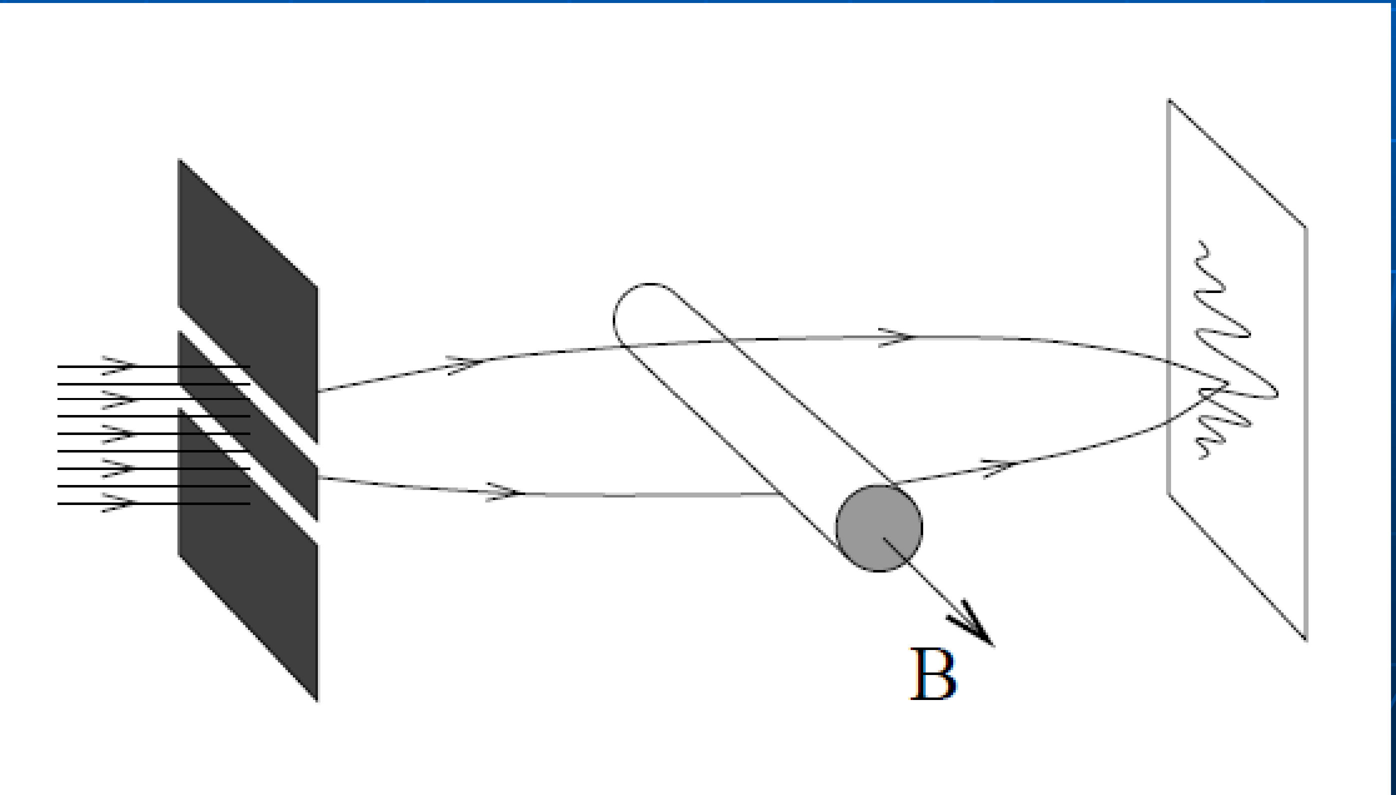
R. Colella, A.W. Overhauser, S. A. Werner, PRL (1975)

Aharonov-Bohm (Ehrenberg-Siday) effect

Theoretical study:

I-Electrons move in field-free region: B = 0 outside the solenoid

II-Classically fields interact locally



Vector potential in classical ED

- in classical ED the field quantities ***B*** and ***E*** can be derived from their potentials

$$\mathbf{B} = \text{rot}\,\mathbf{A}$$

$$\mathbf{E} = -\text{grad}\,\varphi - \dot{\mathbf{A}}$$

and they are invariant under following transformations (gauge choice):

$$\mathbf{A} \rightarrow \mathbf{A}' = \mathbf{A} + \text{grad}\,\Lambda$$

$$\varphi \rightarrow \varphi' = \varphi - \dot{\Lambda}$$

Classically A does not have any direct physical significance

In the absence of a magnetic field

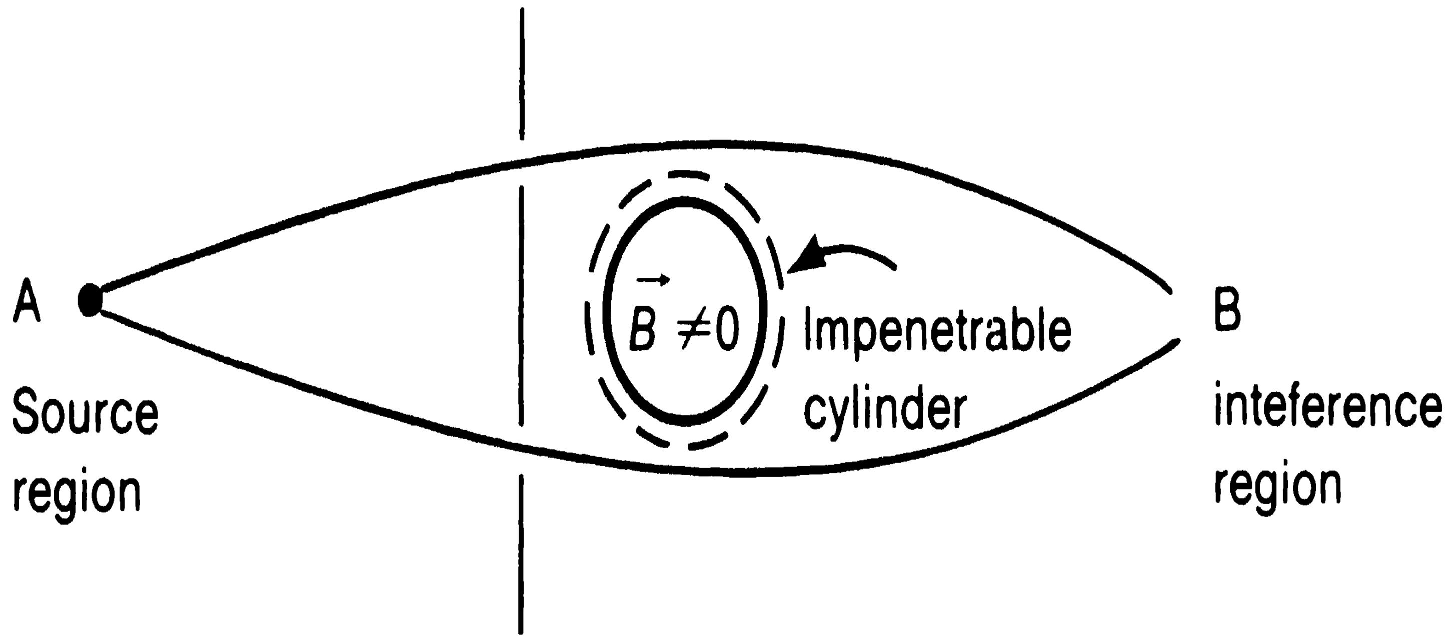
$$\left\{\frac{1}{2m}(-i\hbar\nabla)^2 + V - E\right\}\psi_0 = 0,$$

