We present a general formalism for giving a measure space paired with a separable Hilbert space a quantum version based on a normalized positive operator-valued measure. The latter are built from families of density operators labeled by points of the measure space. We especially focus on various probabilistic aspects of these constructions. Simple phase space examples illustrate the procedure: plane (Weyl-Heisenberg symmetry), half plane (affine symmetry), cylinder (as a co-adjoint orbit of the group of Euclidean displacements in the plane). Interesting applications to quantum cosmology ("smooth bouncing" for Robertson-Walker metric, Bianchi I, II and IX models) will be presented: how can we deal with gravitational singularities on a quantum level through the use of affine coherent state (ACS) quantization instead of canonical quantization. The main issue of ACS approach is the appearance of a quantum centrifugal potential allowing for regularization of the singularity, essential self-adjointness of the Hamiltonian, and unambiguous quantum dynamical evolution.