In the presence of a magnetic field

$$\left\{\frac{1}{2m}(-i\hbar\nabla - e\mathbf{A})^2 + V - E\right\}\psi = 0.$$

The solutions are related by a phase factor

$$\psi(\mathbf{r}) = \exp\left[\frac{ie}{\hbar}\int^{\mathbf{r}} \mathbf{A}(\mathbf{r}')\cdot d\mathbf{r}'\right]\psi_0(\mathbf{r}).$$



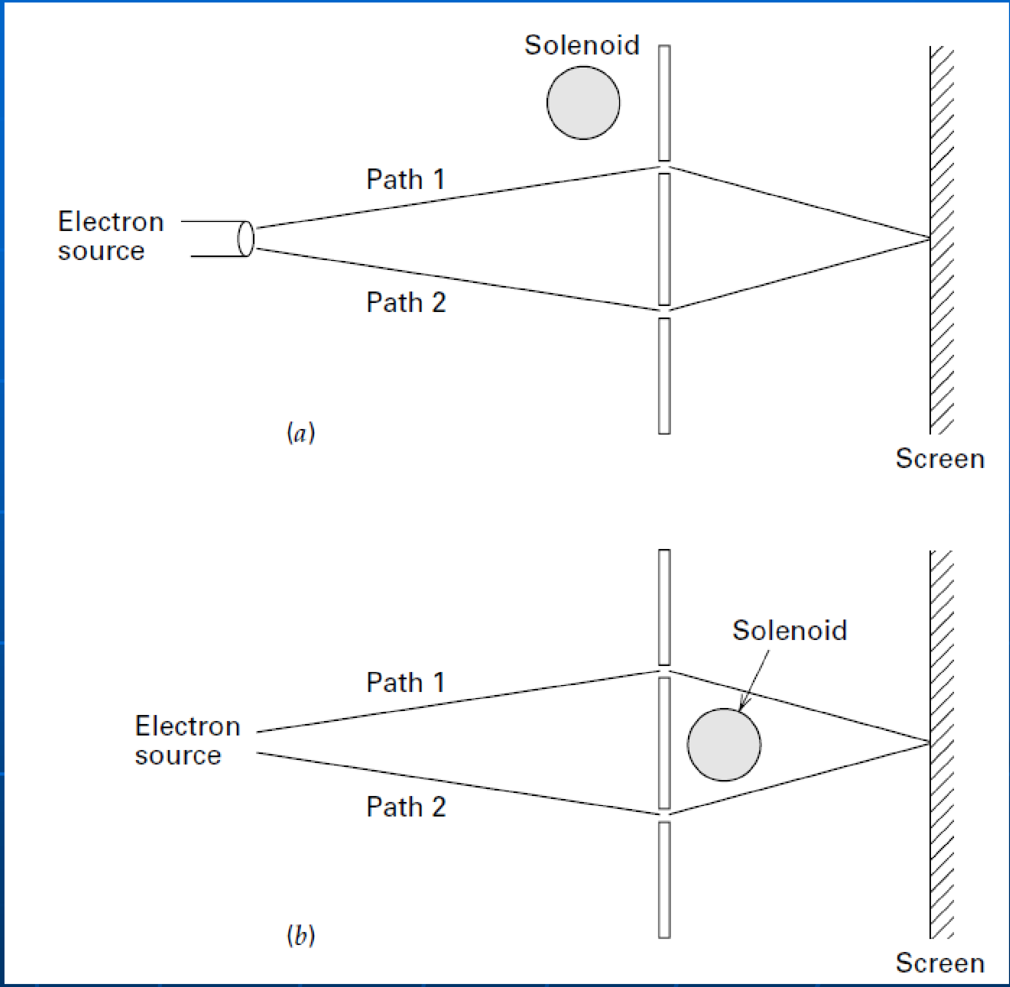
$$\left[\left(\frac{e}{\hbar c}\right)\int_{\mathbf{x}_1}^{\mathbf{x}_N}\mathbf{A}\cdot d\mathbf{s}\right]_{\text{above}} - \left[\left(\frac{e}{\hbar c}\right)\int_{\mathbf{x}_1}^{\mathbf{x}_N}\mathbf{A}\cdot d\mathbf{s}\right]_{\text{below}} = \left(\frac{e}{\hbar c}\right)\oint \mathbf{A}\cdot d\mathbf{s}$$

$$= \left(\frac{e}{\hbar c}\right)\Phi_B,$$

$$\psi = \psi_1 + \psi_2 = R_1 e^{iS_1} + R_2 e^{iS_2} = e^{iS_1}\left(R_1 + e^{i(S_2 - S_1)} R_2\right)$$

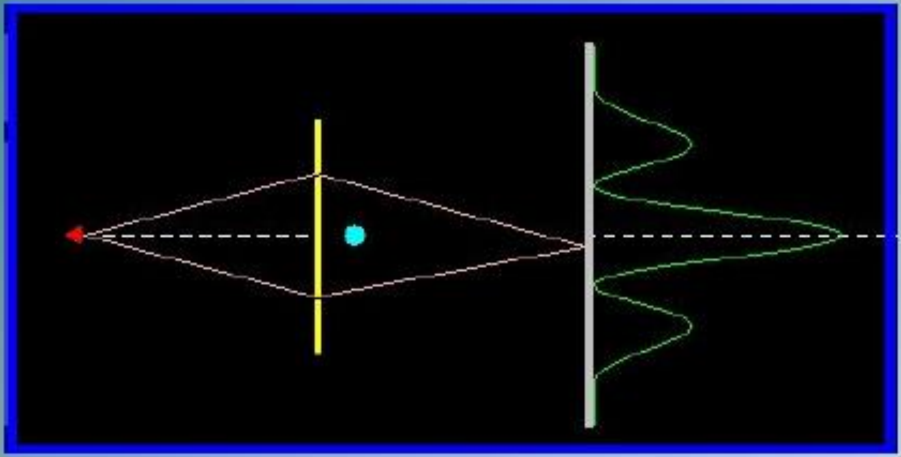
If we want to describe the influence of B NOT as an ACTION AT A DISTANCE (A NON-LOCAL effect) we MUST use the vector potential.

R. P. Feynman

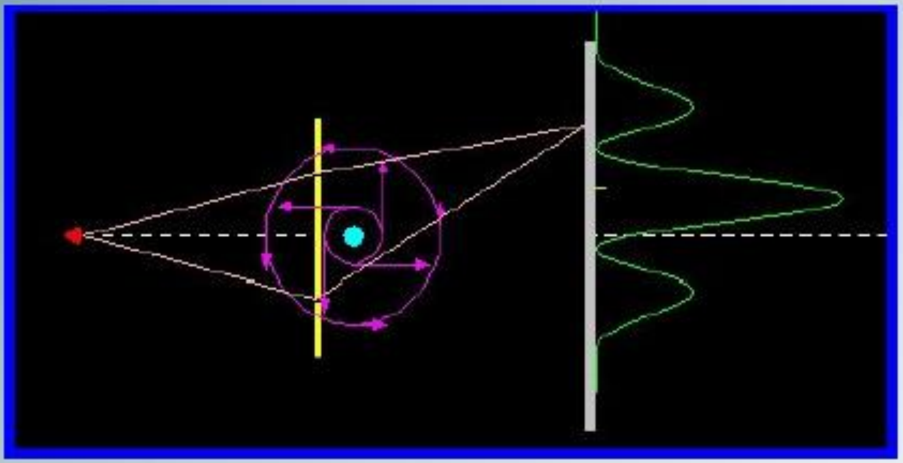


$$\psi = e^{-\frac{ie}{\hbar}\int_1 d\ell' \cdot \mathbf{A}(\mathbf{r}')}\psi_1 + e^{-\frac{ie}{\hbar}\int_2 d\ell' \cdot \mathbf{A}(\mathbf{r}')}\psi_2$$

$$= e^{-\frac{ie}{\hbar}\int_1 d\ell' \cdot \mathbf{A}(\mathbf{r}')} e^{iS_1}\left(R_1 + R_2 e^{i(S_2 - S_1) + ie\Phi/\hbar}\right)$$



- interference pattern in a double-slit experiment when the enclosed flux **B** equals zero

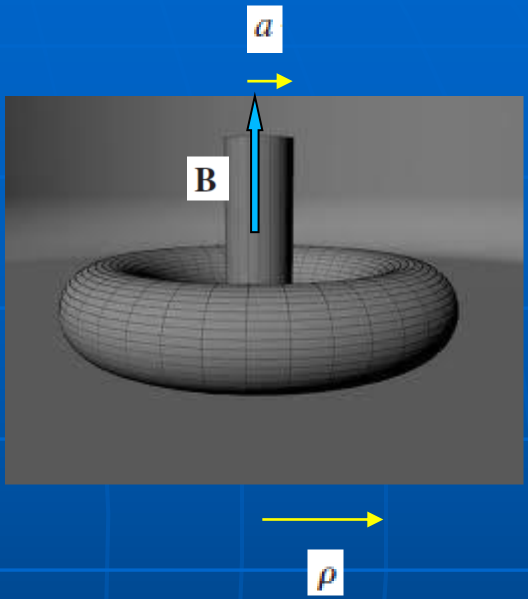


- when a magnetic flux is turned on, the interference pattern gets shifted

Experimentally verified by R. G. Chambers PRL,1960; A. Tonomura et al., PRL, 1982

Energy and angular momentum in A-B effect

$$A_\rho = A_z = 0$$
$$A_\phi = \tfrac{1}{2}\rho B$$



$$\frac{1}{2m_e}(-i\hbar\,\nabla + e\mathbf{A})^2 + V(\rho)$$

$$a \ll \rho$$

Torus is so narrow that ρ is taken as a constant inside it

$$H = \frac{1}{2m_e\rho^2}\left(L_z + \frac{e\Phi}{2\pi}\right)^2 \quad \Longrightarrow \quad E = \frac{\hbar^2}{2m_e\rho^2}\left(m + \frac{e\Phi}{2\pi\hbar}\right)^2$$

$$L_z\psi = m\hbar\,\psi$$

Distances and time intervals in spacetimes

Space-time decomposition into spatial and temporal sections: why?

Measurement : 4d → 3d

I-A-observer B-observable

↓

II-1+3 vs 3+1 formulations

Threading and foliation

Distances and time intervals

1+ 3 : Threading (projection formalism)

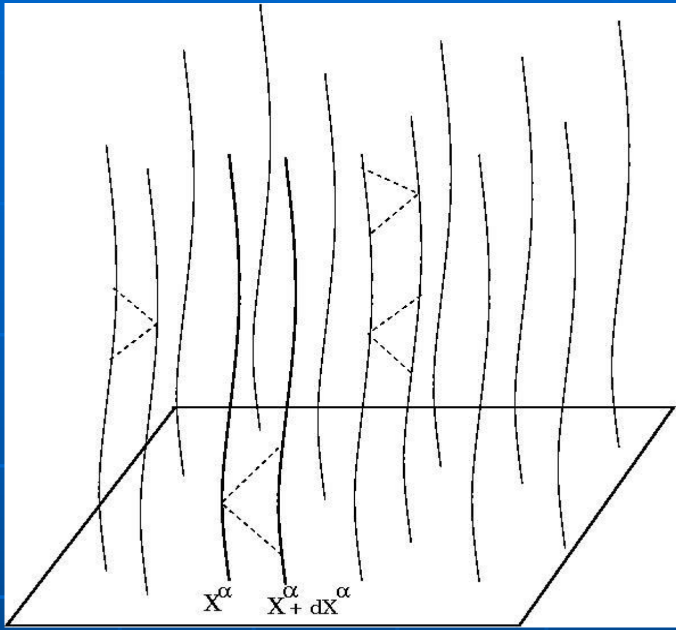
Gravitoelectromagnetism

$$ds^2 = d\tau_{\text{syn}}^2 - dl^2 = h(dx^0 - A_\alpha dx^\alpha)^2 - \gamma_{\alpha\beta}dx^\alpha dx^\beta,$$

$$A_\alpha = -\frac{g_{0\alpha}}{g_{00}}$$

For stationary space-times

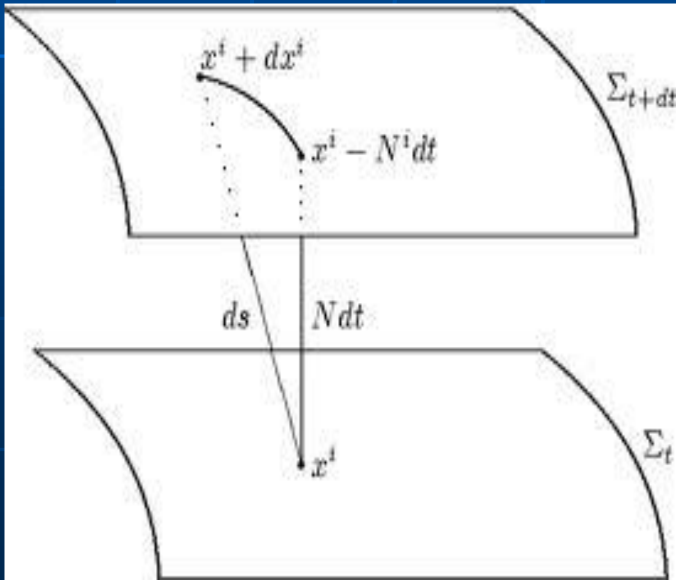
$$dl^2 = \gamma_{\alpha\beta}dx^\alpha dx^\beta = \left(-g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}}\right)dx^\alpha dx^\beta$$



3+1 : Foliation

ADM lapse and shift

$$ds^2 = -N^2 dt^2 + g_{\alpha\beta}\left(dx^\alpha + N^\alpha dt\right)\left(dx^\beta + N^\beta dt\right)$$



Quasi-Maxwell form of EFEs

$$R_{\mu\nu} - {}^1\!/\!{}_2 g_{\mu\nu} R = \frac{8\pi G}{c^2} T_{\mu\nu}$$

$$\nabla \times \mathbf{E}_g = 0, \qquad \nabla \cdot \mathbf{B}_g = 0,$$

$$\nabla \cdot \mathbf{E}_g = 1/2 h B_g^2 + E_g^2 - 8\pi\left[\frac{p+\rho}{1-v^2} - \frac{\rho - p}{2}\right],$$

$$\nabla \times (\sqrt{h}\mathbf{B}_g) = 2\mathbf{E}_g \times (\sqrt{h}\mathbf{B}_g) - 16\pi\left[\frac{p+\rho}{1-v^2}\mathbf{v}\right],$$

$$^{(3)}R\mu\nu = -E_g^{\mu;\nu} + h(B_g^\mu B_g^\nu - B_g^2\gamma^{\mu\nu}) + E_g^\mu E_g^\nu + 8\pi\left[\frac{p+\rho}{1-v^2}v^\mu v^\nu + \frac{\rho - p}{2}\gamma^{\mu\nu}\right].$$

$$\mathrm{f}_g = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(\mathrm{E}_g + \frac{\mathrm{v}}{c} \times e^\phi \mathrm{B}_g\right)$$

$$\mathbf{E}_g = -\frac{\nabla h}{2h} \doteq -\nabla \ln|\xi|, \qquad \mathbf{B}_g = \nabla \times \mathbf{A}.$$

$$h \equiv g_{00} = e^{2\phi}$$

D. Lynden-Bell and M. N-Z, RMP, 1998

Gravitational Aharonov-Bohm effect

Two versions of GA-B effect :

I- Stationary (non-static) space-time

Gravitomagnetic A-B effect

(Rotating mass dist.)

Spacetime has a time-like killing vector field

$$L_\xi g_{\mu\nu} \equiv \xi^\kappa \partial_\kappa g_{\mu\nu} + g_{\kappa\nu}\partial_\mu \xi^\kappa + g_{\mu\kappa}\partial_\nu \xi^\kappa = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0 .$$

g_{0i} are non-zero

Closer analogy to EM A-B effect due to the appearance of **B** g

II-Static space-time *(static mass dist.)*

Gravitoelectric A-B effect

Time-like Killing vector field is (globally) hypersurface orthogonal

$$\xi_\mu \equiv \lambda \partial_\mu \psi$$

GA-B in stationary space-times

When there is a $\mathbf{B}_g$ field one could think of
Gravitomagnetic Aharonov-Bohm effect

Weak field approximation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Einstein field eqs.

$$\partial^\beta \partial_\beta h_{\mu\nu} = (\partial_0^2 - \nabla^2) h_{\mu\nu} = -(16\pi G/c^4) S_{\mu\nu}$$

$$S_{\mu\nu} = T_{\mu\nu} - g_{\mu\nu} T/2$$

$$T_{\mu\nu} = \rho u_\mu u_\nu$$

E. G. Harris, AJP 1995

$$ds^2=(1+\phi/c^2)c^2\,dt^2-(1-\phi/c^2)(dx^2+dy^2+dz^2)$$
$$-2\mathbf{A}\cdot\mathbf{dx}/c.$$

$$2\mathbf{g}=-\nabla\phi$$
$$\mathbf{H}=\nabla\times\mathbf{A}.$$

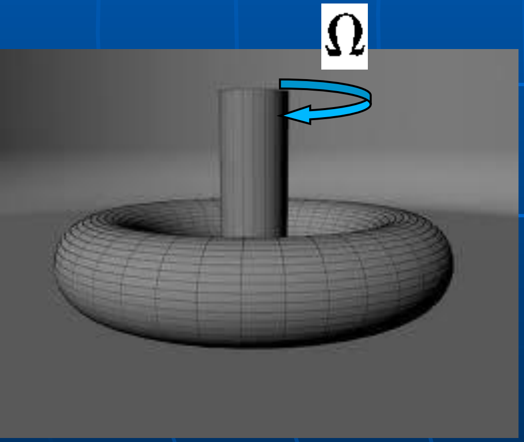
$$\nabla\cdot\mathbf{H}=0,$$
$$\nabla\times\mathbf{g}=0.$$

$$\nabla\cdot\mathbf{g}=-4\pi G\rho,$$
$$\nabla\times\mathbf{H}=(4\pi/c)(-4G\rho\mathbf{u}).$$

$$H=8\pi G\rho\Omega(r_0^2-r^2)/c$$

H = 0 for r > r_0

$$\Phi=4\pi^2 G\rho\Omega r_0^4/c$$



$$ds^2=(1+\phi/c^2)(dx^0)^2-(1-\phi/c^2)[dr^2+r^2d\theta^2+dz^2]$$
$$-2\Phi d\theta\,dx^0/2\pi c^2,$$

$$E^2=M^2c^4+\hbar^2c^2\{(n_r\pi/a)^2+(n_z\pi/b)^2$$
$$+(1/r_{\text{av}}^2)[(m-FE/\hbar c)^2-1/4]\}.$$

Solving Klein-Gordon eqn.
for a scalar field of mass M

$$F=(1+\phi)^{-1/2}\Phi/2\pi c^2$$

Globally stationary but locally static space-times

Now the possibility arises that, while the vector field ξ may be globally stationary in some region of space-time, it satisfies $\xi_\mu \equiv \lambda \partial_\mu \psi$ only locally. That is, in any subregion ot the entire region in question, ξ_μ is hypersurface orthogonal, but there exists no global (single-valued) function ψ for the entire region which satisfies $\xi_\mu \equiv \lambda \partial_\mu \psi$ Such a region of space-time will be called globally stationary but locally static.

For a globally stationary but locally static space-time The integral of the gravitomagnetic potential over a closed path is non- zero

$$\oint_C A_{g_\mu} dx^\mu = \oint_C \mathbf{A}_g . d\mathbf{x} \neq 0$$

Van Stockum solution of a rigidly rotating dust cylinder has such a property.

Stachel PRD, 1981

GA-B in Static space-times

The metric $g_{\mu\nu}$ and Riemann curvature tensor $R^{\mu}_{\alpha\nu\beta}$ play roles analogous to those of the potentials and field strengths respectively in electromagnetism

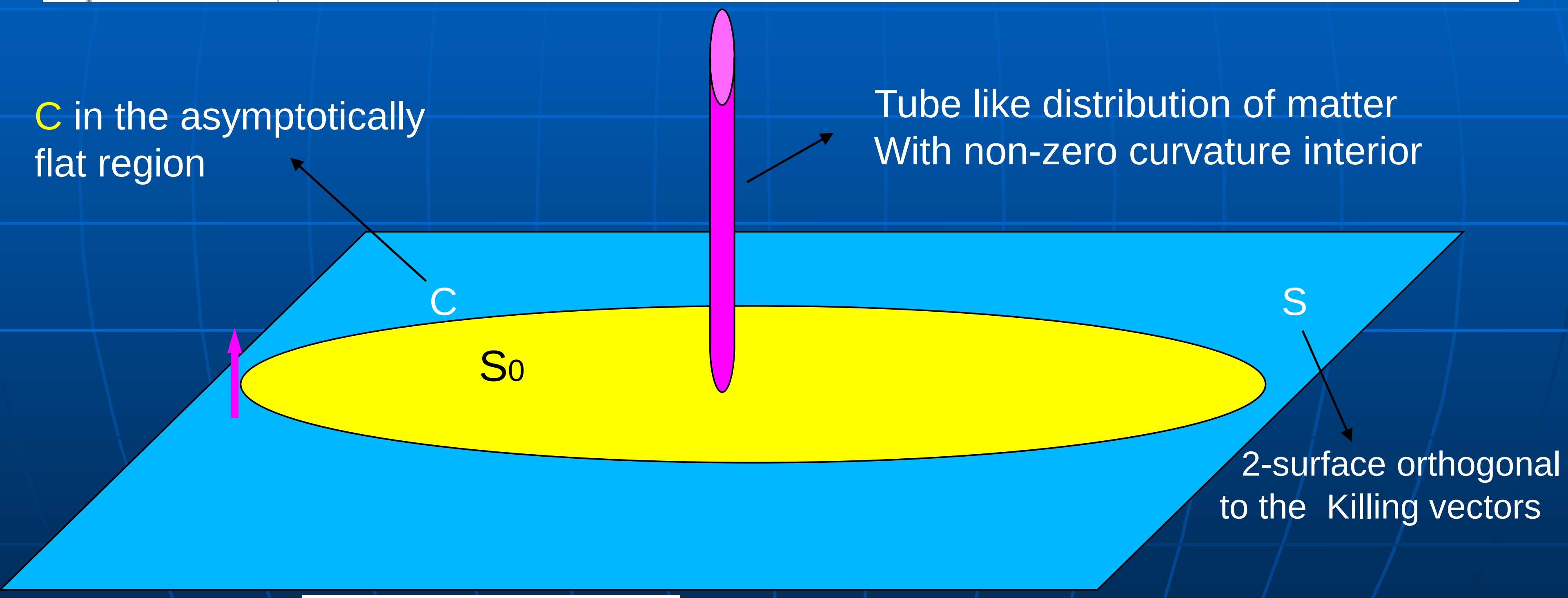
Newton :	$g^{ij}\Phi_{,ij} = 0$
Maxwell :	$g^{\mu\nu}\Phi_{\rho,\mu\nu} = 0$
Einstein :	$g^{\mu\nu}g_{\rho\sigma,\mu\nu} + \cdots = 0$

Particles constrained to move in a region where the Riemann tensor vanishes may nonetheless exhibit physical effects arising from non-zero curvature in a region from which they are excluded

Ford & Vilenkin's approach

Space-time of a Tube like distribution of matter with a time-like and a space-like Killing vector fields generating time translation and translations along the z-axis.

assume the existence of two Killing vector fields, t^μ which is time-like and z^μ which is space-like and generates translations along the direction of the tube. Thus the matter distribution is stationary and uniform along z^μ. We also assume that the space is asymptotically flat in a direction perpendicular to z^μ (i.e. at large distances from the



Gauss-Bonnet formula

$$\alpha = \int_{S_0} K \, da$$

S is asymptotically a conical surface rather than a plane

Consider the static metric

$$ds^2 = -D^2(r,\theta)\,dt^2 + A^2(r,\theta)\,dr^2 + B^2(r,\theta)\,d\theta^2 + E^2(r,\theta)\,dz^2$$

Metric of the 2-surface is

$$ds_S^2 = A^2(r,\theta)\,dr^2 + B^2(r,\theta)\,d\theta^2.$$

Choose the coordinates so that

$A \to 1$ and $B \sim r$ as $r \to 0$

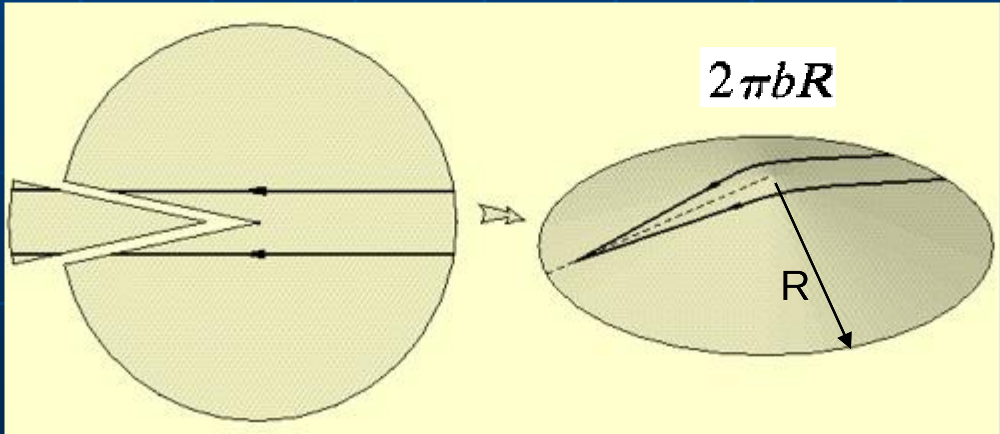
$A \to 1$ and $B \to br$ as $r \to \infty$

$0 < b < 1$

If we integrate K over S, using the above asymptotic forms of A and B, we find

$$1 - b = \frac{1}{2\pi}\int K\sqrt{g}\,d^2x$$

$$\alpha = 2\pi(1-b)$$



$\Delta = \pi b$ angle subtended at the apex

The Gaussian curvature K of S in terms of the Riemann tensor of the 4-dim space-time

$$^{(2)}R^{i}_{\ jkl} = {}^{(4)}R^{i}_{\ jkl}$$

$$K = \tfrac{1}{2}\,{}^{(2)}g^{ik}\,{}^{(2)}g^{jl}\,{}^{(4)}R_{ijkl}$$

In general it does not seem to be possible to express K in terms of the four-dimensional Ricci tensor, and hence of the energy–momentum tensor of the source, $T_{\mu\nu}$. However, in the case that the gravitational field is sufficiently weak that the linearised theory may be applied, such an expression can be given. Let the metric be

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad \text{where } \eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$$

Assuming the source to be independent of t and z

$$K = 8\pi(T_{11} + T_{22} - \tfrac{1}{2}T)$$

For a dust source with only $\rho = T_{00}$

i.e mass per unit length of the tube

→

$$1 - b = 4\pi \int \rho \, \mathrm{d}^2 x$$

Gaussian curvature & global effects

$$ds^2 = d\tau_{syn}^2 - dl^2 = h(dx^0 - A_\alpha dx^\alpha)^2 - \gamma_{\alpha\beta} dx^\alpha dx^\beta$$

$$\gamma_{\alpha\beta} = -g_{\alpha\beta} + \frac{g_{0\alpha} g_{0\beta}}{g_{00}}$$

$$\xi^a \doteq (1, 0, 0, 0)$$

$$h_{ab} = -g_{ab} + u_a u_b$$

$$\gamma_{\alpha\beta} \doteq h_{\alpha\beta} \doteq -g_{\alpha\beta} + \frac{1}{\xi_0} \xi_\alpha \xi_\beta$$

To relate the Gaussian curvature of the 2-surface to the gravitoelectromagnetic fields of the underlying spacetime we introduce, using the spacelike Killing vector η, a projection tensor from Σ_3 to S as follows

$$\tilde{h}_{\alpha\beta} = \gamma_{\alpha\beta} - n_\alpha n_\beta \;\; ; \;\; \alpha, \beta = 1, 2, 3$$

where now $n_\alpha = \frac{\eta_\alpha}{|\eta|}$ is the unit vector normal to S.

Choosing a preferred coordinate system in which $\eta \doteq \partial_z$, i.e η^α takes the following form

$$\eta^\alpha \doteq (0, 1, 0), \quad x^1 = r, \; x^2 = z, \; x^3 = \phi,$$

M.N-Z and A. Parvizi, PRD, 2013

for two-surface S with metric

$$\tilde{g}_{ij} = \gamma_{ij} \quad ; \quad i,j = 1,3$$

the Gaussian curvature K is given by

$$K = \frac{1}{2}\,\tilde{g}^{ik}\tilde{g}^{jl}\,{}^{(2)}R_{ijkl} = \frac{1}{2}\,{}^{(2)}R \quad ; \quad i,j,k,l = 1,3.$$

$${}^{(2)}R = 2K = {}^{(3)}R - 2\frac{\eta^\alpha \eta^\beta}{|\eta|^2}\,{}^{(3)}R_{\alpha\beta}$$

$${}^{(3)}R^{\mu\nu} = -E_g^{\mu;\nu} + \frac{1}{2}h(B_g^\mu B_g^\nu - B_g^2\gamma^{\mu\nu}) + E_g^\mu E_g^\nu + 8\pi[\frac{p+\rho}{1-v^2}v^\mu v^\nu + \frac{\rho - p}{2}\gamma^{\mu\nu}]$$

$$ds^2 = g_{tt}dt^2 - g_{rr}dr^2 - g_{zz}dz^2 - g_{\phi\phi}d\phi^2$$

$$K = 4\pi\rho + \mathbf{E}_g \cdot \nabla ln|\eta|$$

Static tube-like dust space-time

In asympt. Flat region $E_g \to 0 \text{ as } r \to \infty$

Cylindrically symmetric dust spacetime

Consider matching of a cylindrically symmetric static dust solution, at a given radius R, to an exterior (vacuum) asymptotically flat static solution, both of the general type (12) but now all metric components are functions only of r. As an explicit example one can think of the interior solution introduced by Teixeira and Som [22], representing counter rotating dust particles with net zero angular momentum and the following energy-momentum tensor[30]

$$T_b^a = \frac{1}{2}\rho(u_a u^b + v_a v^b), \tag{16}$$

where $u^a \doteq (u^0, 0, 0, \omega)$ and $v^a \doteq (u^0, 0, 0, -\omega)$ are the four velocities of the counter rotating particles with $u^a u_a = v^a v_a = 1$. This interior spacetime is matched to the well-known exterior Levi-Civita metric [23]

$$ds^2 = r^{4\sigma} dt^2 - r^{-4\sigma}\left[r^{8\sigma^2}(dr^2 + B^2 dz^2) + C^2 r^2 d\phi^2\right], \tag{17}$$

in which B and C are scaling parameters and σ, for small values, could be interpreted as the effective gravitational mass per unit proper length [31]. Now two cases could be considered:

(I) The closed path of the parallel transported particle encircles the nonvacuum cylindrical region (Fig. 1) so that the rotation angle (1) is given by

$$\alpha = \int_{S_0 = S_0^{\text{ext}} \cup S_0^{\text{int}}} K da = \int_{S_0^{\text{ext}}} (K da)^{\text{ext}} + \int_{S_0^{\text{int}}} (K da)^{\text{int}}, \qquad (18)$$

where the upper indices show the region of space-time in which the quantities are calculated.

$$K^{\text{ext}} = (\mathbf{E}_g \cdot \nabla \ln|\eta|)^{\text{ext}}$$

$$K^{\text{int}} = \frac{4\pi\rho}{1 - v^2} + (\mathbf{E}_g \cdot \nabla ln|\eta|)^{\text{int}}$$

$$v^2 = \frac{g_{\phi\phi}\omega^2}{g_{\phi\phi}\omega^2 - 1}$$

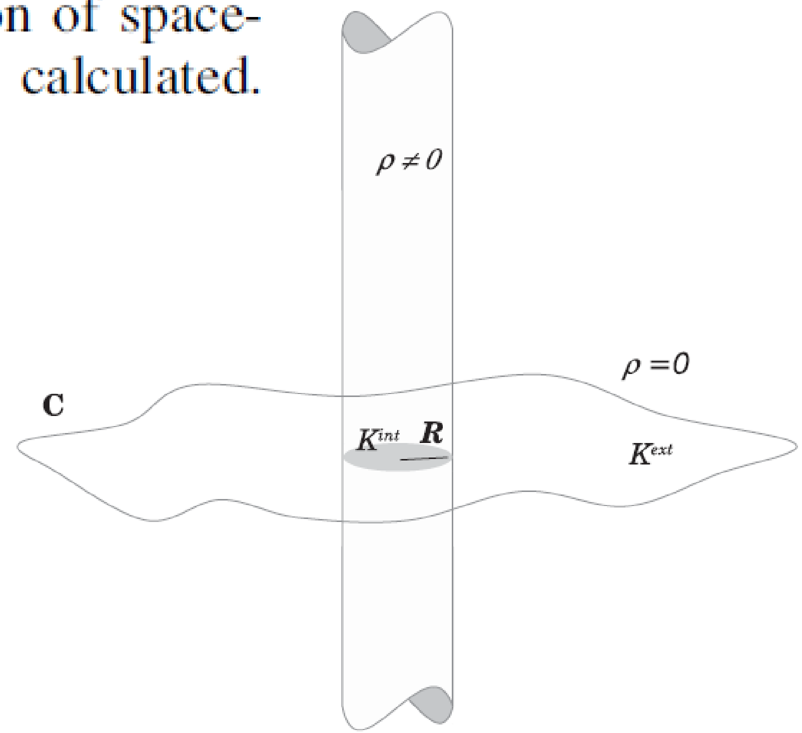


FIG. 1. Closed path C of a particle encircling the nonvacuum cylindrical region at the asymptotically flat region.

(II) In this case the closed path of the parallel transported particle does not encircle the nonvacuum cylindrical region (Fig. 2), so that the rotation angle (1) is given by

$$\alpha = \int_{S_0^{ext}} (Kda)^{ext} = \int (\mathbf{E}_g \cdot \nabla \ln |\eta| da)^{ext}, \qquad (22)$$

which is obviously of the COW-type effect leading to a gravitationally induced phase shift on the transported particle 'a-

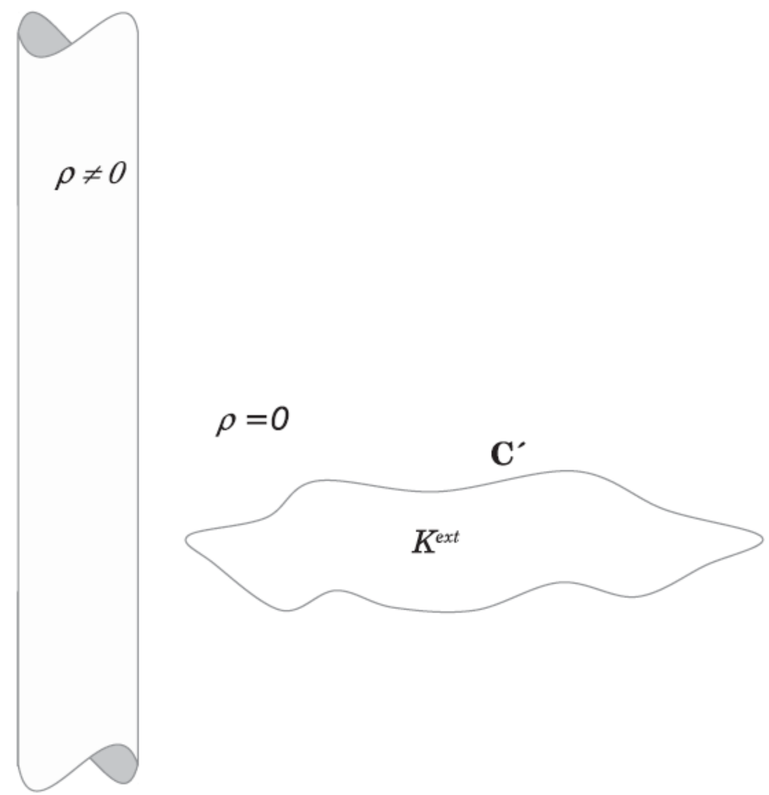


FIG. 2. Closed path C' of a particle which does not encircle the nonvacuum cylindrical region.

Experiment proposal

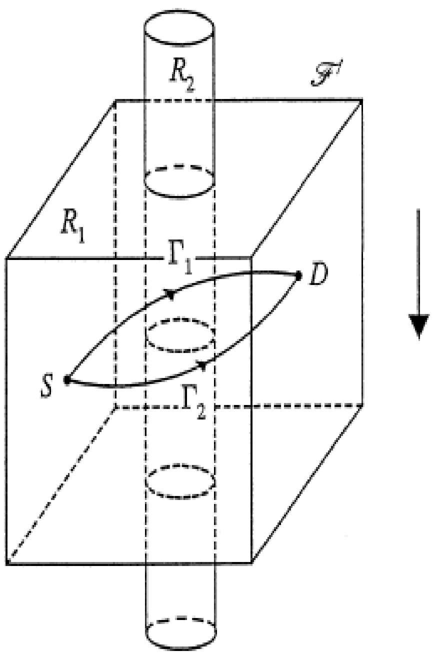


Fig. 1. A hypothetical interference experiment carried out in a freely falling (indicated by arrow), non-rotating reference frame $\mathcal{F}'$. Here S denotes a beam of nonrelativistic coherent particles that is split into two parts which travel over the paths Γ_1 and Γ_2 around the cylindrical region R_2. The particles are prevented from entering region R_2. When the particles are brought together an interference pattern is produced at D. In the freely falling reference frame $\mathcal{F}'$ the region R_1 is not simply connected. An interference experiment performed in this frame will produce a phase shift, $\Delta\theta$, relative to a frame at rest in the uniform gravitational field; this phase shift originates from a gravitoelectromagnetic AB effect as discussed in the text.

Aust. J. Phys., 1994, **47**, 245–52

An Experiment to Test the Gravitational Aharonov–Bohm Effect

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Abstract

The gravitational Aharonov–Bohm (AB) effect is examined in the weak-field approximation to general relativity. In analogy with the electromagnetic AB effect, we find that a gravitoelectromagnetic 4-vector potential gives rise to interference effects. A matter wave interferometry experiment, based on a modification of the gravity-induced quantum interference experiment of Colella, Overhauser and Werner (COW), is proposed to explicitly test the gravitoelectric version of the AB effect in a uniform gravitational field.

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Force-Free Gravitational Redshift: Proposed Gravitational Aharonov-Bohm Experiment

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We propose a feasible laboratory interferometry experiment with matter waves in a gravitational potential caused by a pair of artificial field-generating masses. It will demonstrate that the presence of these masses (and, for moving atoms, time dilation) induces a phase shift, even if it does not cause any classical force. The phase shift is identical to that produced by the gravitational redshift (or time dilation) of clocks ticking at the atom's Compton frequency. In analogy to the Aharonov-Bohm effect in electromagnetism, the quantum mechanical phase is a function of the gravitational potential and not the classical forces.

Yet another Gravitational analogue of the electrostatic A-B effect:
Gravitostaic A-B effect

$$\phi_G = \omega_C \int (\Delta U/c^2) dt$$

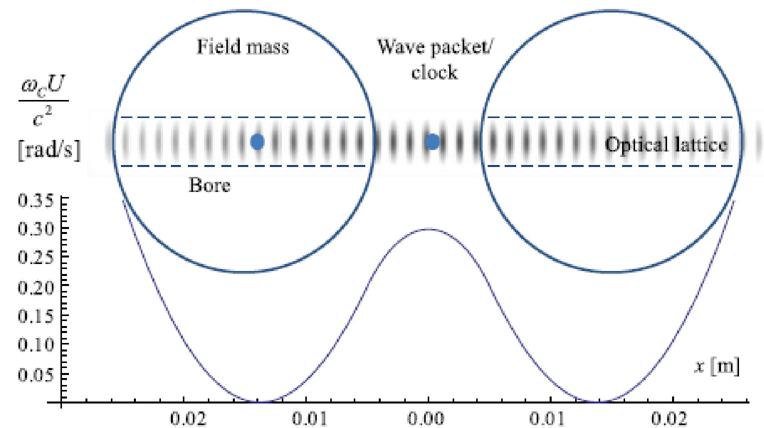

FIG. 1 (color online). Setup. The source masses (radius $R = 1$ cm, density $\rho = 10$ g/cm^3) are separated by $L = 3$ cm. Wave packets are at saddle points of the potential $U(x)$, separated by $s = 1.38$ cm. The gravitational phase shift in rad/s is plotted for cesium atoms, for which $\omega_C/(2\pi) = 3 \times 10^{25}$ Hz. For $L = 3R$, the gravitational potential difference is $\Delta U = 1.11\rho G s^2$. $L = 2.61R$, $s = 1.14R$ yields the largest ΔU for a given s, $\Delta U = 1.17 G\rho s^2$.

Conclusions

1-Gravitoelectric A-B within full EFEs i.e NO weak field approx.

2-GA-B and COW effect both as geometrical effects and under the same flag!

Thank you for your attention